Breaking the Sub-Exponential Barrier in Obfustopia

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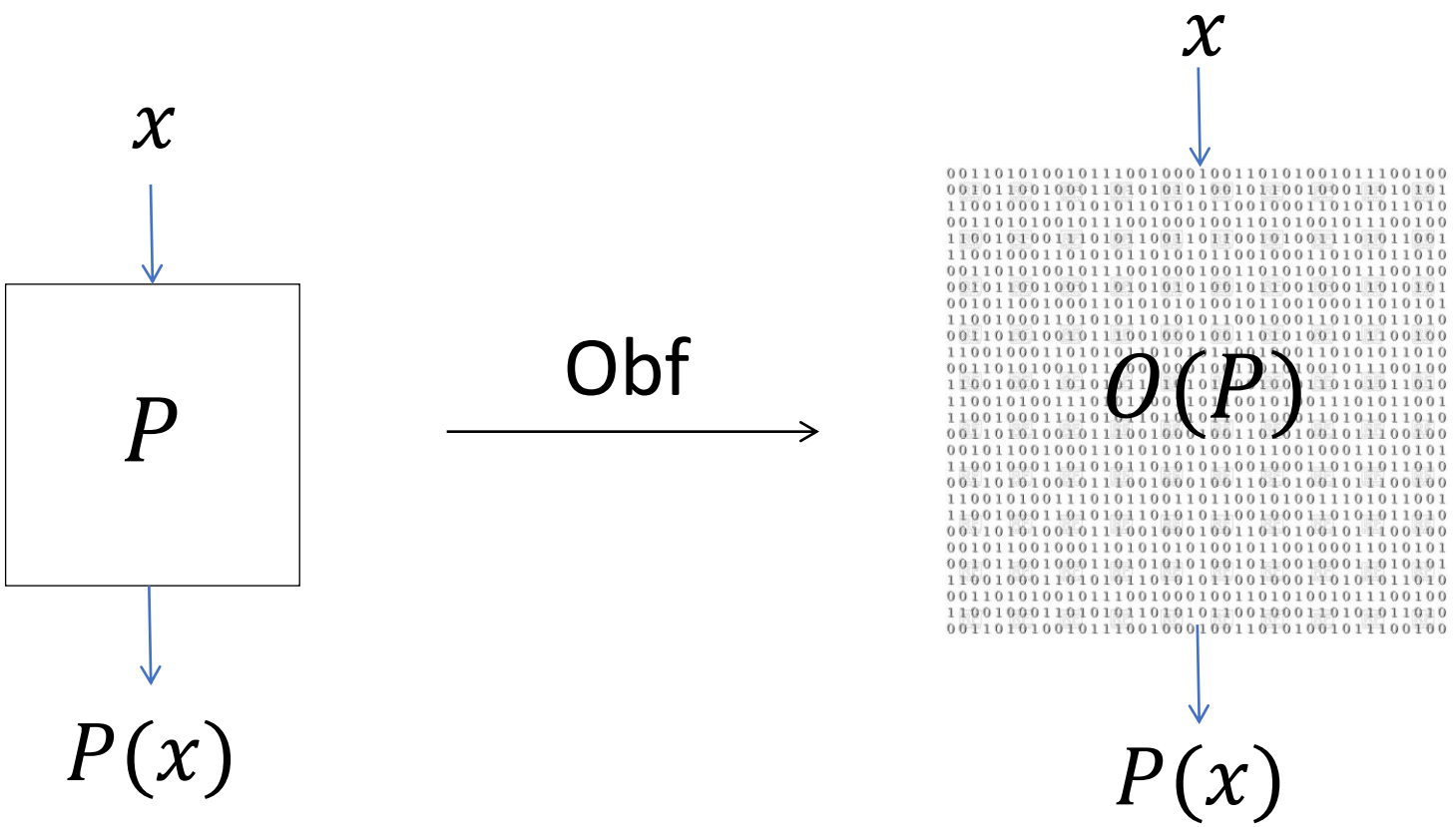
Akshayaram Srinivasan, University of California, Berkeley
Mark Zhandry, Princeton University

