# Non-malleable Randomness Encoders and their Applications

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**Sruthi Sekar**

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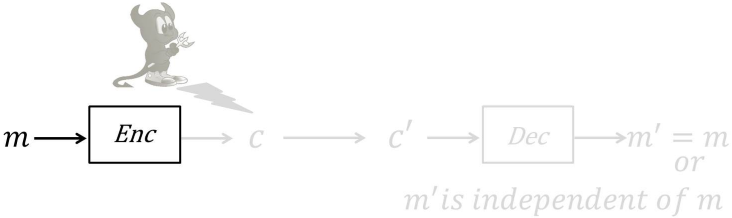

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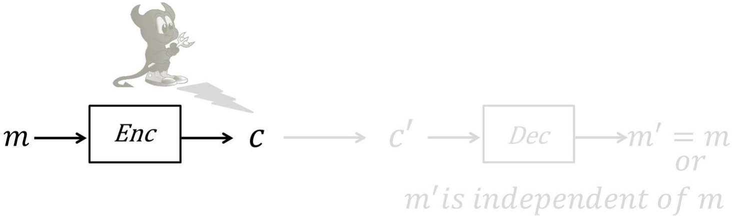

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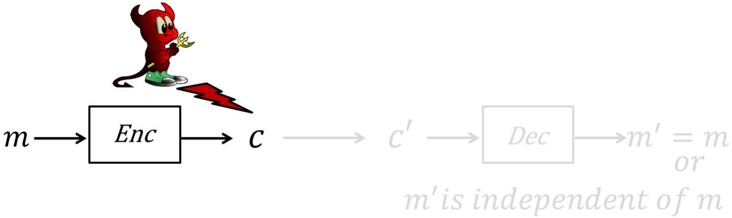

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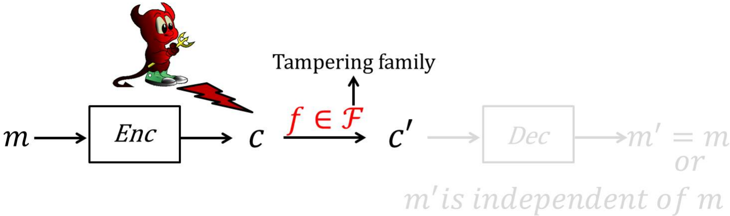

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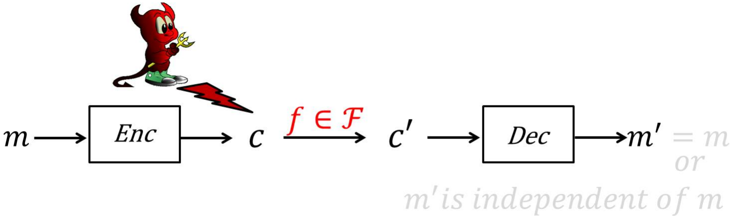

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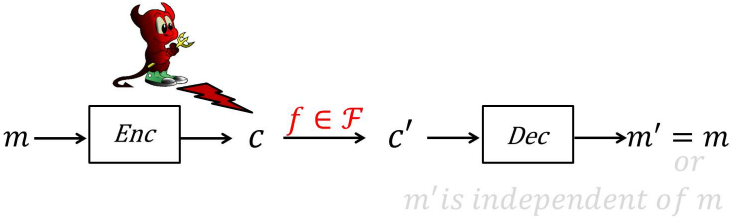

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# An Application: Related Key Attacks

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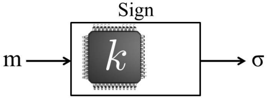

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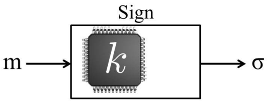


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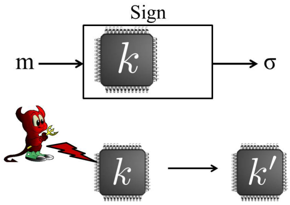


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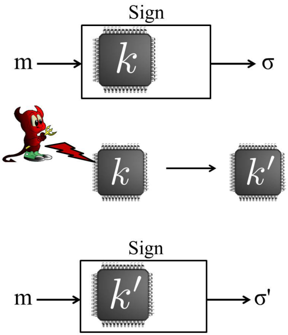


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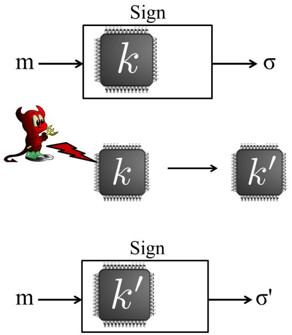


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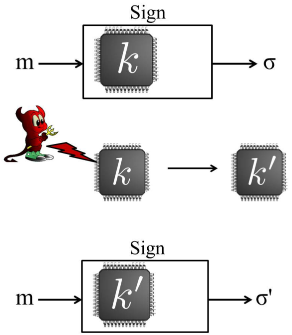


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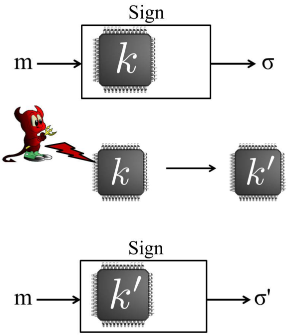


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**Non-malleable codes**

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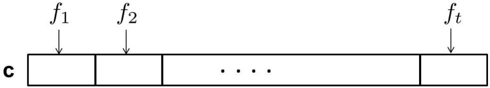
- **Tampering family**: Commonly studied tampering family is the *t-split-state family*:




$$\mathcal{F}_t = \{(f_1, \cdots, f_t) : f_i : \{0,1\}^{n/t} \to \{0,1\}^{n/t} \text{ for each } i\}$$

