# Simple Proofs of Sequential Work

Bram Cohen

Krzysztof Pietrzak


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# Outline

- What Proofs of Sequential Work
- How Sketch of Construction & Proof
- Why Sustainable Blockchains

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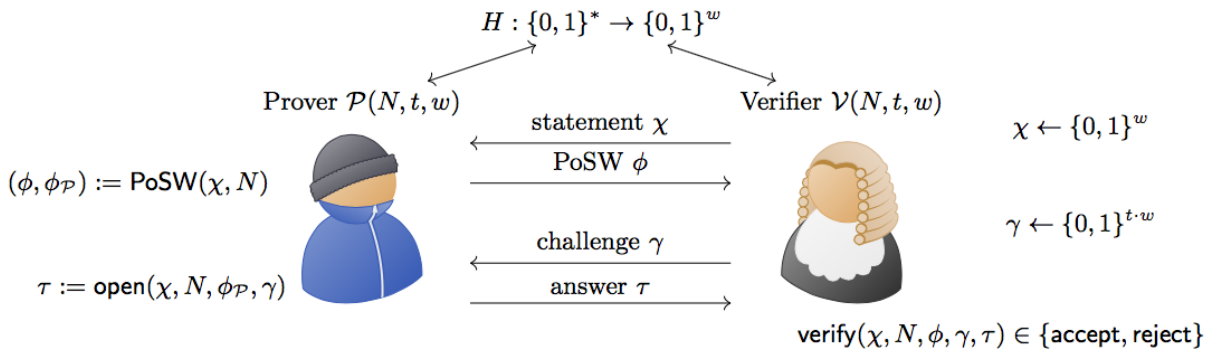

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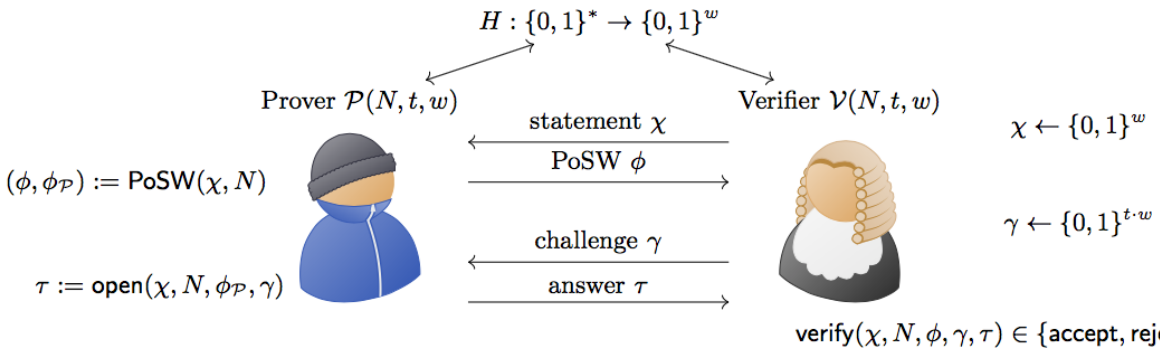


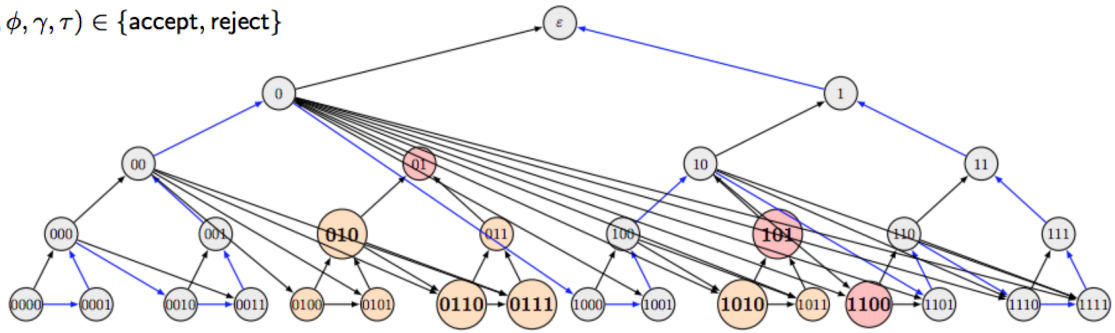

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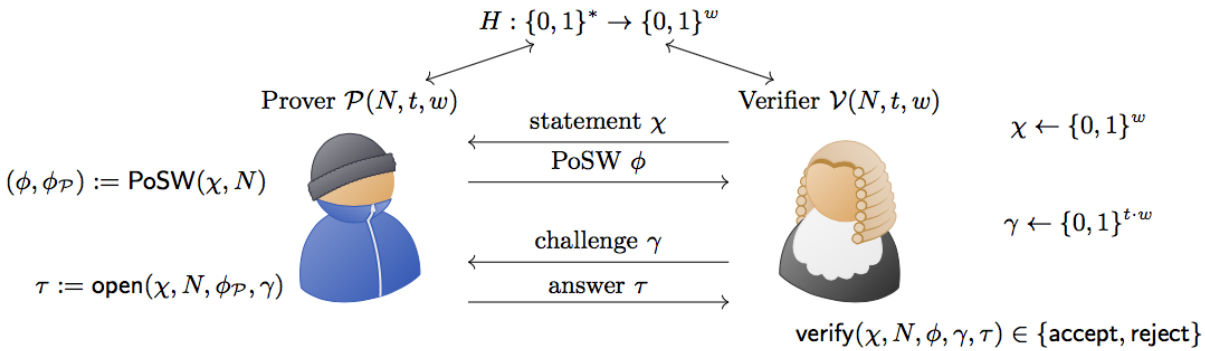


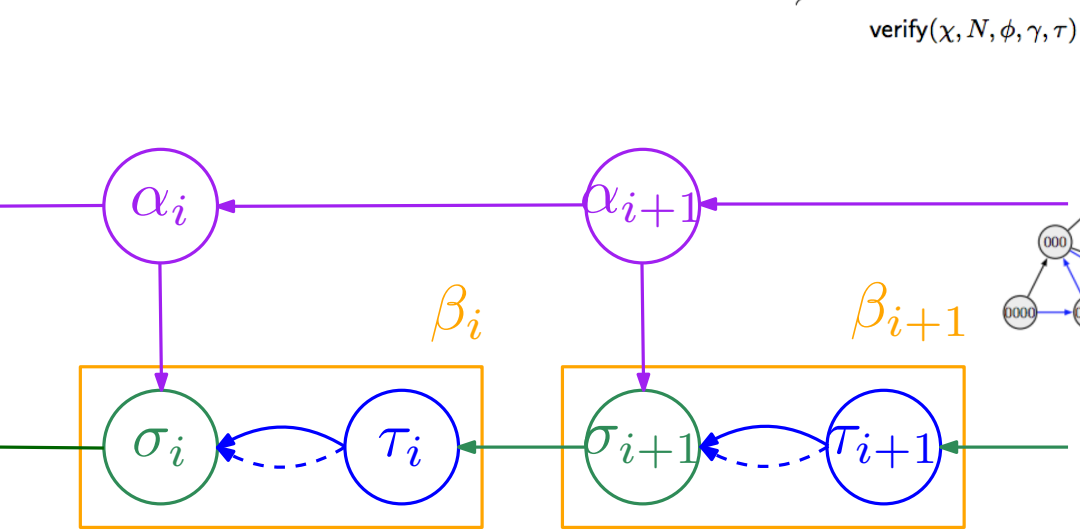


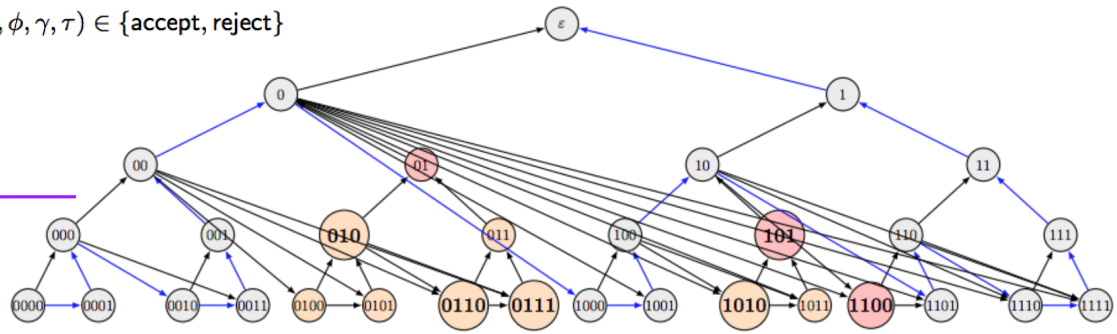

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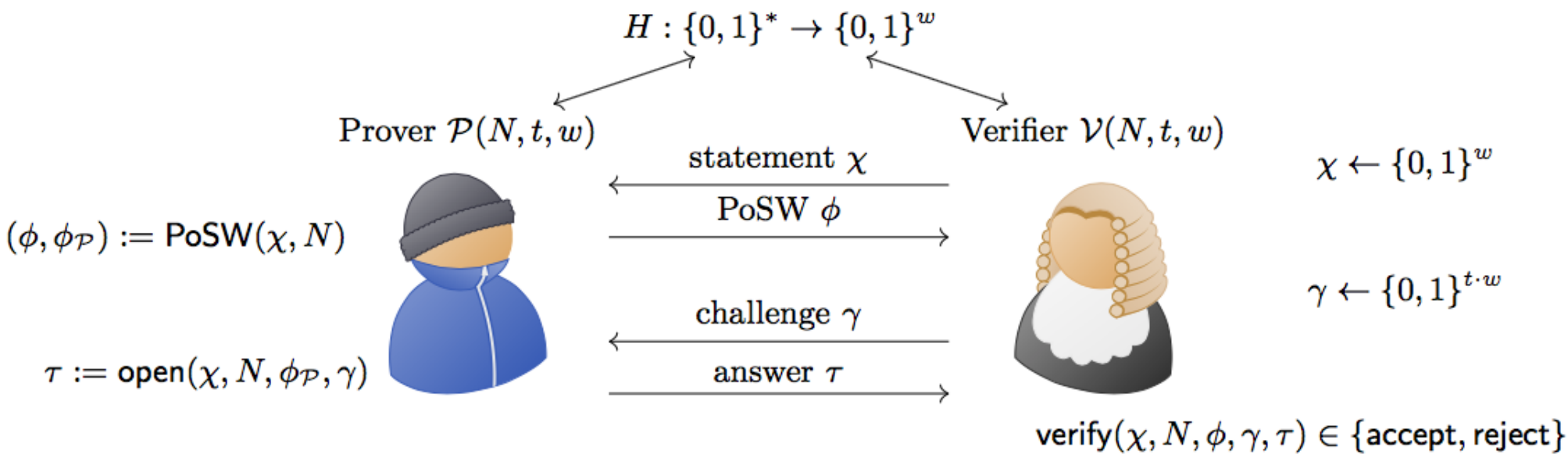