Program Obfuscation

[Barak-Goldreich-Impagliazzao-Rudich-Sahai-Vadhan-Yang 01]



Indistinguishability Obfuscation

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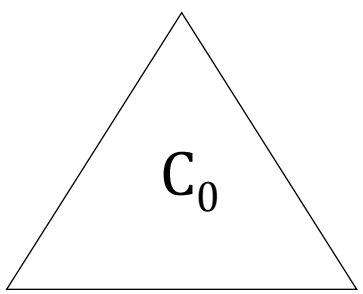

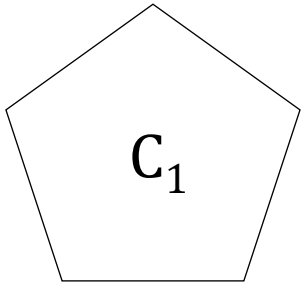

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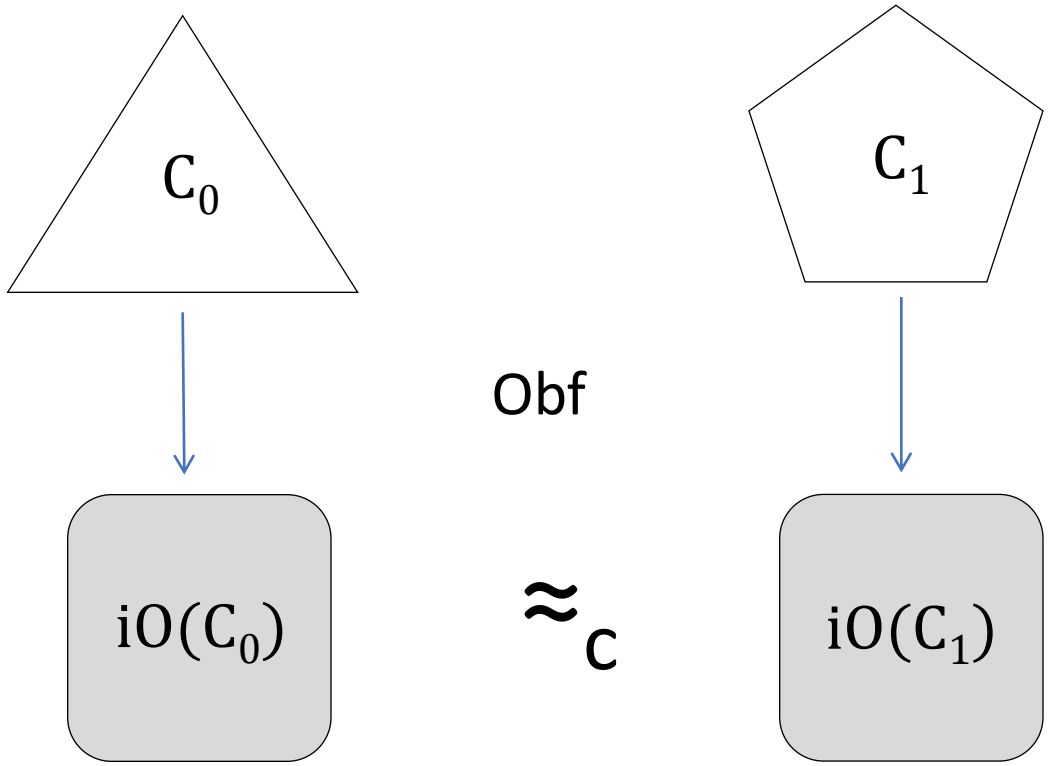