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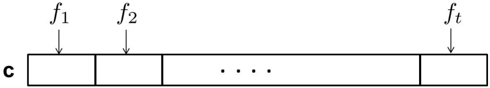


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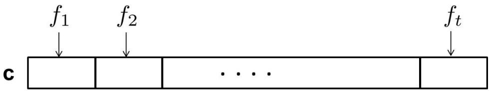


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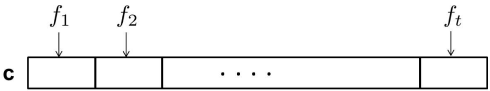


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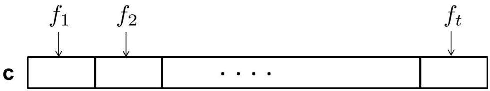


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*Holy Grail*: Build **optimal rate** NMCs for $\mathcal{F}_2$

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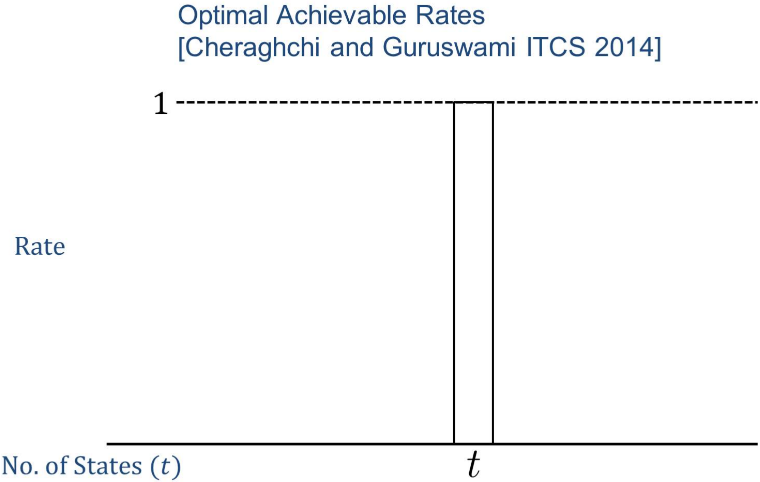

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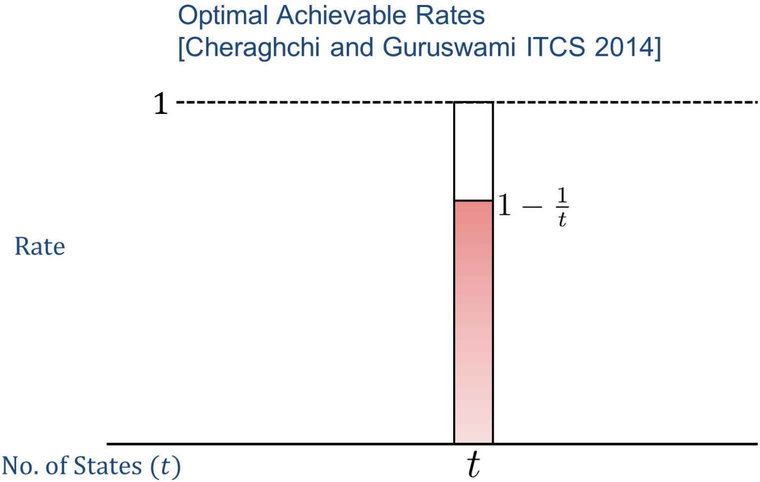

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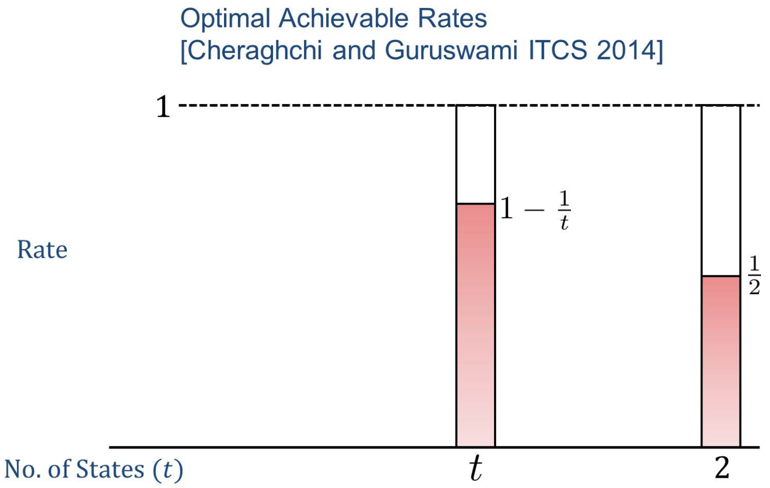

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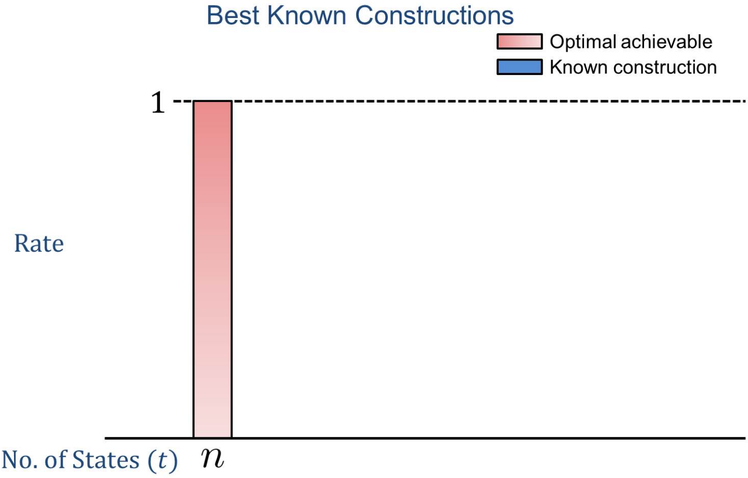