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Ronald L. Rivest*, Adi Shamir**, and David A. Wagner***

Revised March 10, 1996

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solution computed with two exponentiation given $p, q$:

$$e \leftarrow 2^T \mod \phi(N) \quad , \quad x^{2^T} = x^e \bmod N$$

conjectured to require $T$ sequential squarings given only $N$

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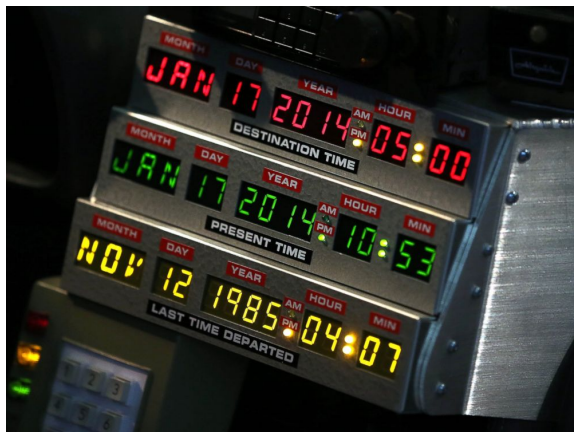
sequential computation $\sim$
computation time $\Rightarrow$
"send message to the future"



# Publicly Verifiable Proofs of Sequential Work

Mohammad Mahmoody* Tal Moran† Salil Vadhan‡

February 18, 2013

# PoSW vs. Time-Lock Puzzles

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## Public vs. Private

- Public-coin $\Rightarrow$ Publicly verfiable
- Private-coin $\Rightarrow$ Designated verifier

# Proofs of Sequential Work

aka. Verifiable Delay Algorithm

Prover $\mathcal{P}$

statement $\chi$

Time $T \in \mathbb{N}$

Verifier $\mathcal{V}$

$\chi \leftarrow$ 

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$\tau = \tau(\chi, T)$

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verify$(\chi, T, \tau) \in$
accept/reject

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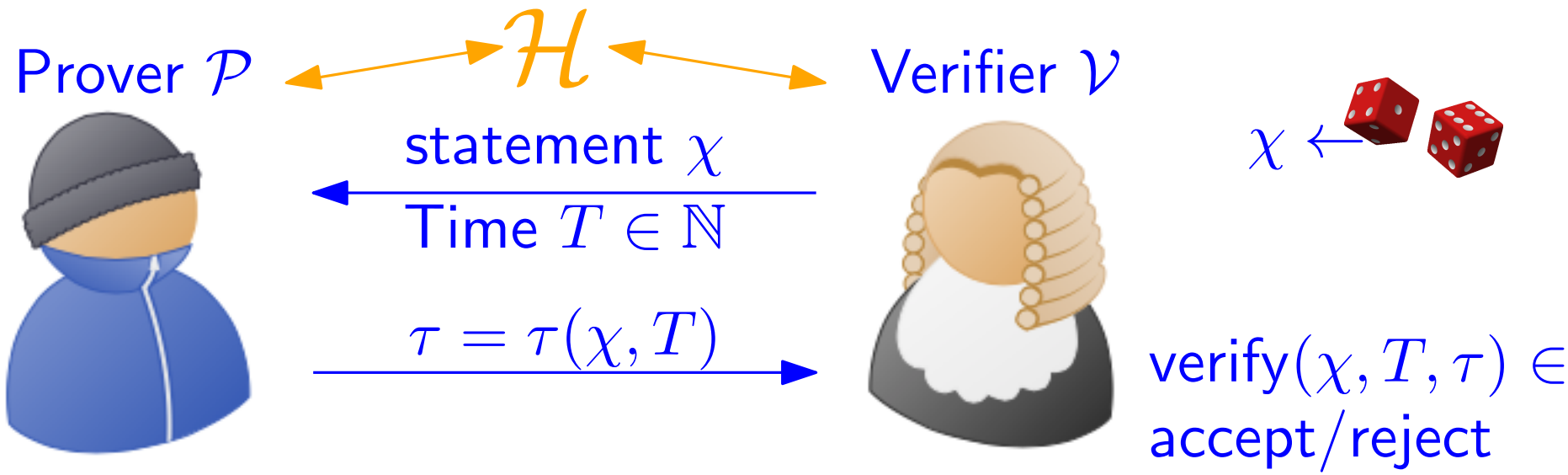


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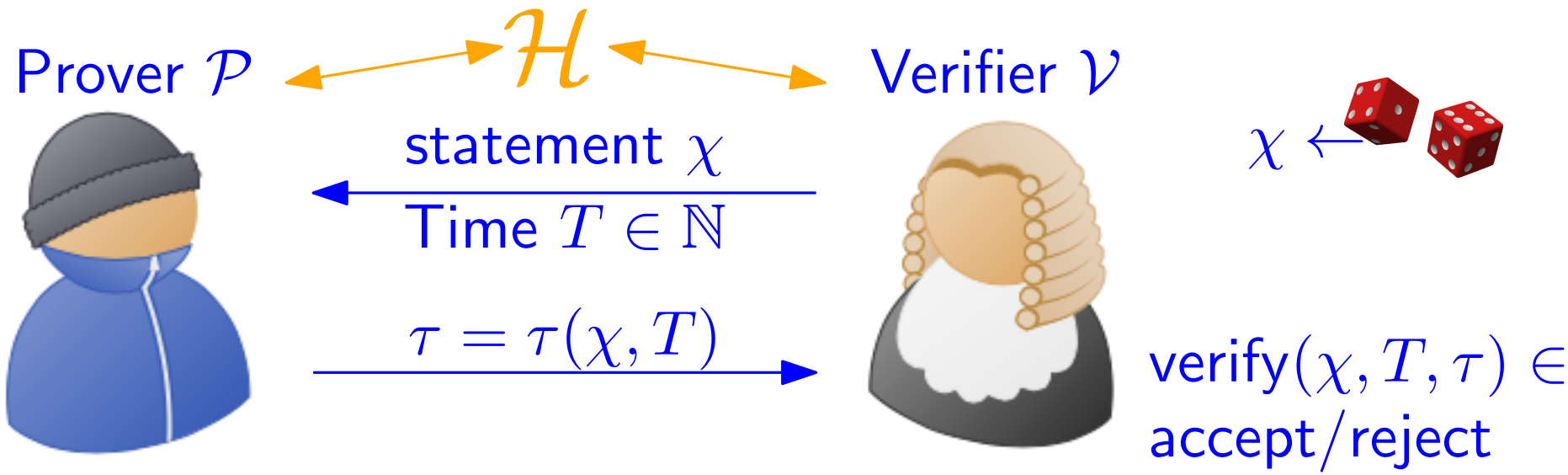


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**Completeness:** $\tau(c, T)$ can be computed making $T$ queries to $\mathcal{H}$

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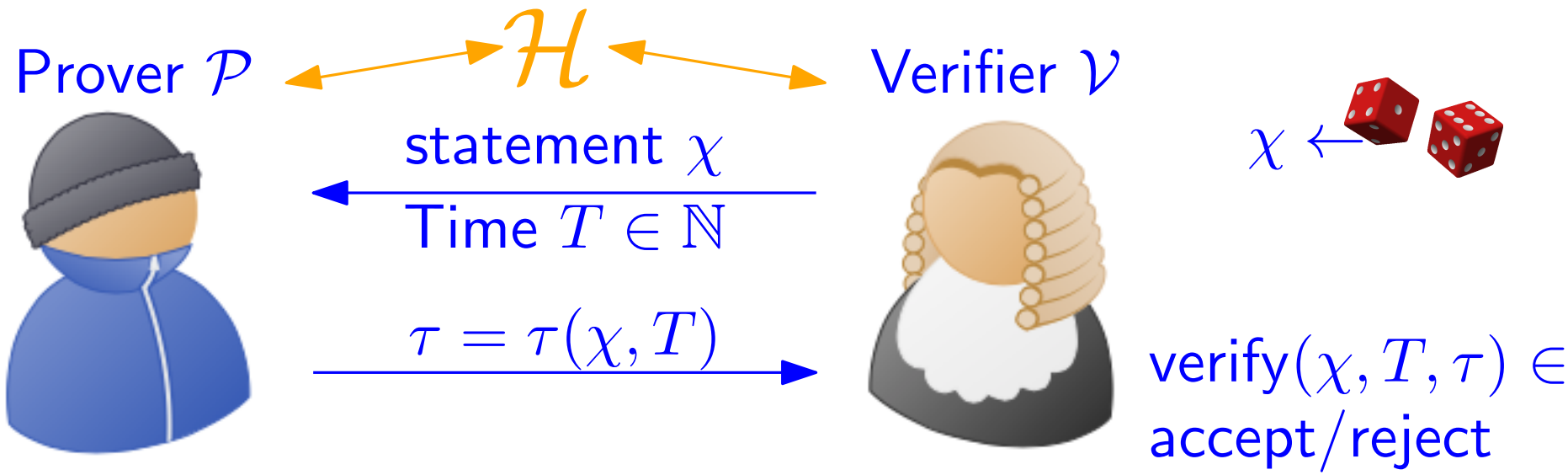


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massive parallelism useless to generate valid proof faster $\Rightarrow$ prover must make almost $T$ sequential queries $\sim T$ time

# Three Problems of the [MMV'13] PoSW

1) **Space Complexity :** Prover needs massive (linear in T) space to compute proof.
2) **Poor/Unclear Parameters** due to usage of sophisticated combinatorial objects.
3) **Uniqueness :** Once an accepting proof is computed, many other valid proofs can be generated (not a problem for time-stamping, but for blockchains).

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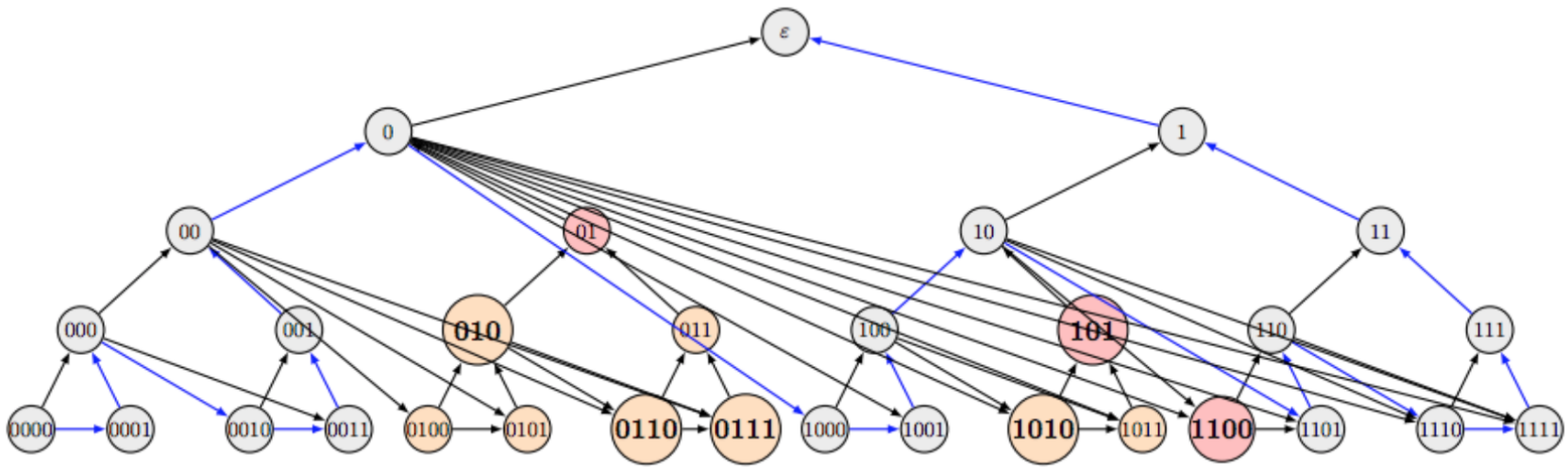
# New Construction

1) Prover needs only $O(log(T))$ (not $O(T)$) space, e.g. for $T = 2^{42}$ ($\approx$ a day) that's $\approx 10KB$ vs. $\approx 1PB$.
2) Simple construction and proof with good concrete parameters.
3) Awesome open problem!