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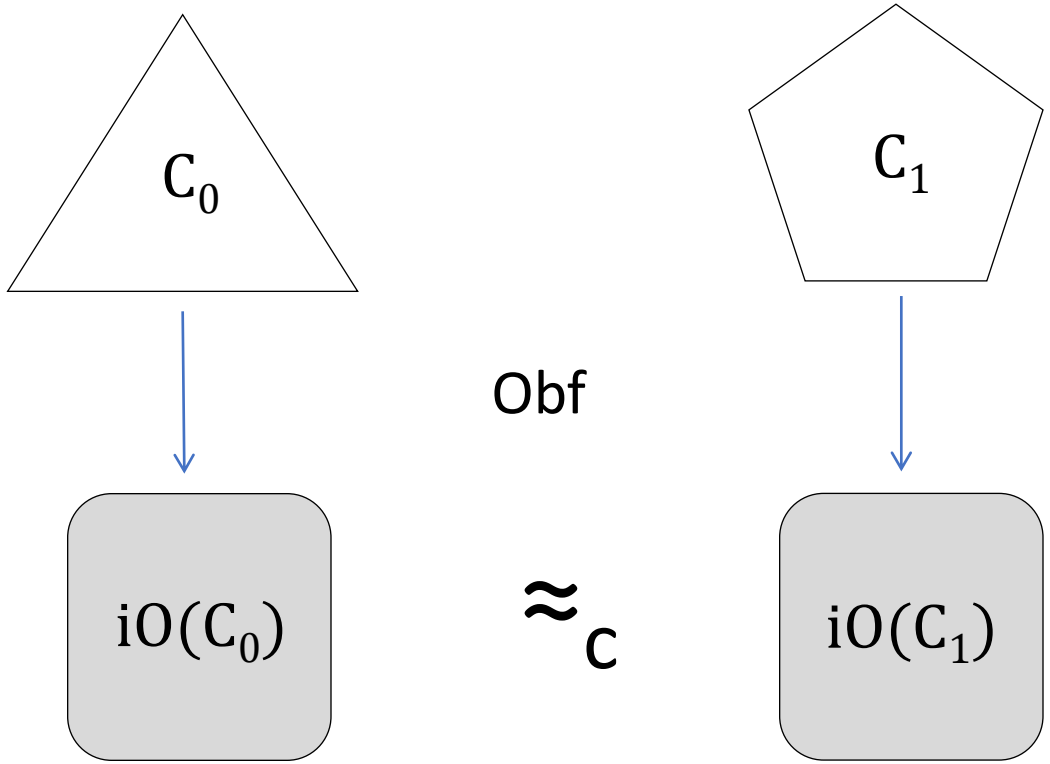
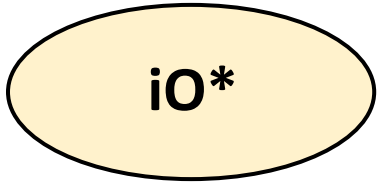


Several constructions of iO: [GGHRSW13], [BGKPS14], [AGIS14], [BR14],[GLSW15], [AJ15], [BV15], [AJS15], [BMSZ16], [L16], [LV16], [GMMS**S**Z16], [L17], [LT17] …

Applications of iO



* + other assumptions

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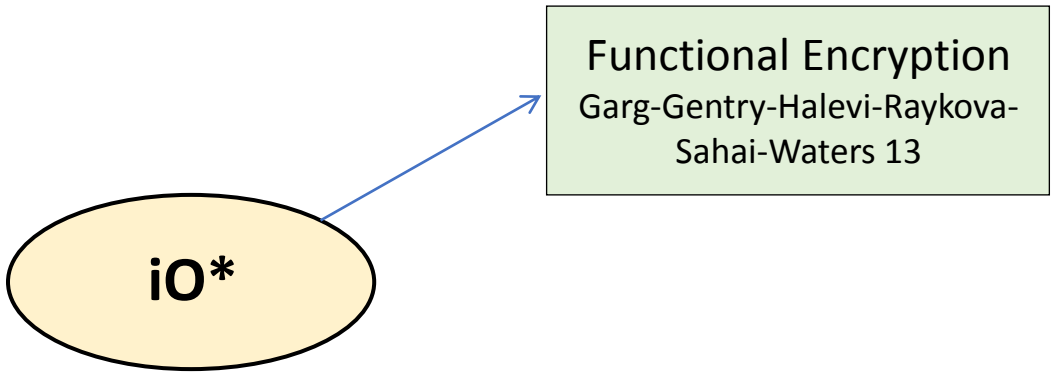