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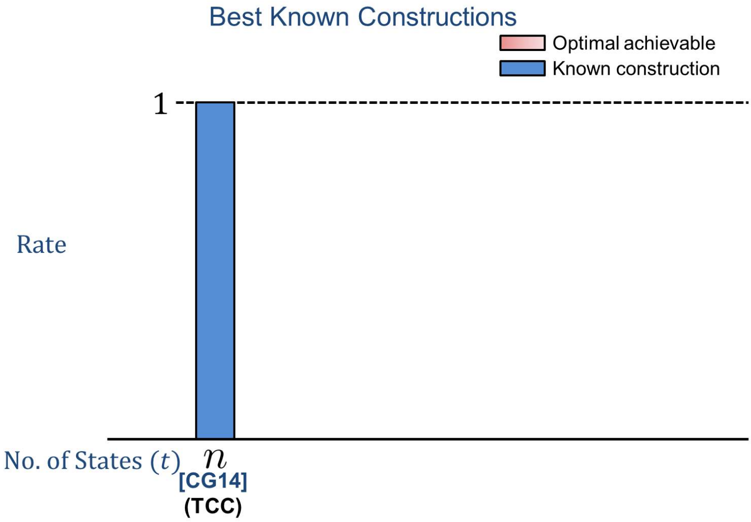

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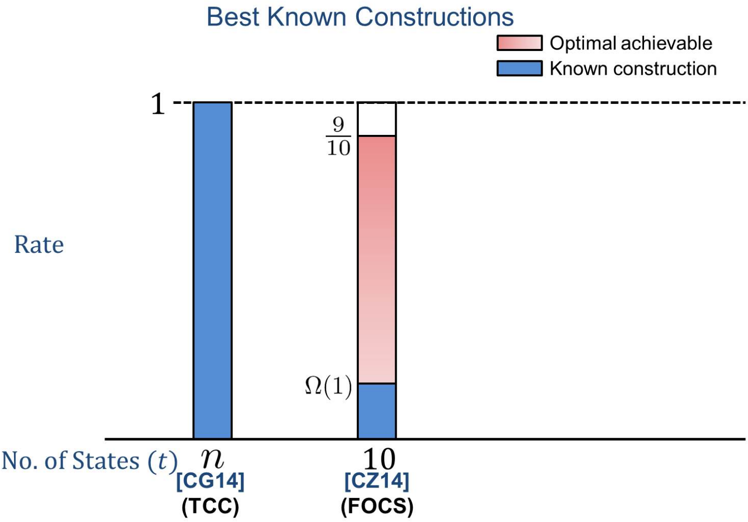

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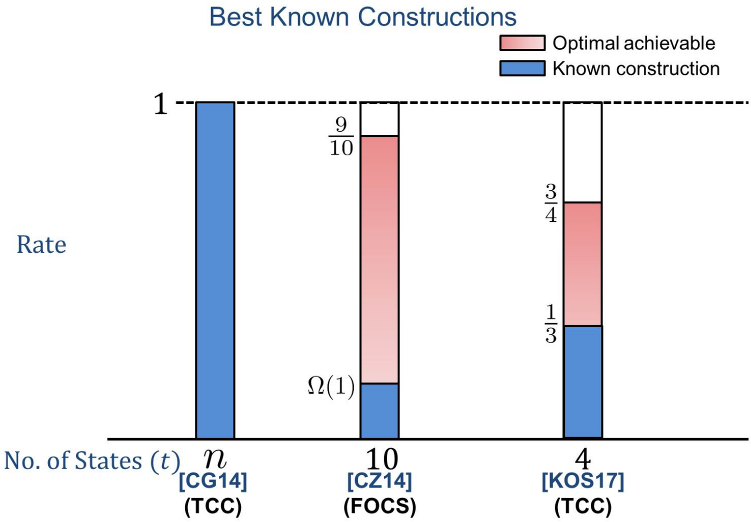

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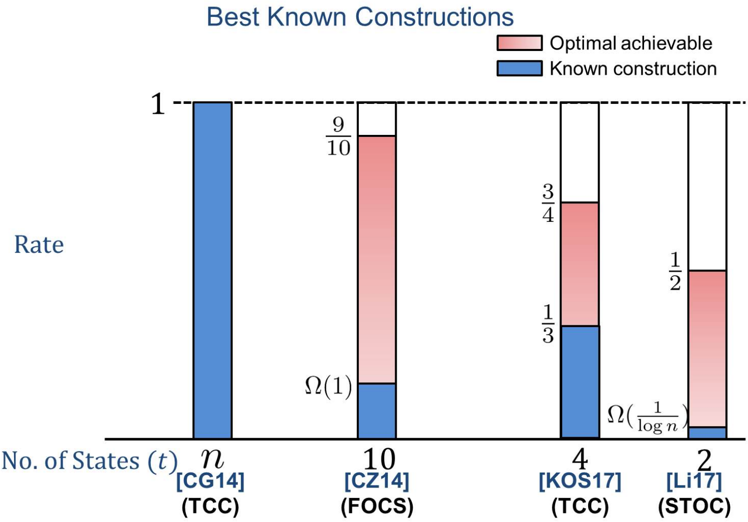

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*This work*: 2-state, $1/2$-rate NMRE

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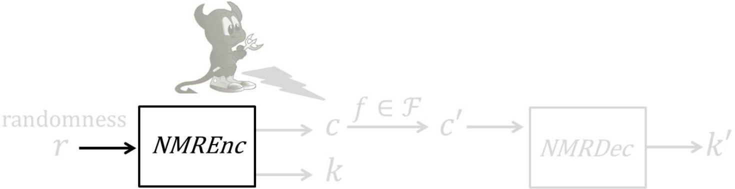