# Construction and Proof Sketch



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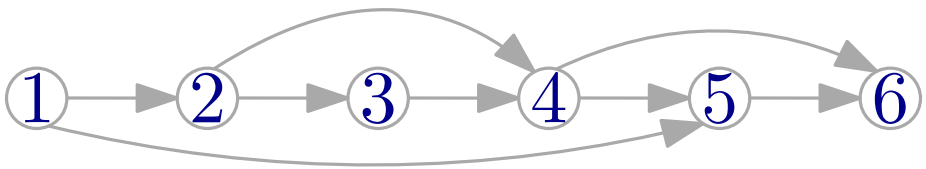
**Depth-Robust** Graphs (only [MMV'13])




DAG $G = (V, E)$ is $(e, d)$ **depth-robust** if after removing any $e$ nodes a path of length $d$ exists.

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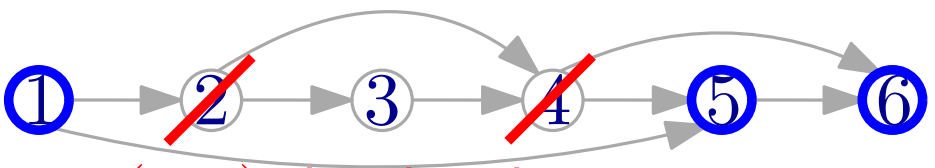


is $(2, 3)$ depth-robust

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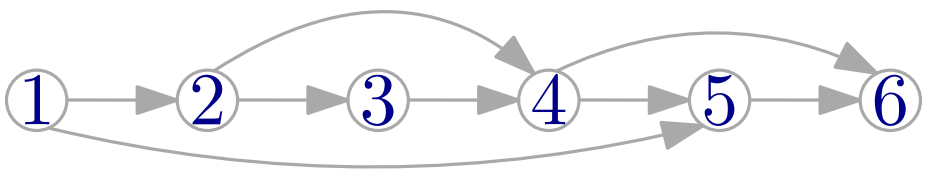


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## Graph **Labelling**

label $\ell_i = \mathcal{H}(\ell_{parents(i)})$, e.g. $\ell_4 = \mathcal{H}(\ell_3, \ell_4)$

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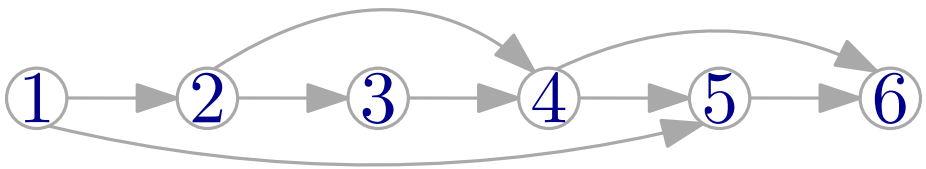


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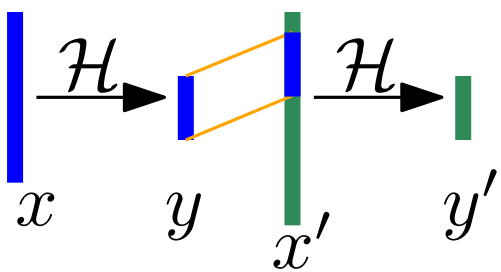
## Random Oracles are **Sequential**




queries $y = \mathcal{H}(x), y' = \mathcal{H}(x')$ where $y \subseteq x' \Rightarrow$ query $x'$ was made after $x$

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Prover $\mathcal{P}$ $\leftrightarrow$ $\mathcal{H}$ $\leftrightarrow$ Verifier $\mathcal{V}$

statement $\chi$

Time $T = 6$

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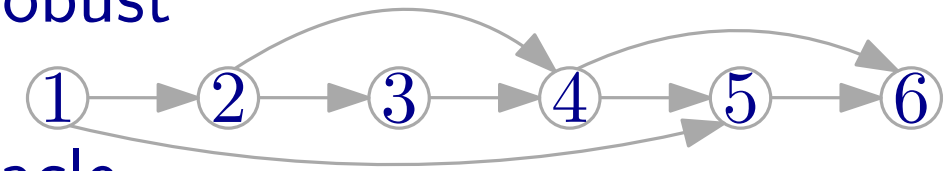
- Define "fresh" random oracle $\mathcal{H}_\chi(\cdot) \equiv \mathcal{H}(\chi \| \cdot)$



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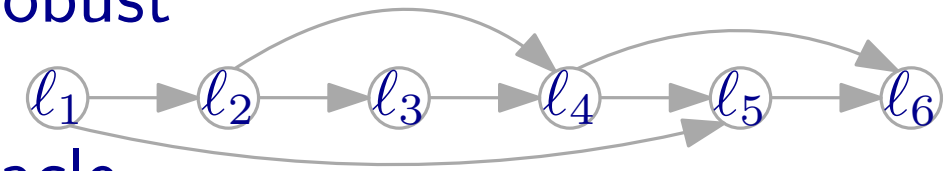


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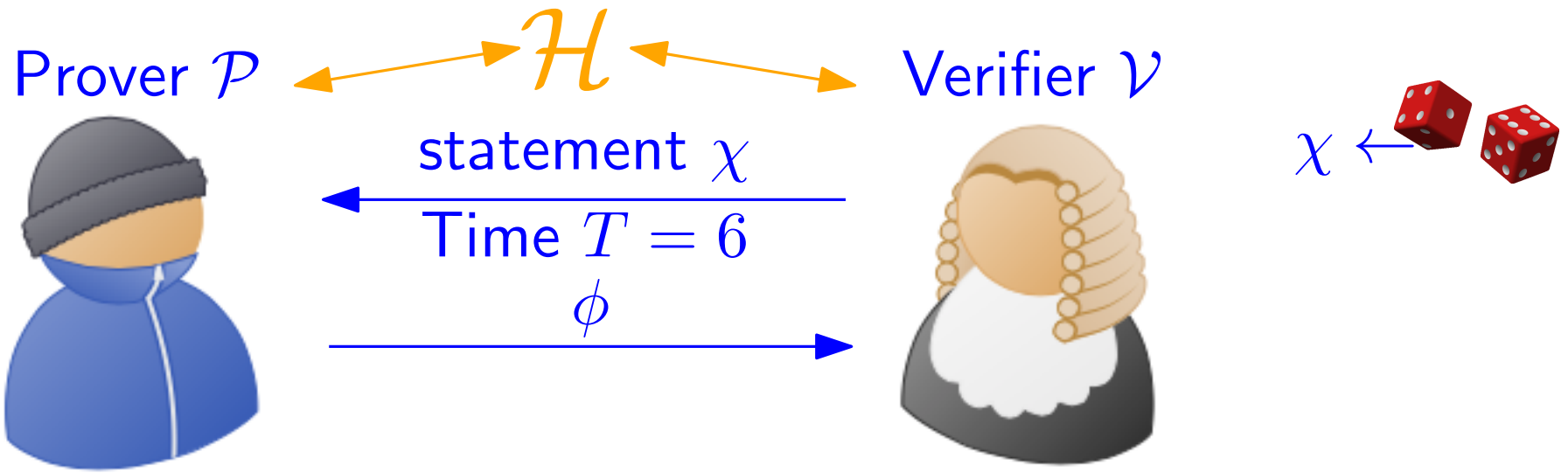


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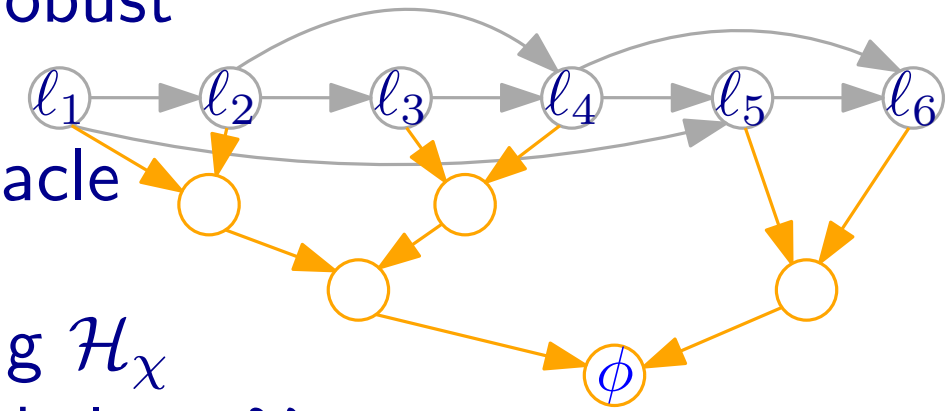