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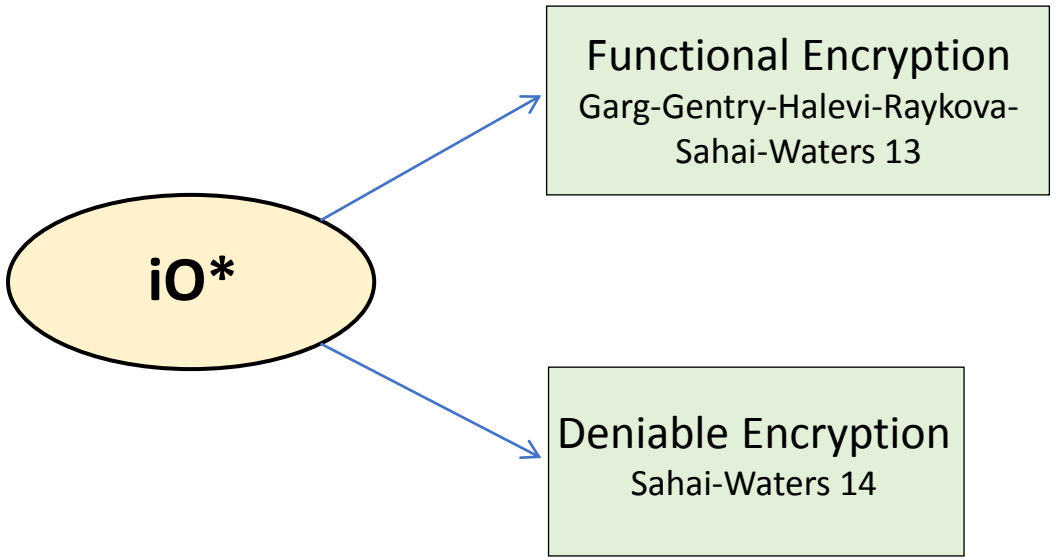

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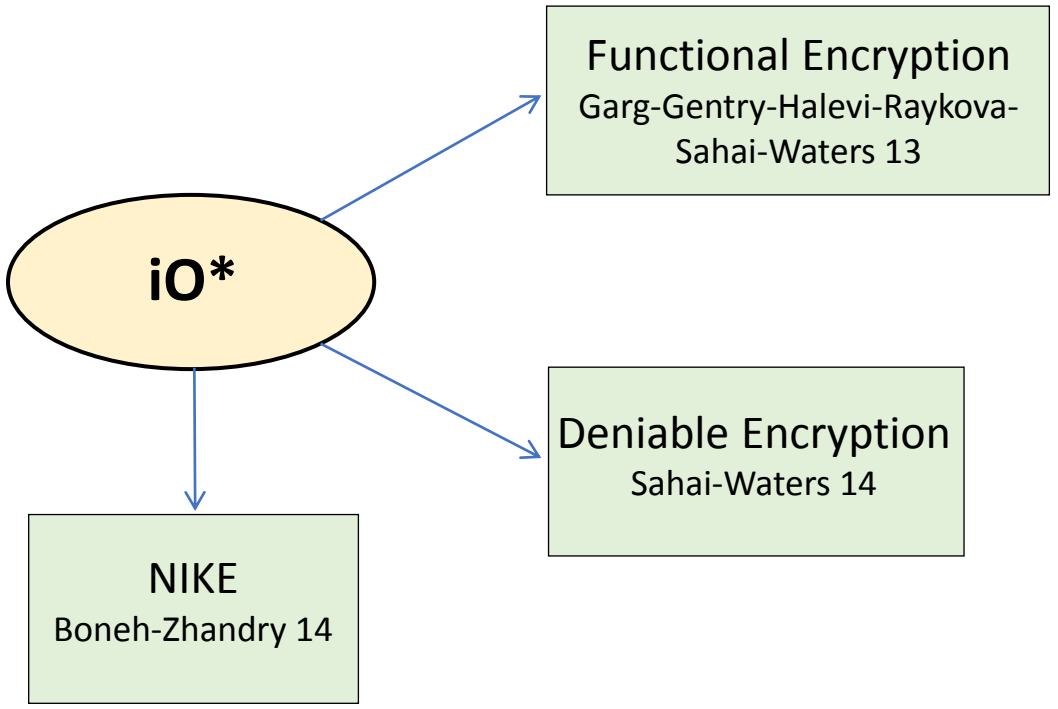