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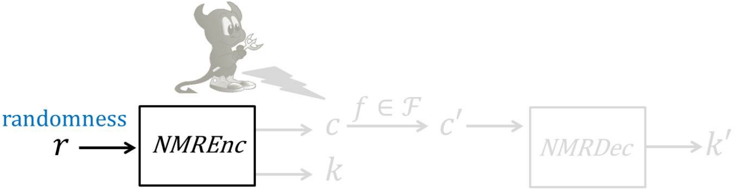

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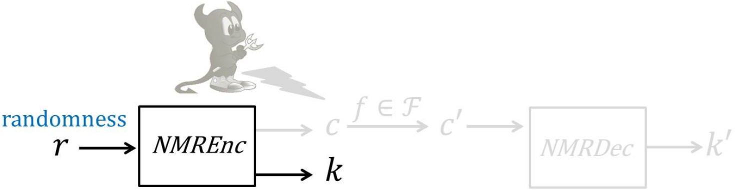


- A *random message* $k$ is generated

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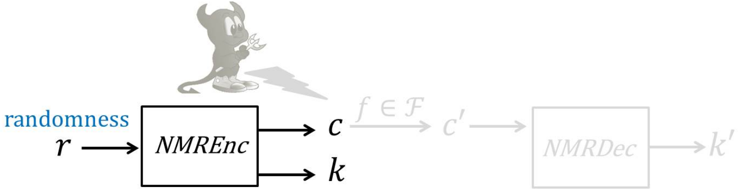


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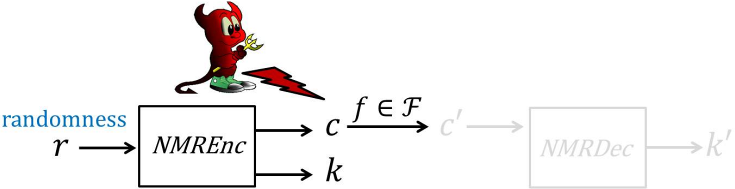


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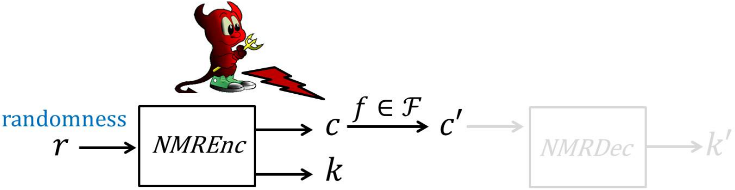


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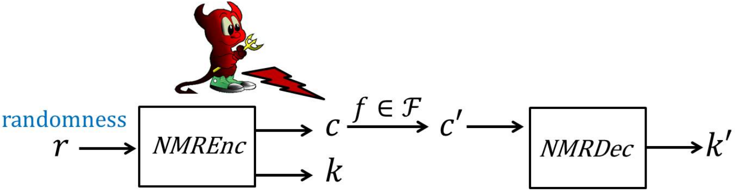


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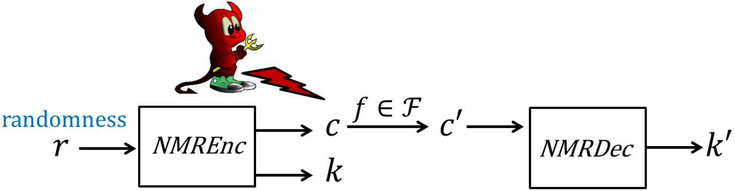


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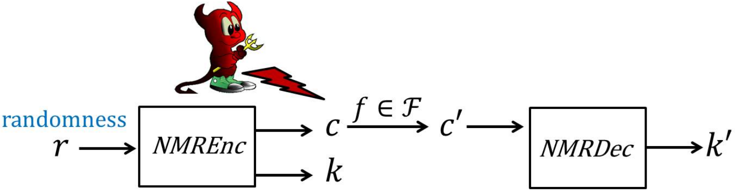


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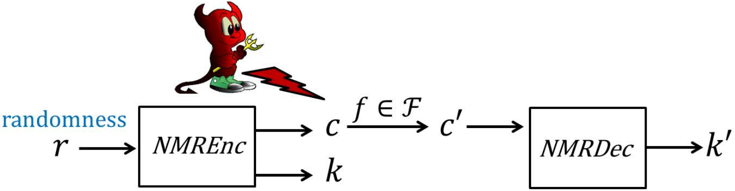


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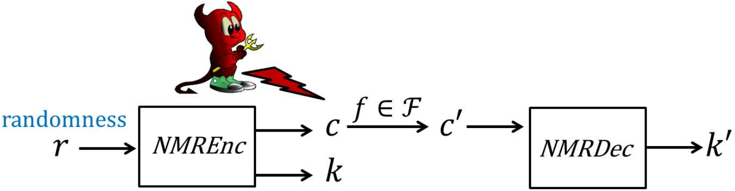


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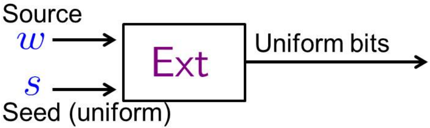
  - or $k$ looks uniform, even given $k'$.
- Any NMC is by default a secure NMRE.