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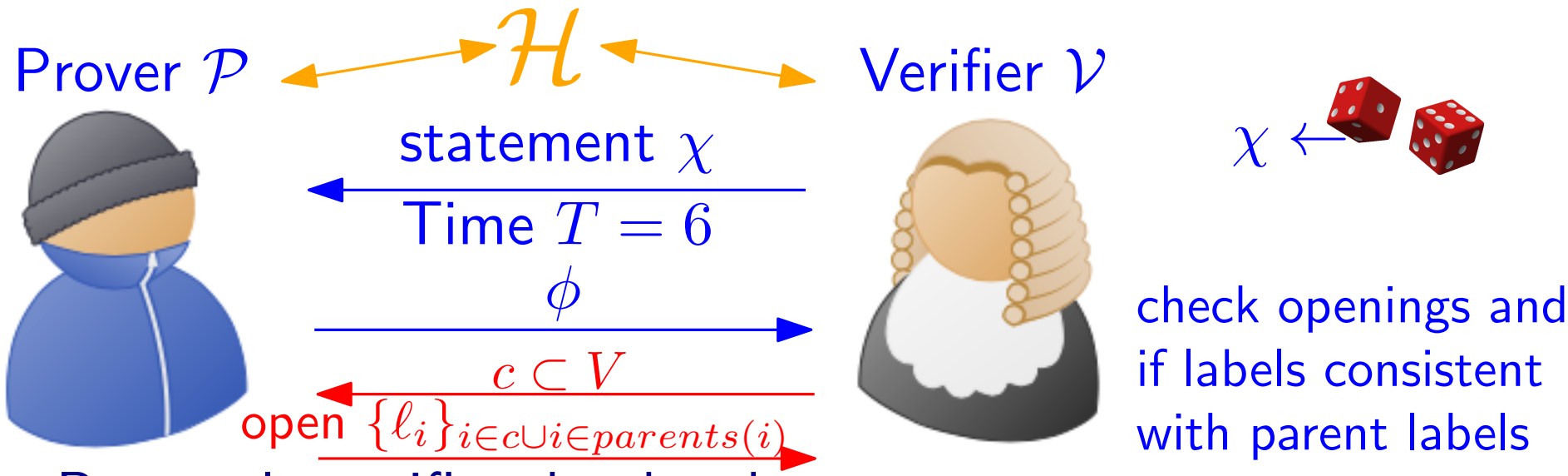


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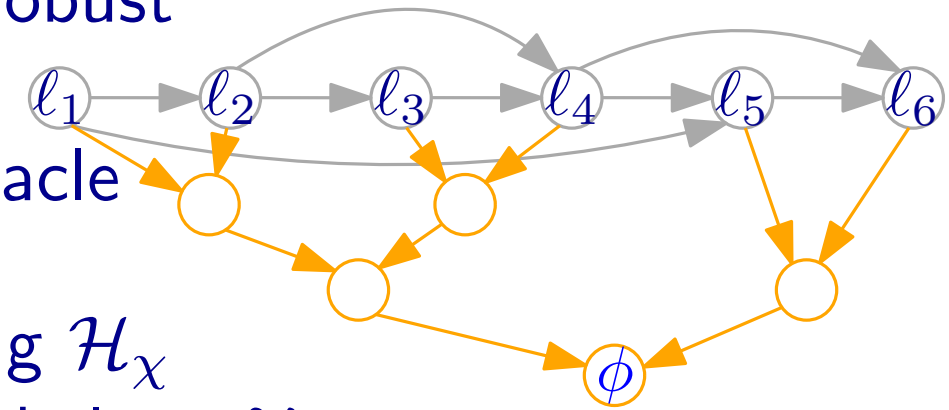
(interaction can be removed using Fiat-Shamir)



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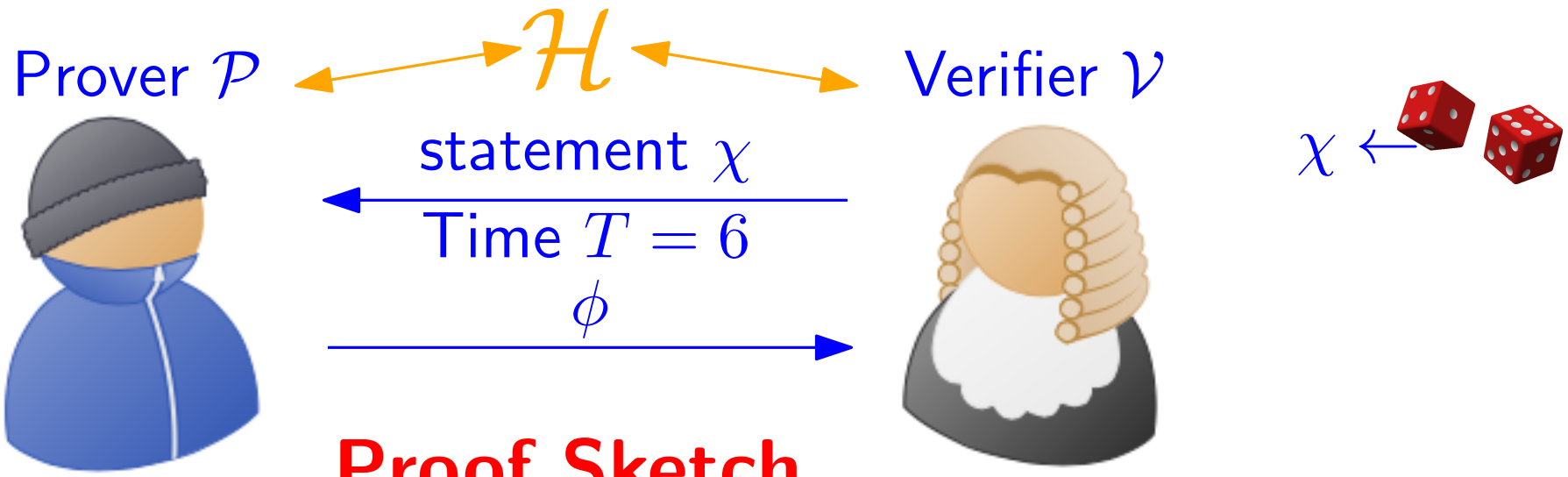


## Proof Sketch

- $G$ is $(e, d)$ depth-robust
- $\phi$ commits $\tilde{\mathcal{P}}$ to labels $\{\ell'_i\}_{i \in V}$
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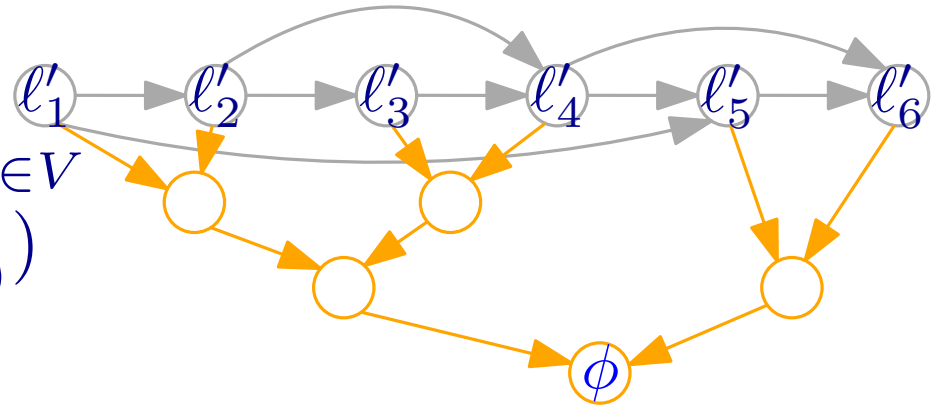

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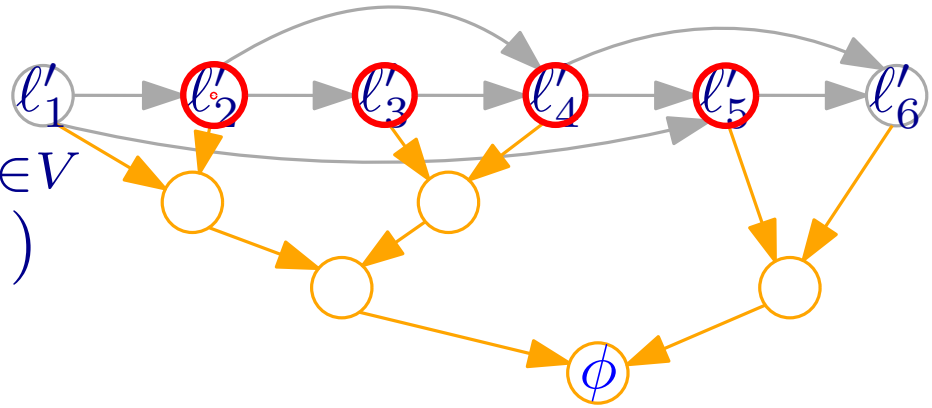


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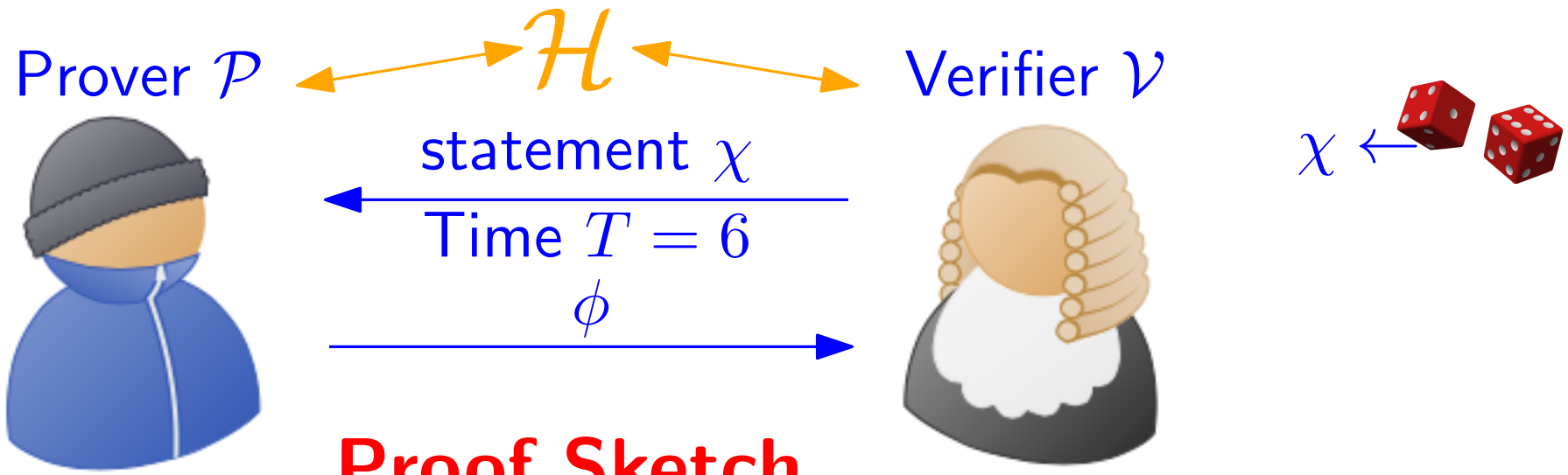


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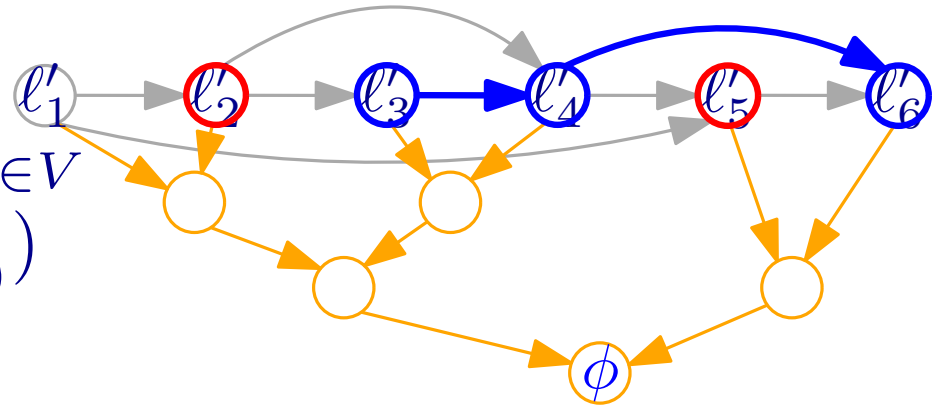