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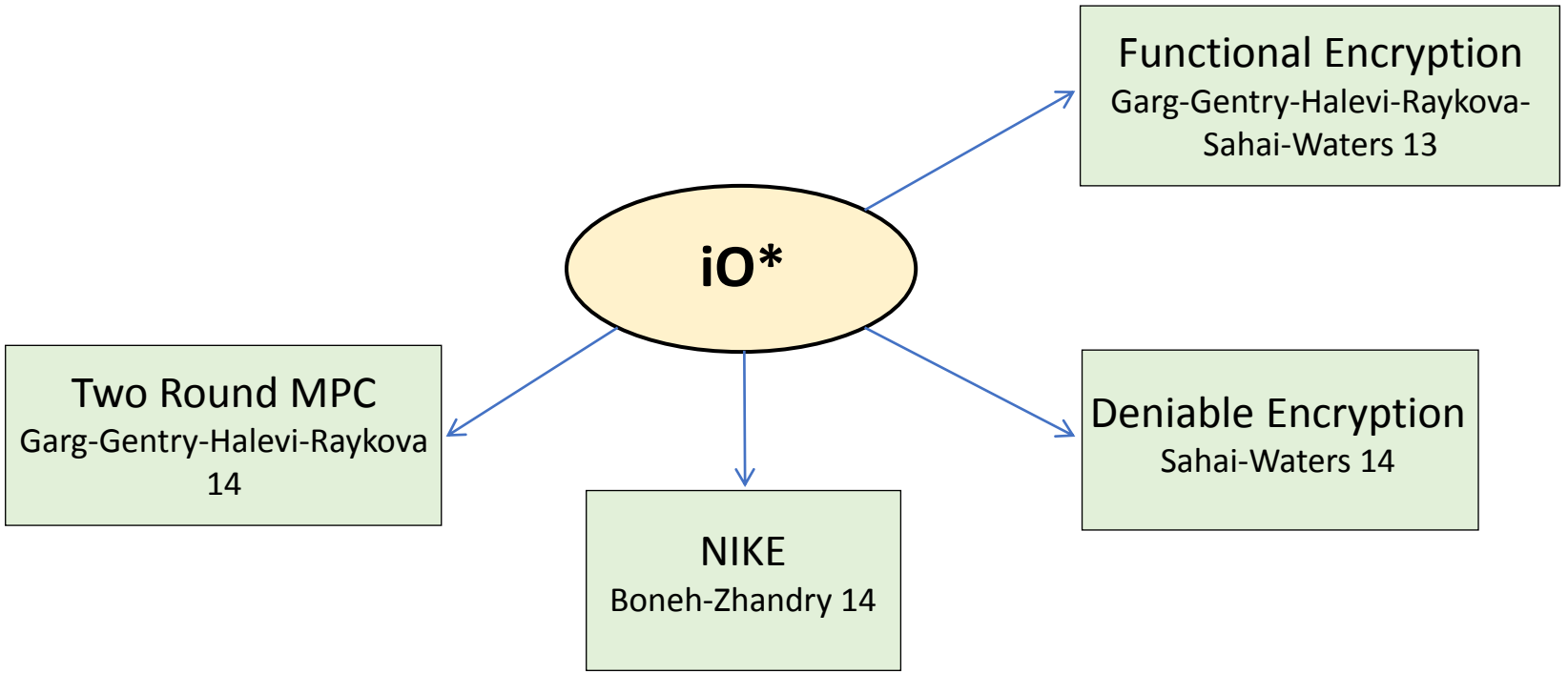