# Journey Ahead

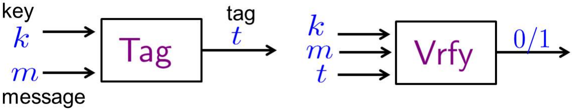
- Building blocks
- Motivating the construction
- Our construction
- Security proof

Converts non-uniform source string to a uniform string




$$S, \mathsf{Ext}(W; S) \approx S, U$$

MAC is composed algorithms $(\mathsf{Tag}, \mathsf{Vrfy})$:



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A Non-malleable code $(\mathsf{NMEnc}, \mathsf{NMDec})$ w.r.t. to $\mathcal{F}_2$

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- Can be any 2-state NMC.
  Specific instantiation: [Li17]

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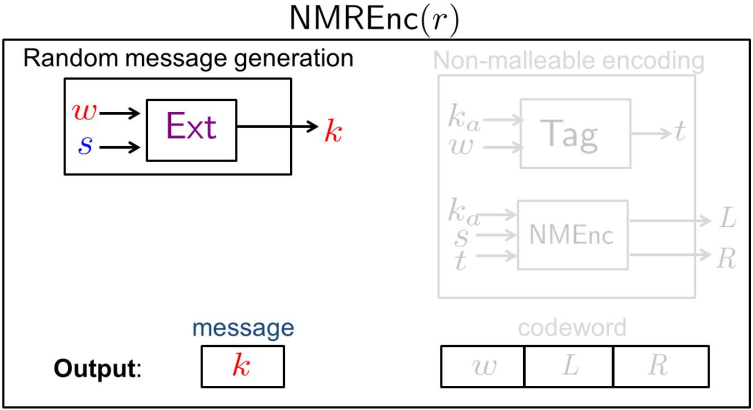
## Two-state NMC

A Non-malleable code $(\mathsf{NMEnc}, \mathsf{NMDec})$ w.r.t. to $\mathcal{F}_2$


- Can be any 2-state NMC.
  Specific instantiation: [Li17]
- Used to encode short messages only

# Motivating our construction



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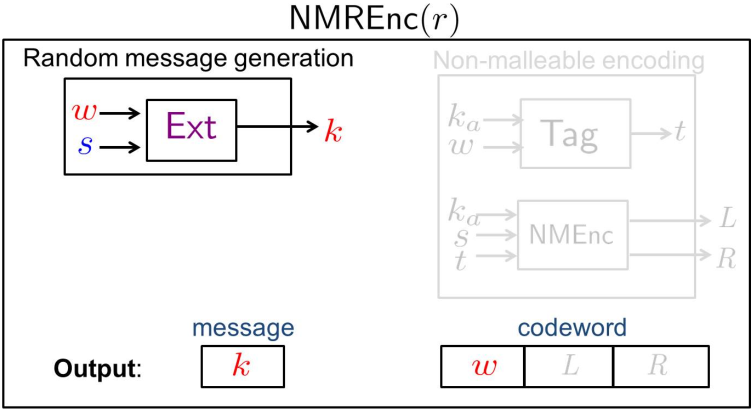

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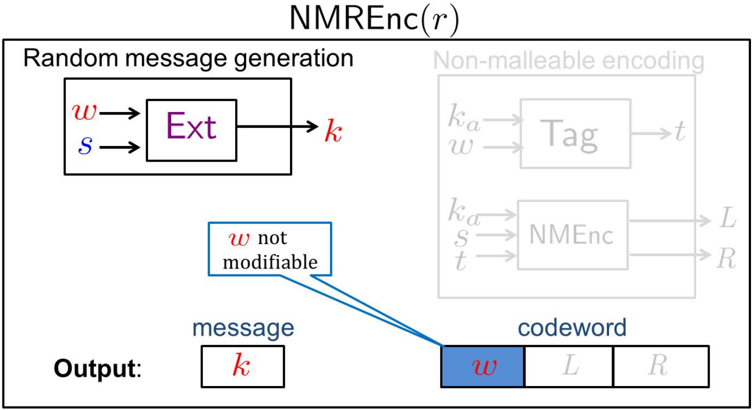

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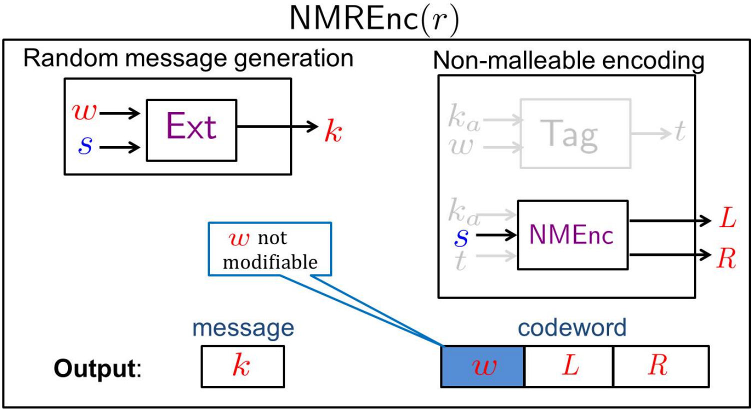

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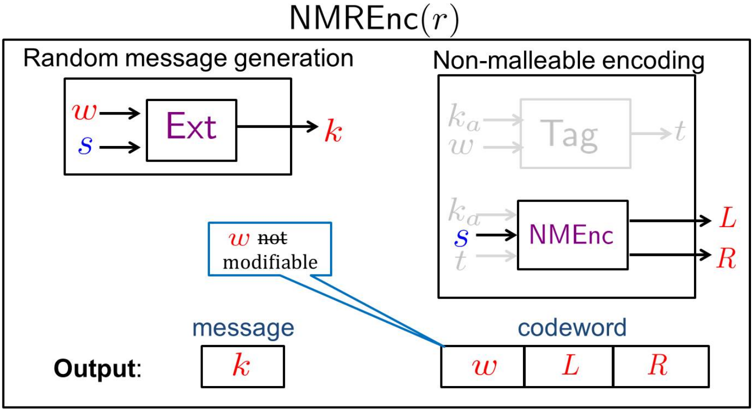