- Case 1: $\geq e$ bad nodes $\Rightarrow$ will fail opening phase whp.
- Case 2: Less than $e$ bad labels $\Rightarrow \exists$ path of good nodes (by $(e, d)$ depth-robustness) $\Rightarrow \tilde{P}$ made $d$ sequential queries (by sequantality of RO)

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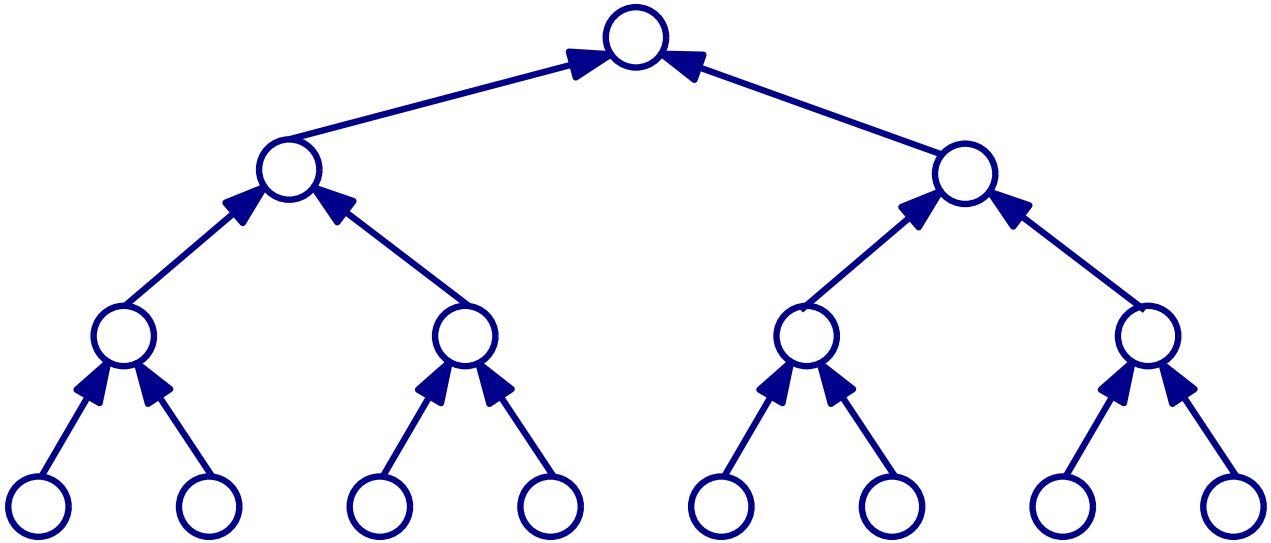

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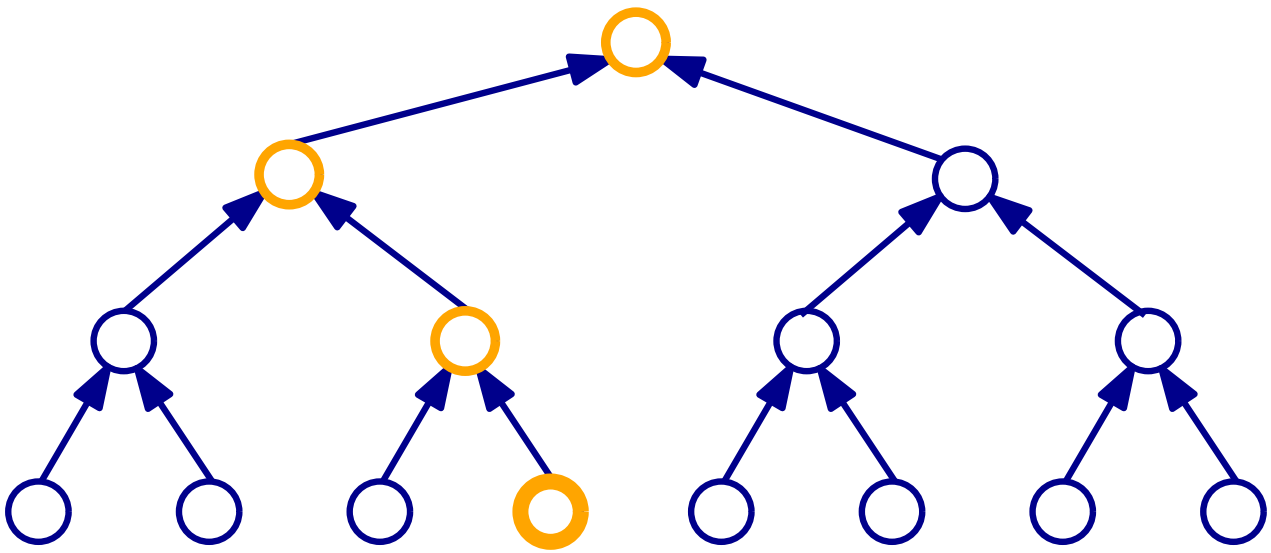

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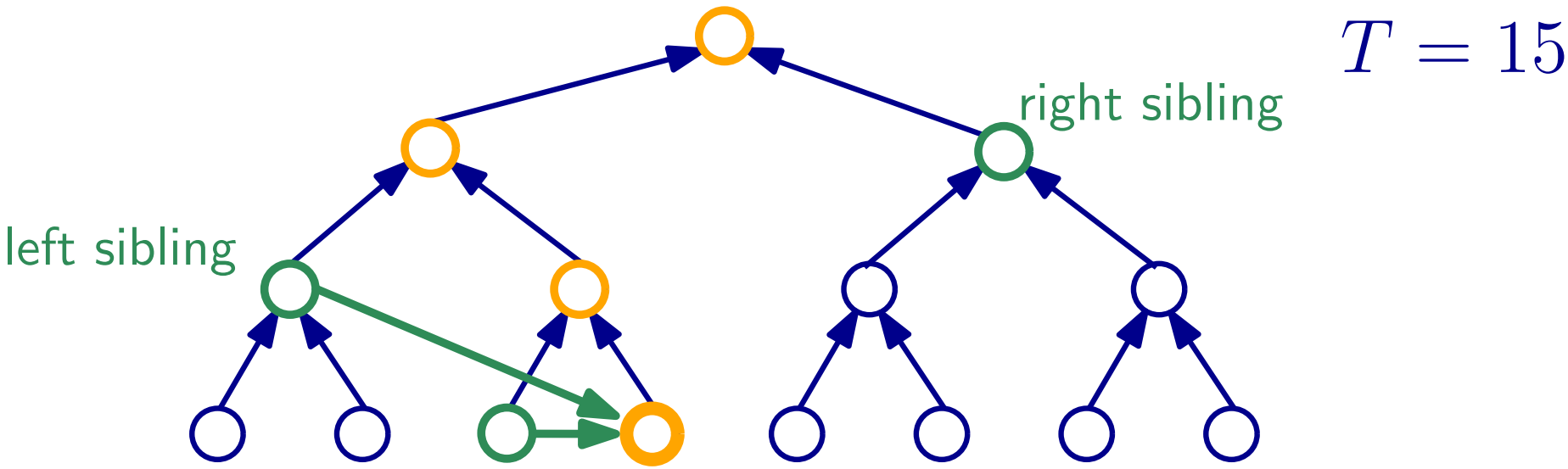


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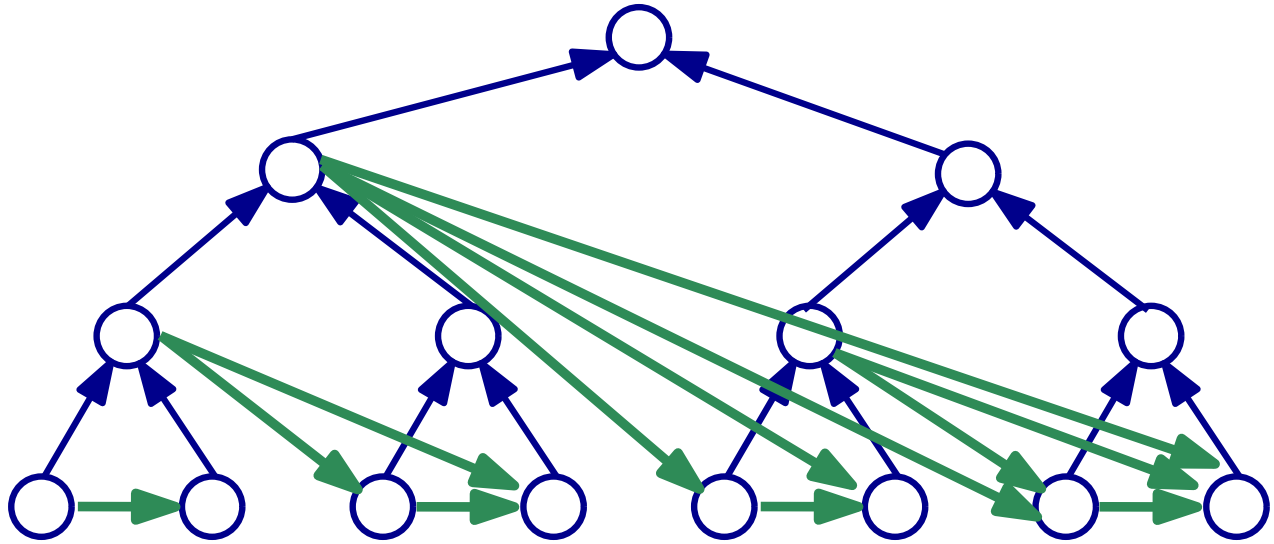