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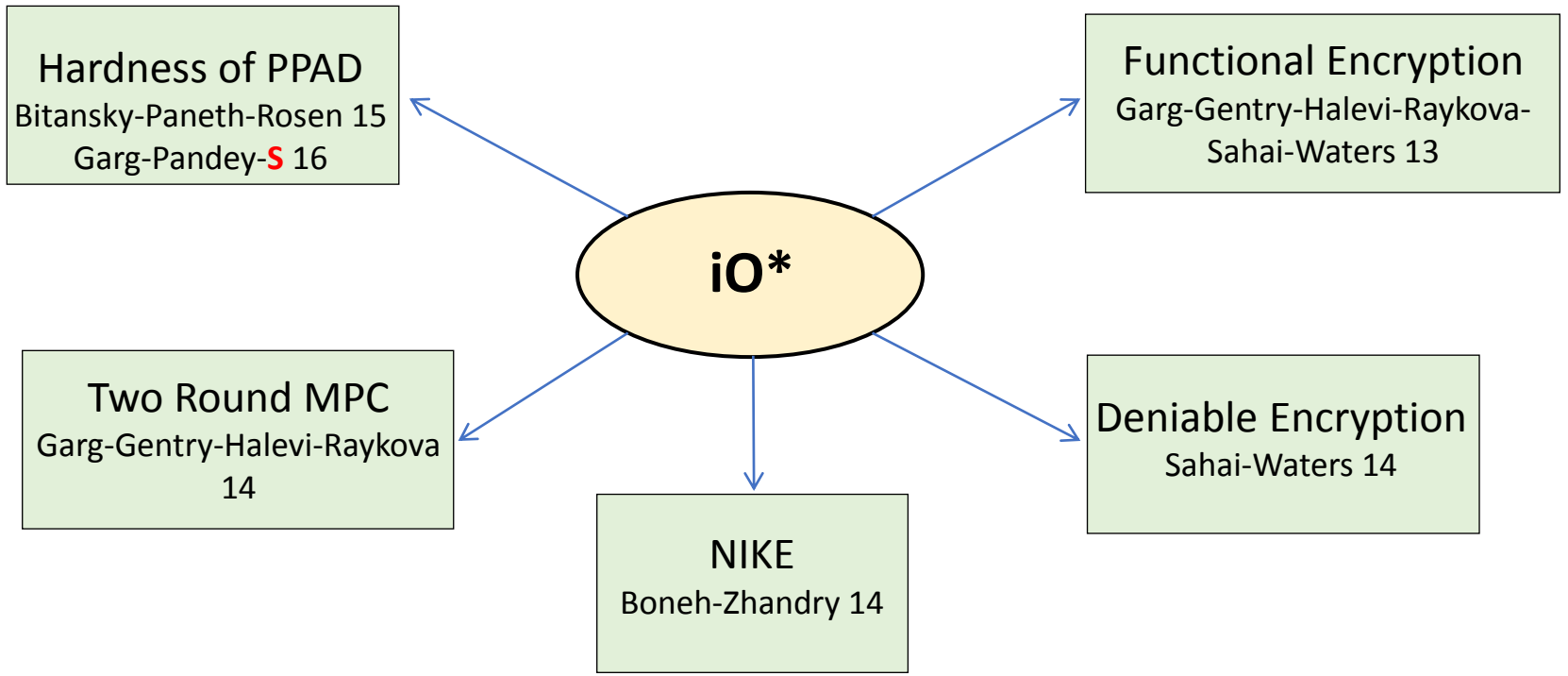