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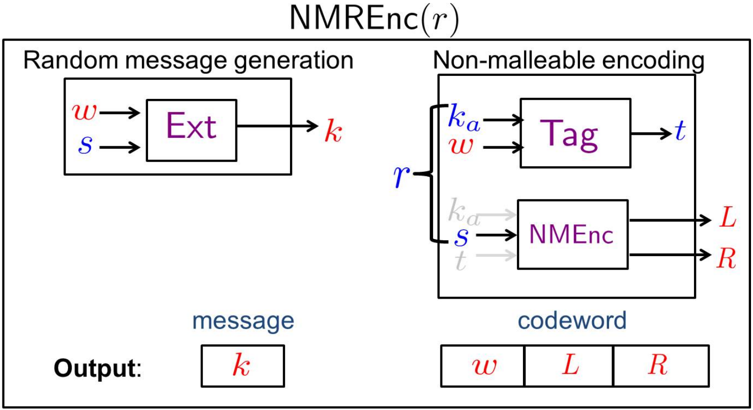

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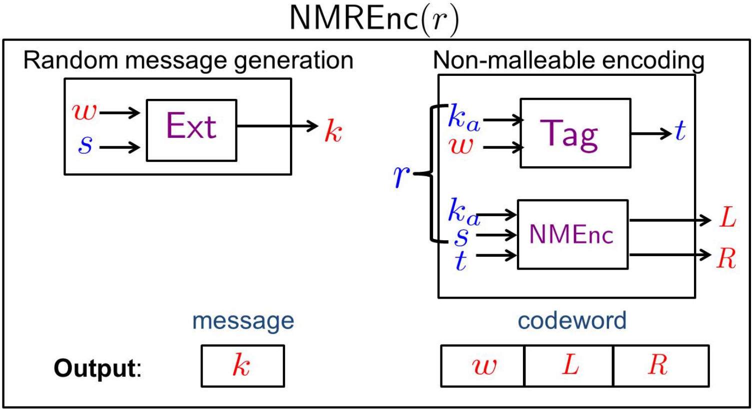

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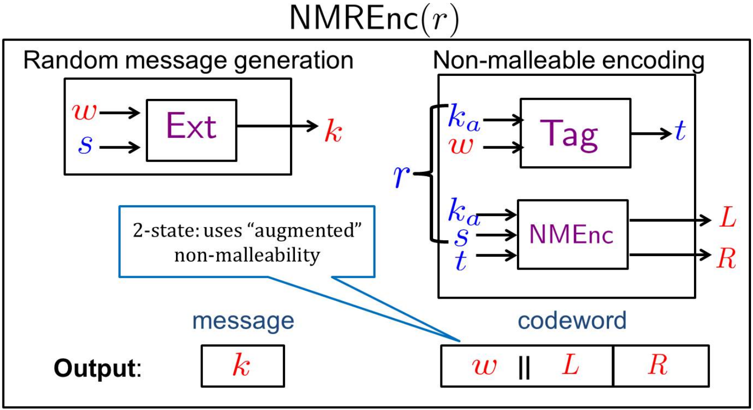


# Security Proof

Goal: Build a simulator $\mathsf{NMRSim}_{f,g}$, similar to NMCs.

$\mathsf{NMRSim}_{f,g}$

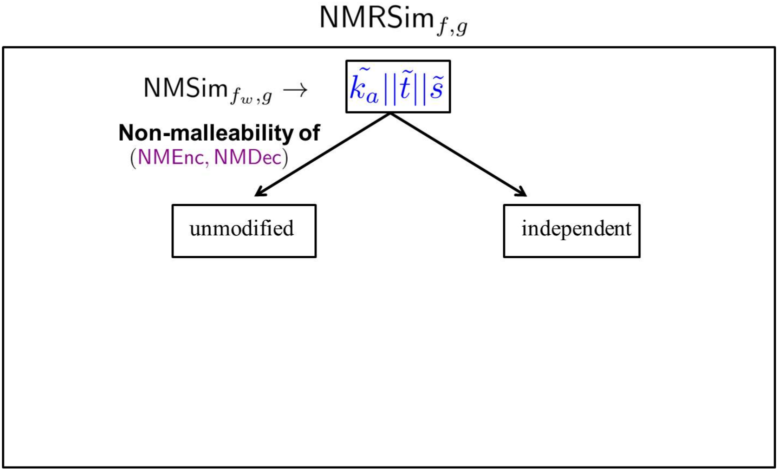
# Security Proof

To do this, we use the simulator for NMC, $\mathsf{NMSim}$ in black box.

$\mathsf{NMRSim}_{f,g}$

$\mathsf{NMSim}_{f_w,g} \rightarrow \boxed{\tilde{k_a}||\tilde{t}||\tilde{s}}$

# Security Proof



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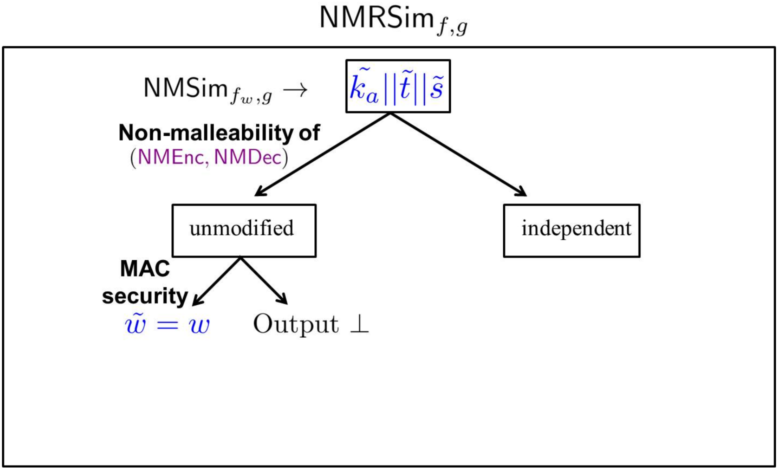