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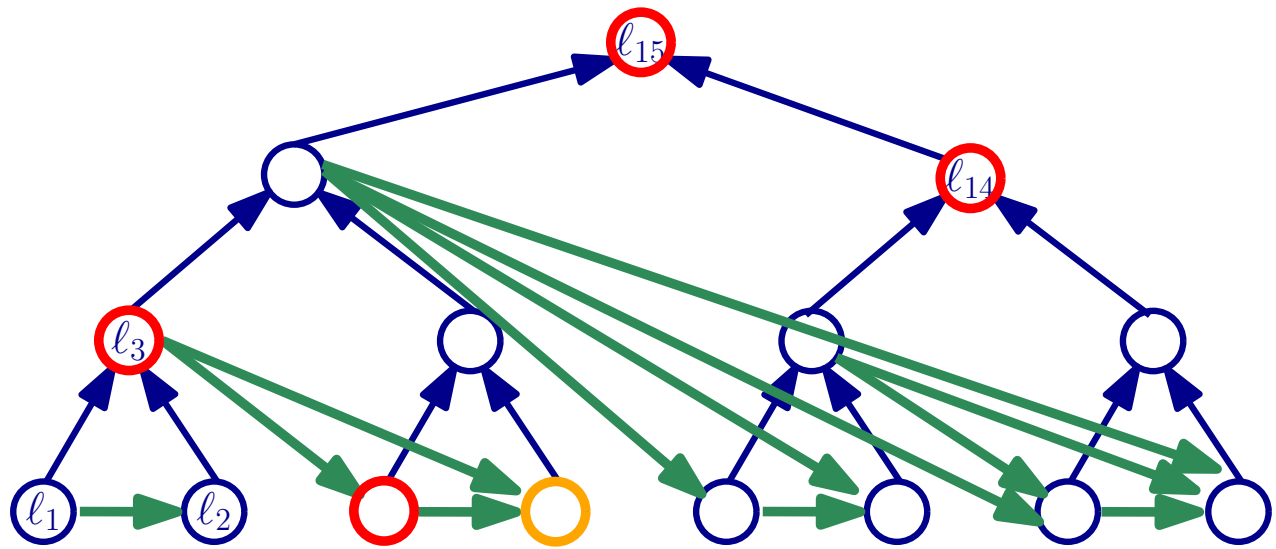


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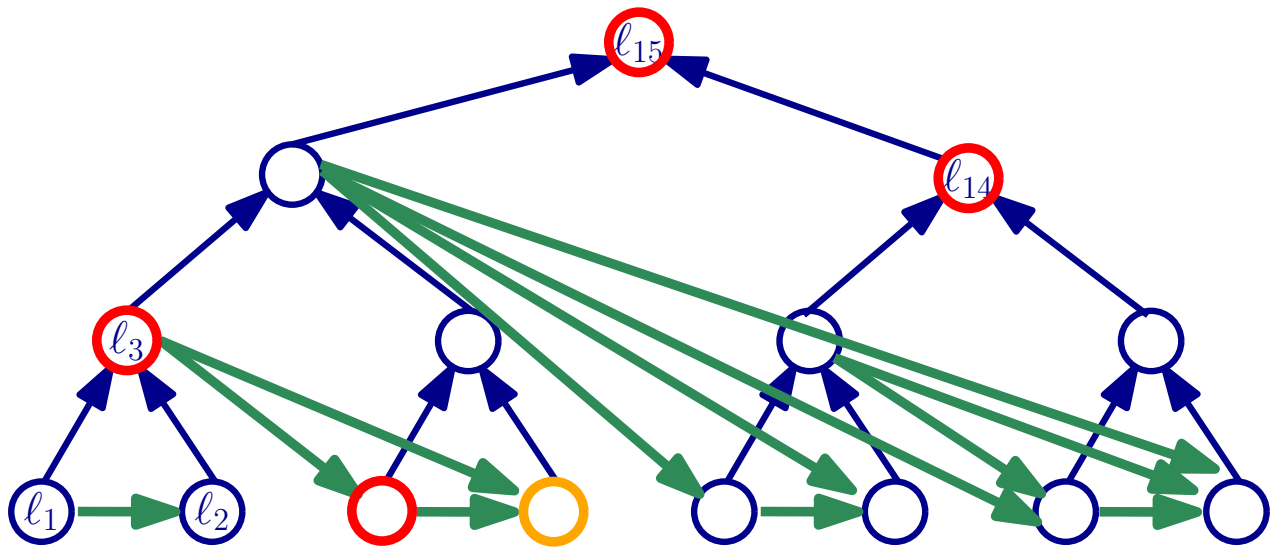


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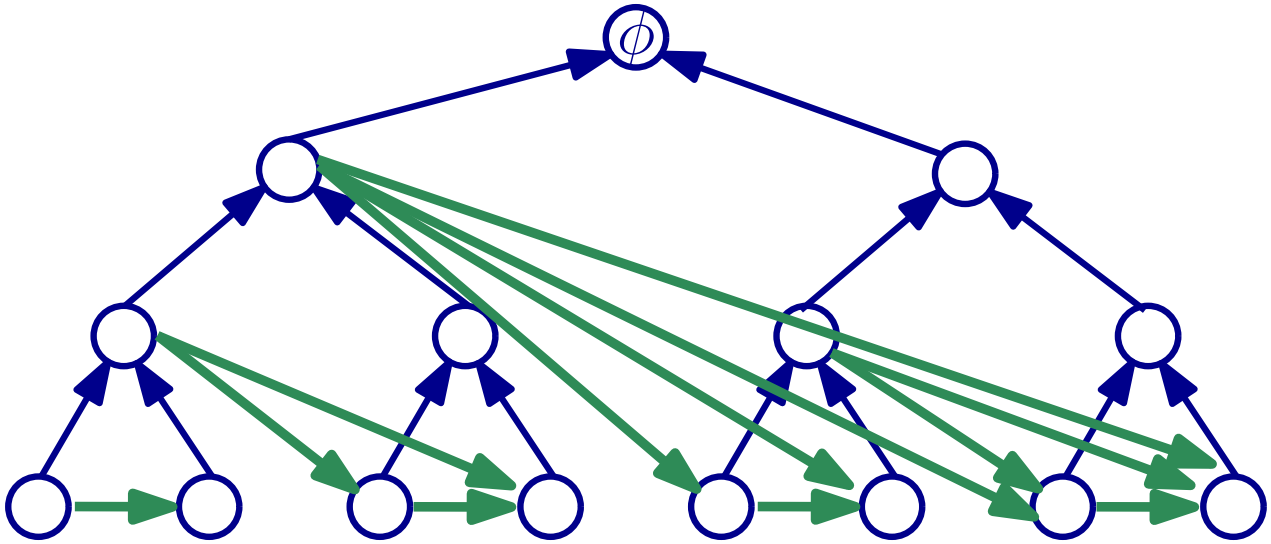
**Optimally Efficient Accountable Time-Stamping**

PKC'00

Ahto Buldas*[1], Helger Lipmaa*[1], and Berry Schoenmakers**[2]

# The New Construction




$T = 15$

**Proof Sketch**

# The New Construction

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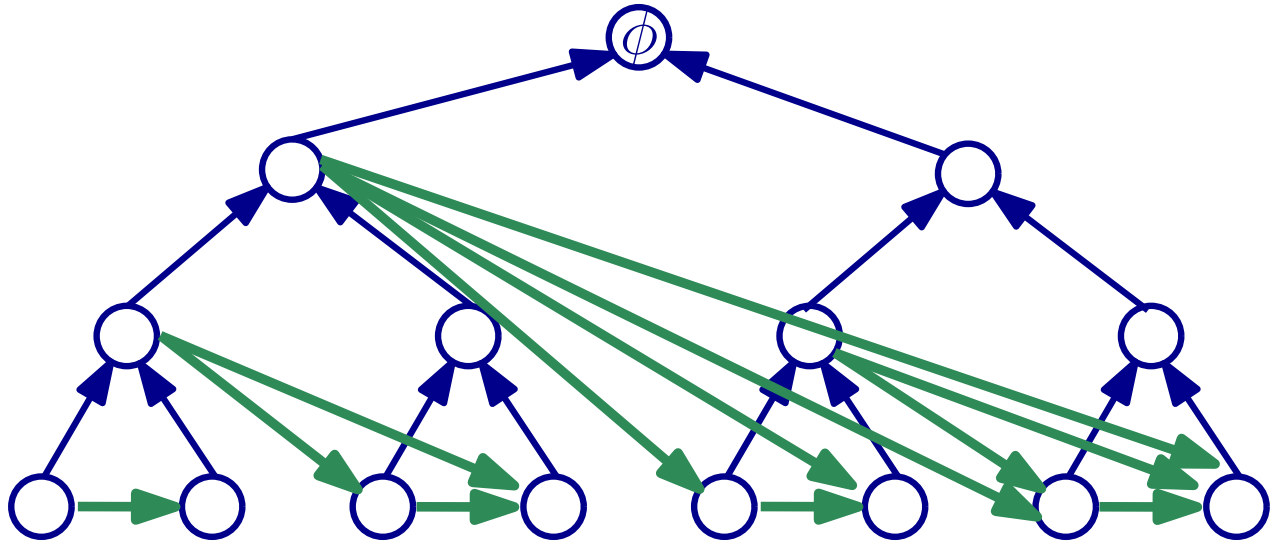


## Proof Sketch

- $\tilde{P}$ committed to labels $\ell'_i$ after sending $\phi = \ell_{15}$.
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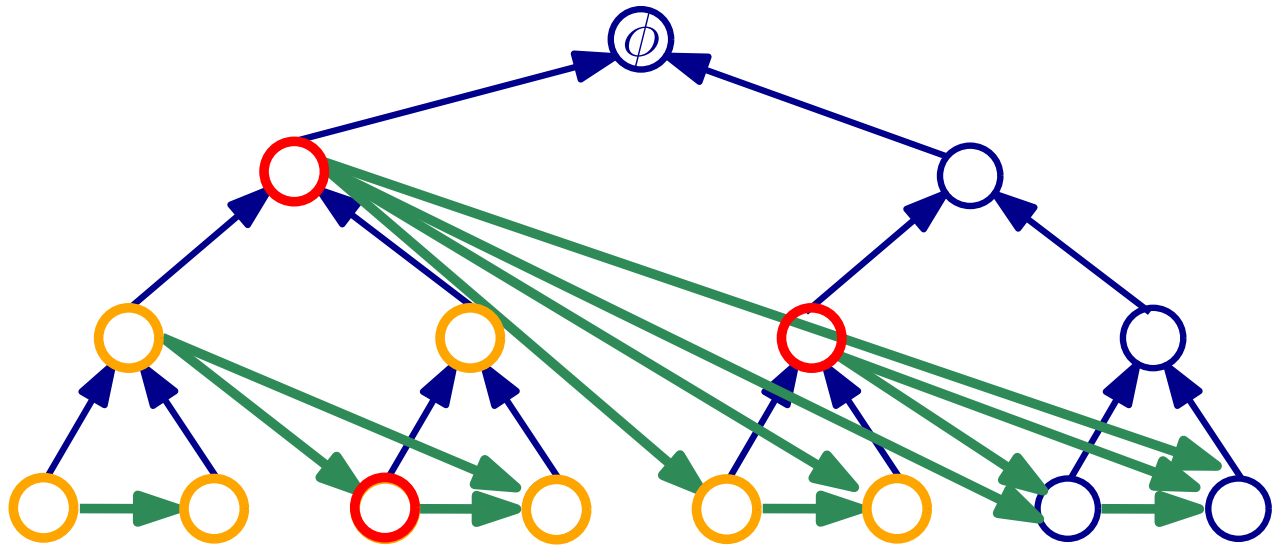


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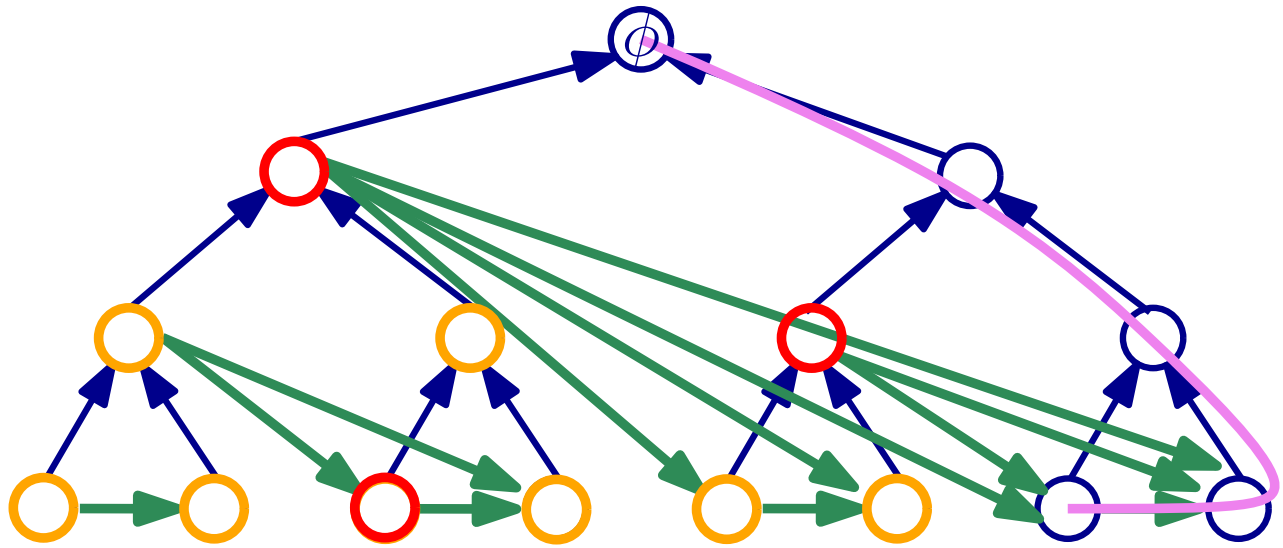


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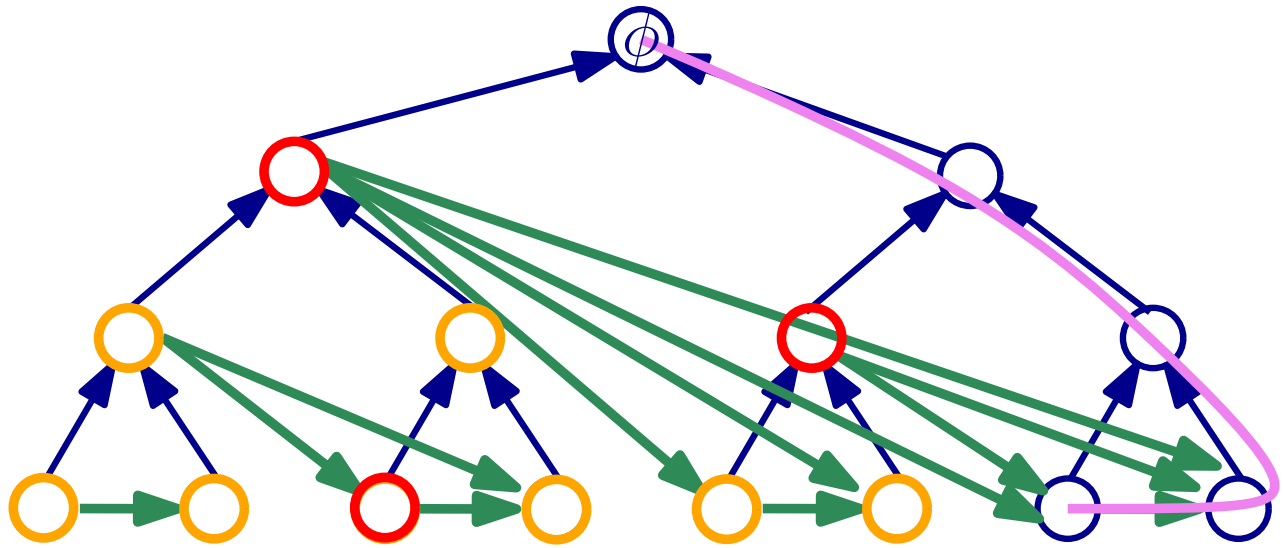

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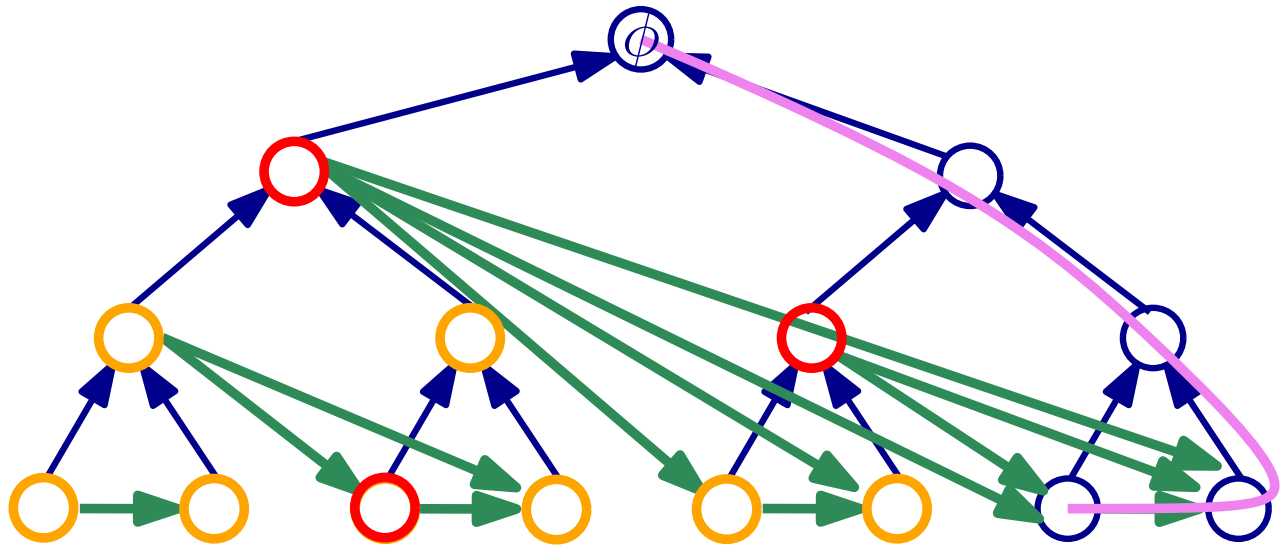


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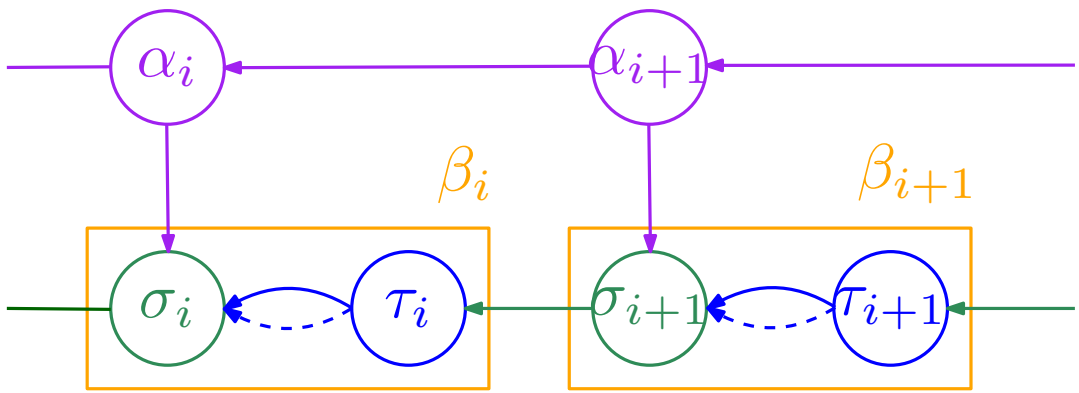
Theorem: $\tilde{P}$ made only $T(1-\epsilon)$ sequential queries
$\Rightarrow$ will pass opening phase with prob. $\leq (1-\epsilon)^{\#\text{of challenges}}$

why we care

# Sustainable Blockchains



# Mining Bitcoin (Proofs of Work)

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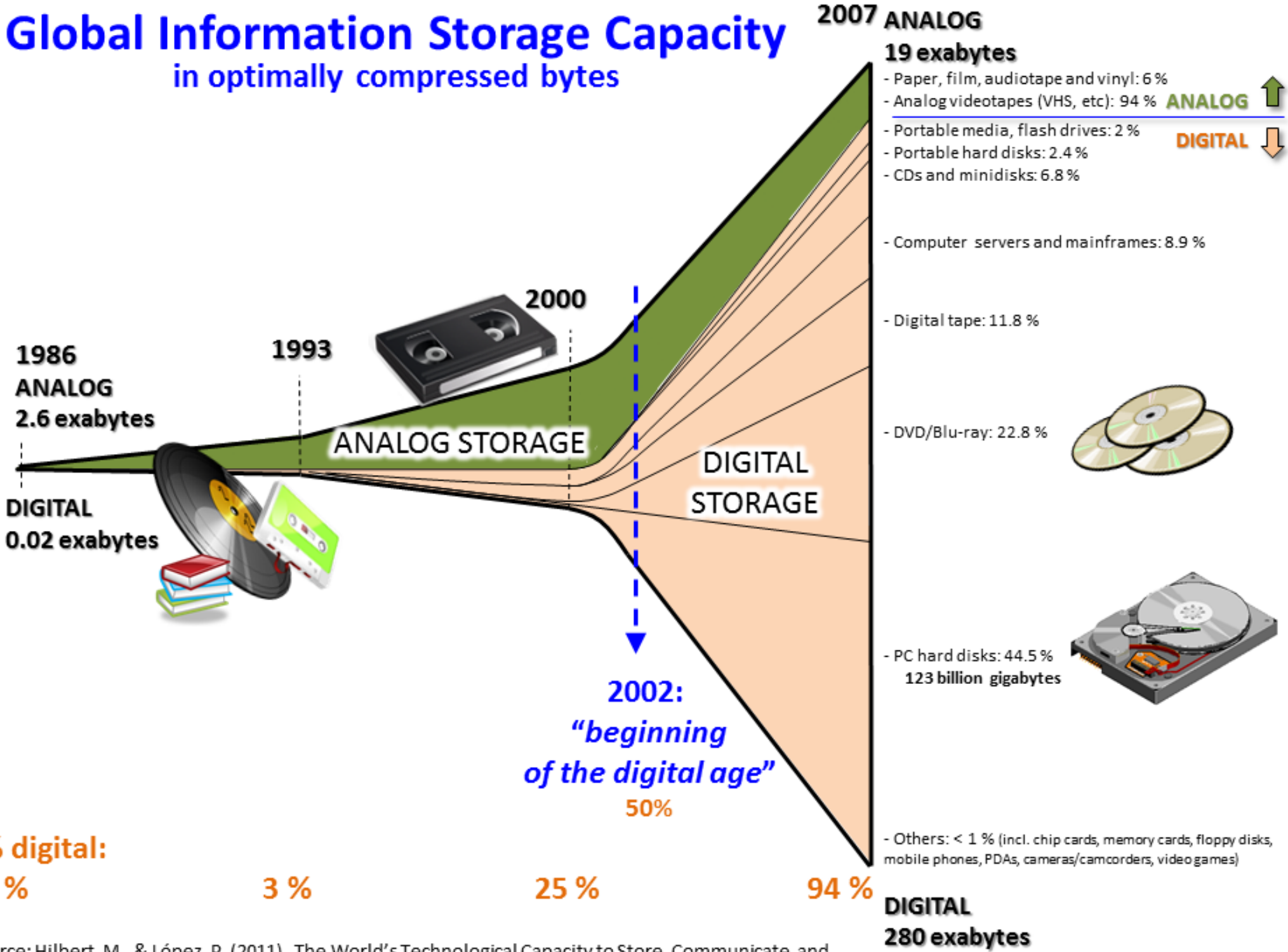
# Can we have a more "sustainable" Blockchain?