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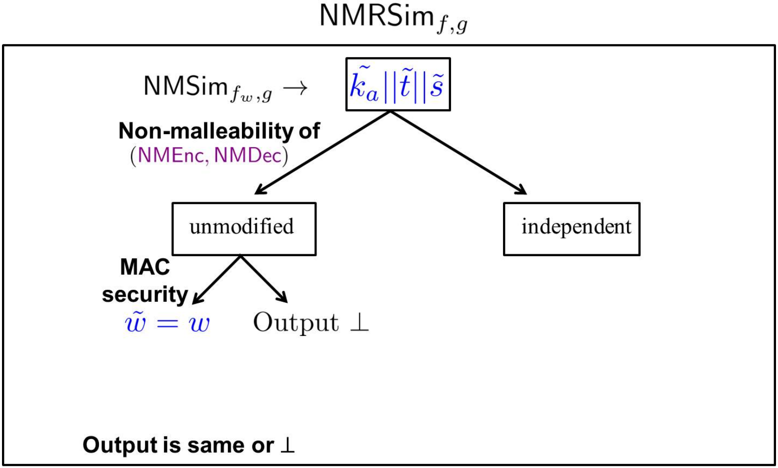

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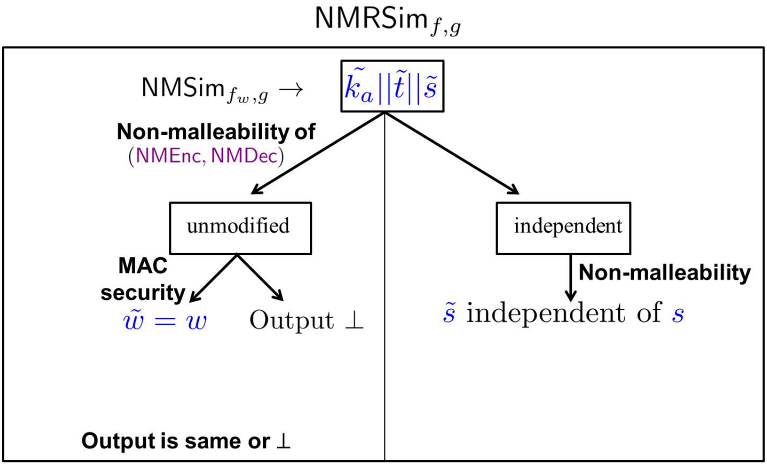

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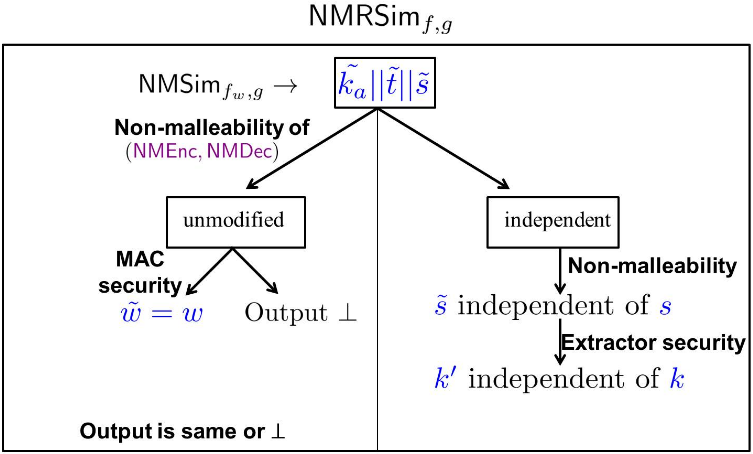

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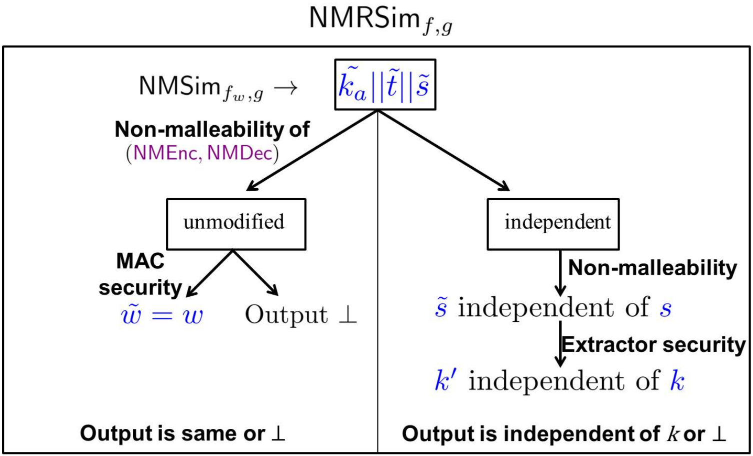
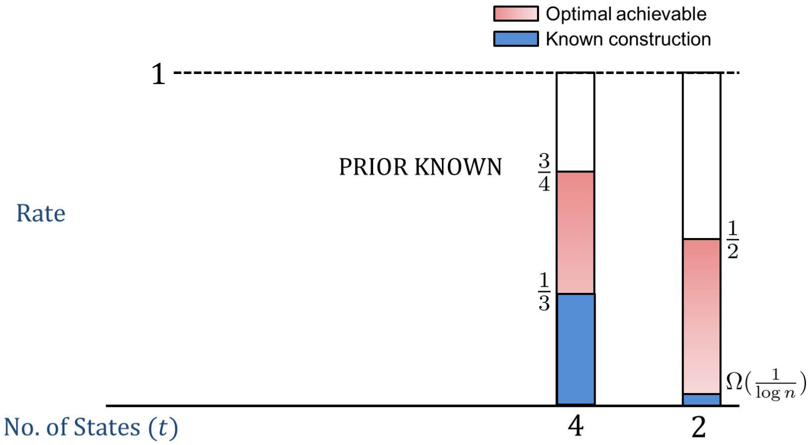