# Global Information Storage Capacity

## in optimally compressed bytes




Source: Hilbert, M., & López, P. (2011). The World's Technological Capacity to Store, Communicate, and Compute Information. *Science*, 332(6025), 60 –65. http://www.martinhilbert.net/WorldInfoCapacity.html

# Proofs of Work

computation as resource

prob. of solving PoW **first** $\sim$ fraction of hashing power

dynamics

proof of work hardness set so blocks appear $\approx$ every 10 minutes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# Proofs of Space and Time

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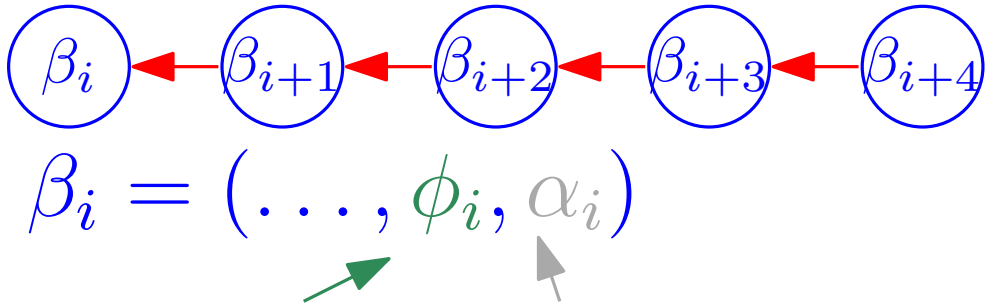
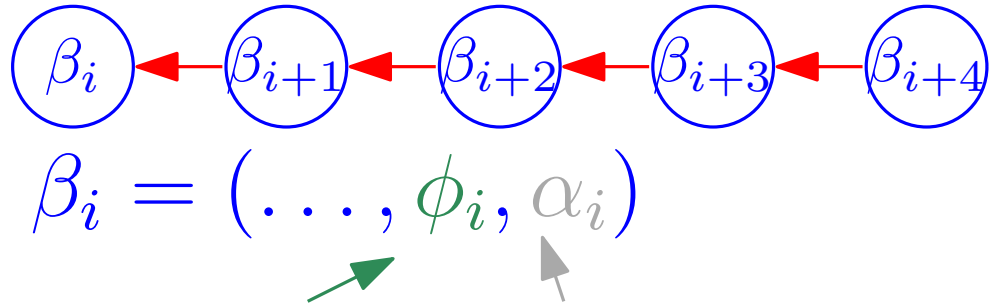
Run PoSW on top of PoSpace for $T \sim$ quality of PoSpace to "finalize" block

bitcoin




$\phi_i$ : proof of work on challenge $hash(\beta_{i-1})$ transactions

bitcoin

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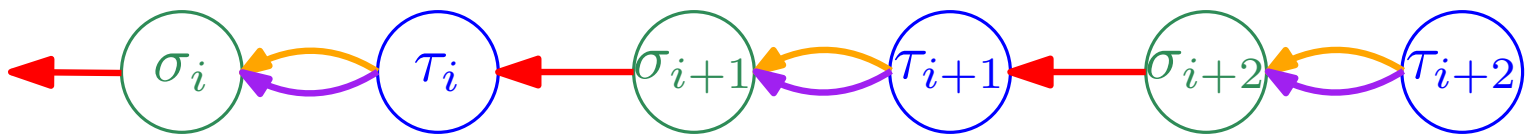
chia

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$\tau_i$ : proof of sequential work on challenge $hash(\sigma_{i-1})$
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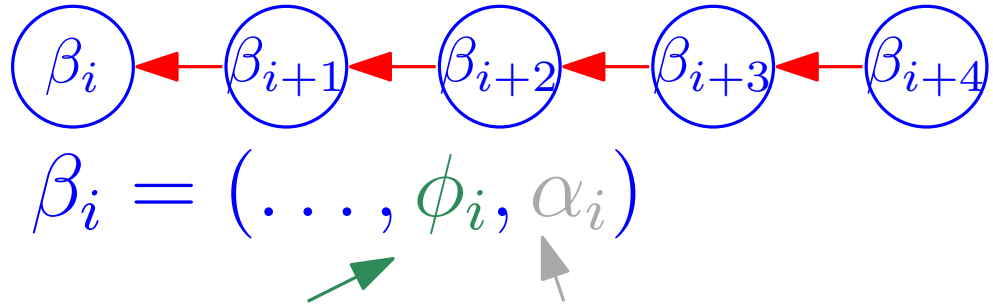
bitcoin

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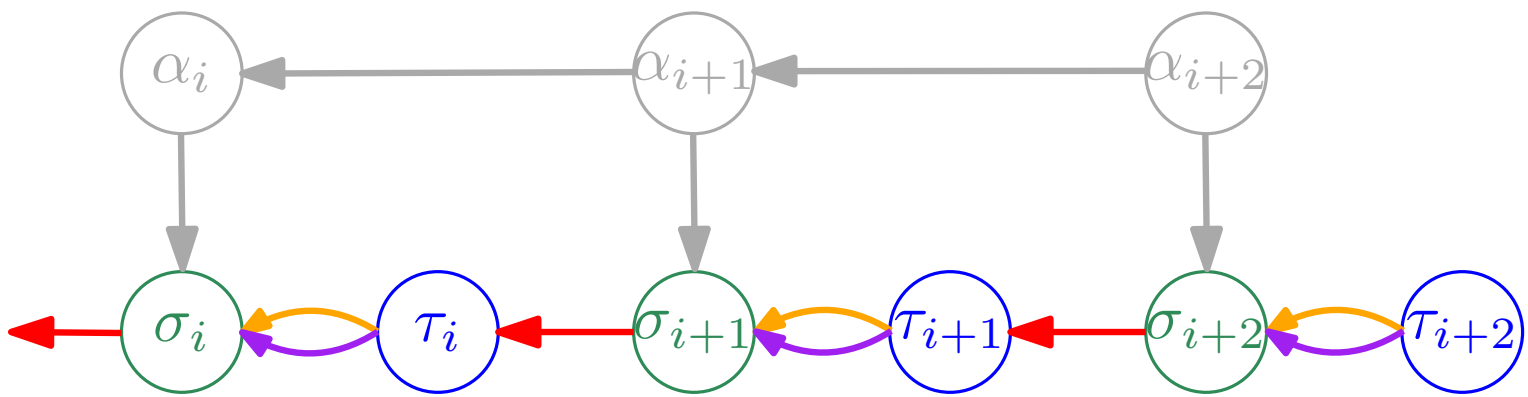


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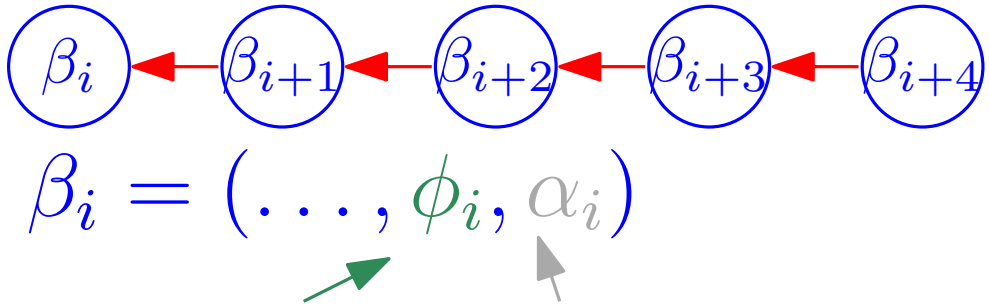


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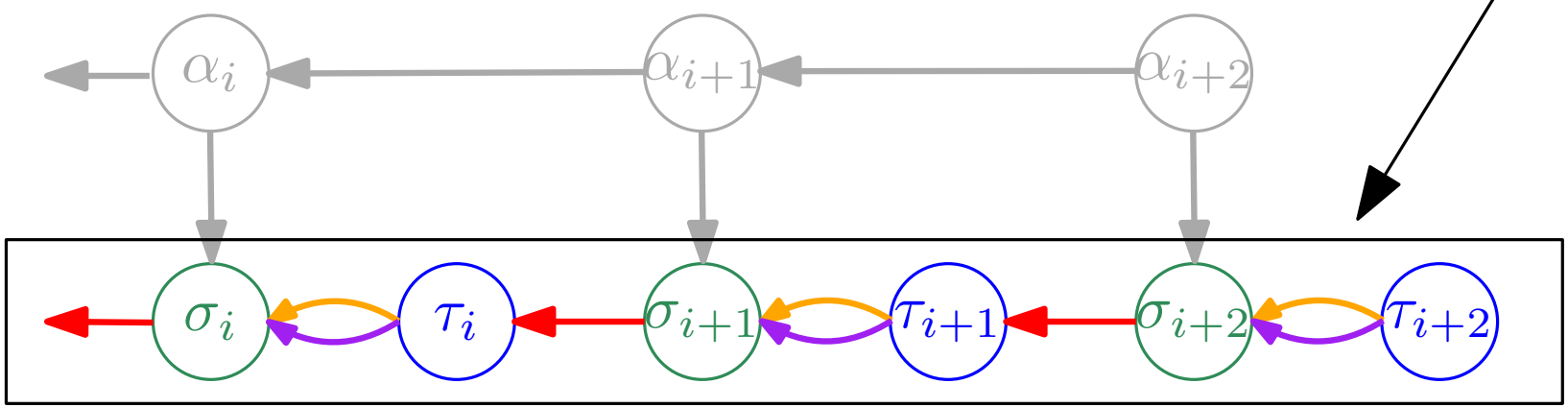
NOTHING TO GRIND HERE!

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