# Application of NMRE: Constant Rate 3-state NMCs



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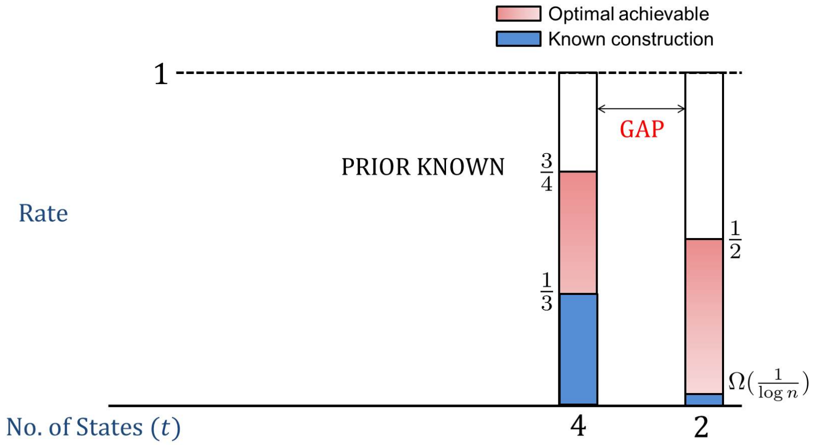

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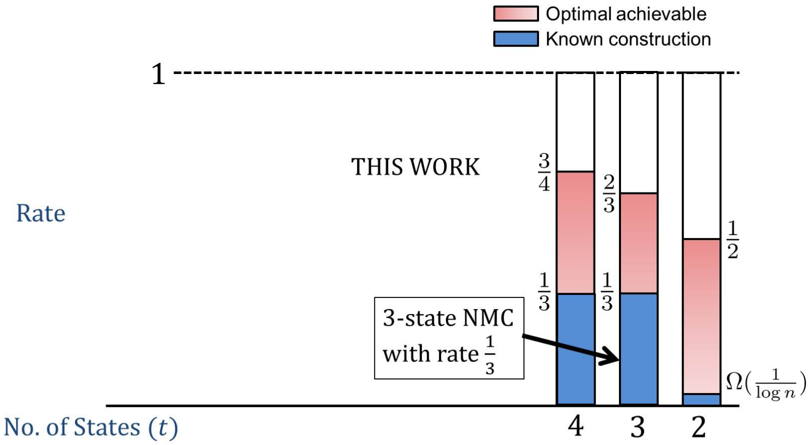

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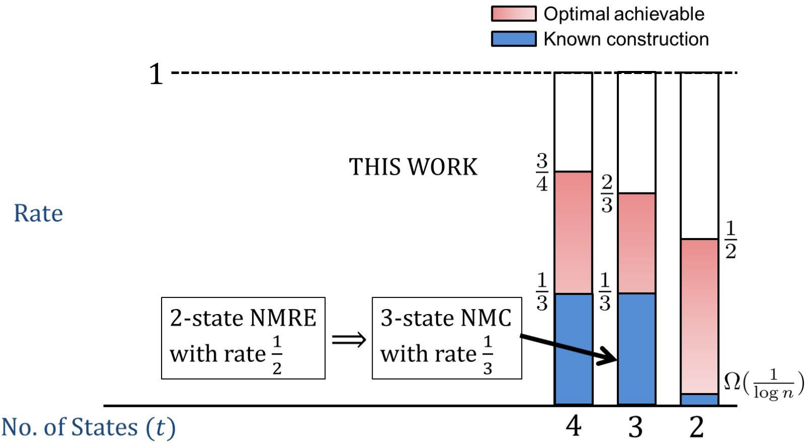

# Conclusion

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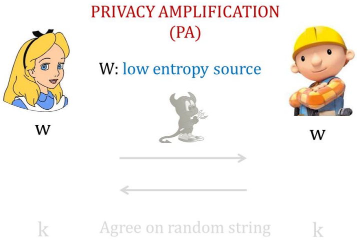
- Other applications of NMREs

# Subsequent Work



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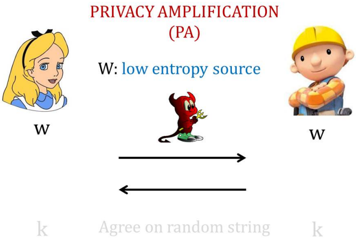

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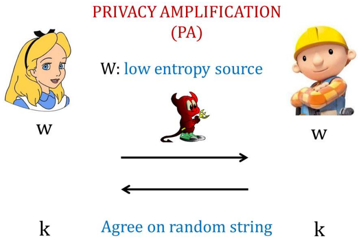

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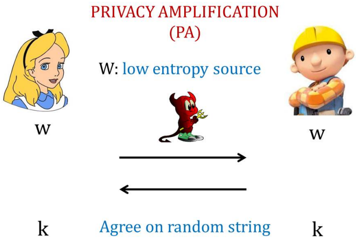


Holy Grail for PA: Build 2-round protocol with entropy loss $\Theta(\lambda)$ and requiring a min-entropy of $\mathcal{O}(\lambda + \log n)$

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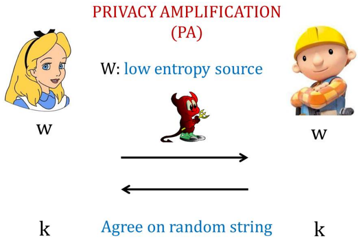


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Our Result: An "augmented" 2-state constant rate NMRE with optimal error $\implies$ 8-round PA protocol with optimal entropy loss and min-entropy requirement.

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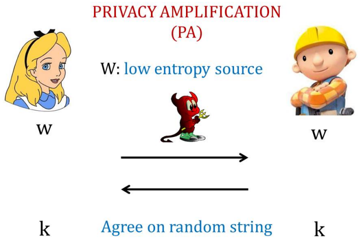


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(Joint work with: Eshan Chattopadhyay, Bhavana Kanukurthi, Sai Lakshmi Bhavana Obbattu)
(https://eprint.iacr.org/2018/293)

THANK YOU!!