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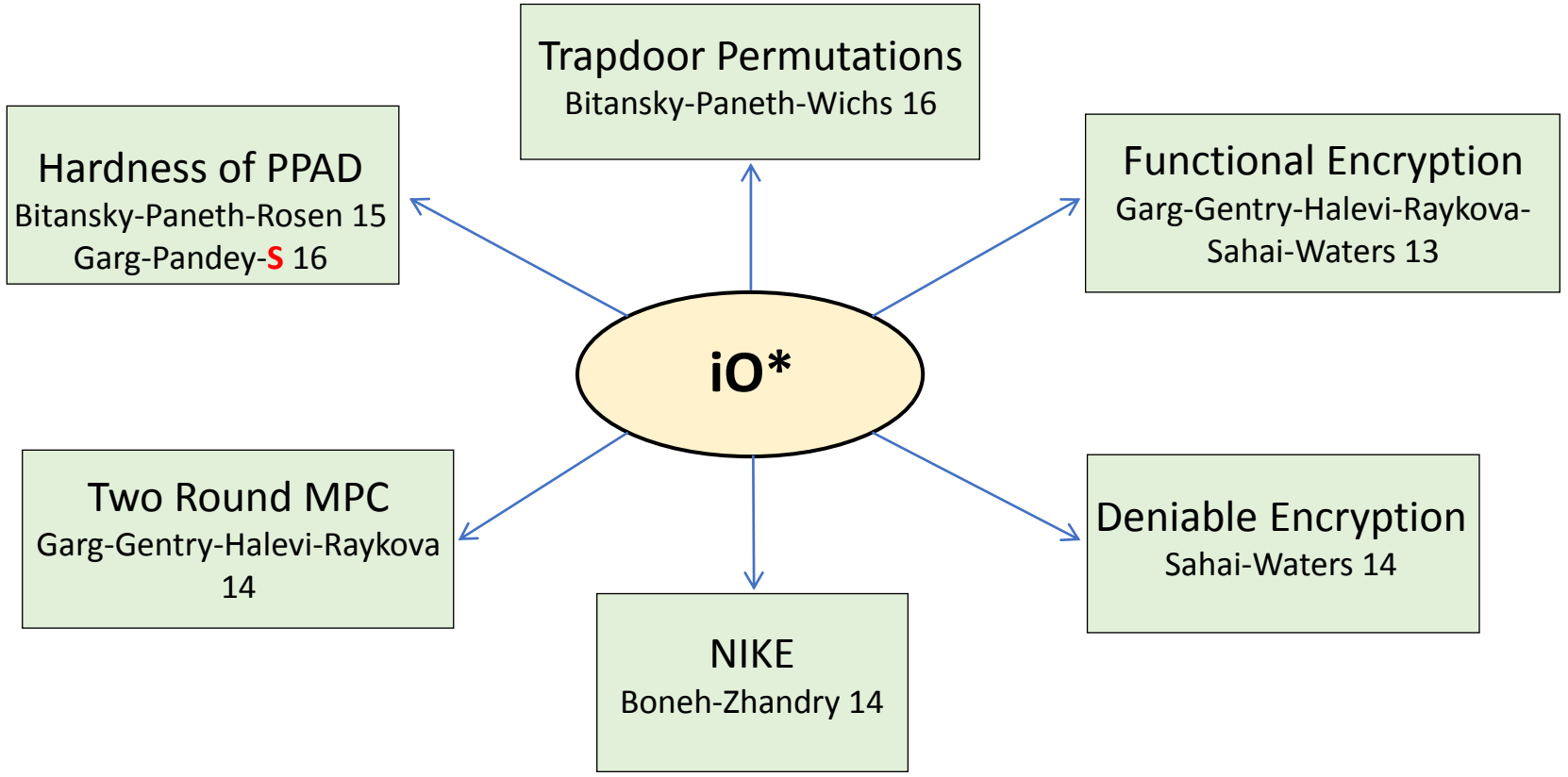

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Sub-Exponential Barrier

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Can we break the sub-exponential barrier?

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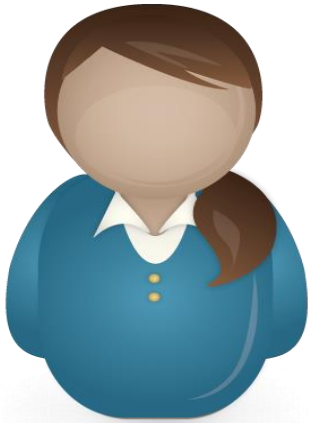
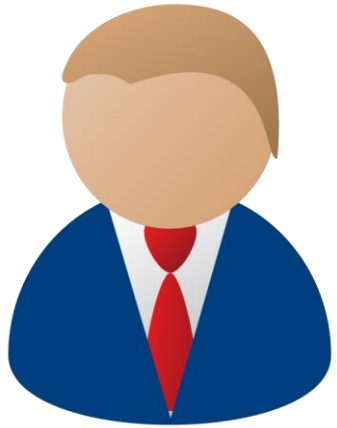
- How to use this technique to construct NIKE?

Functional Encryption [Boneh-Sahai-Waters 11]



Data D



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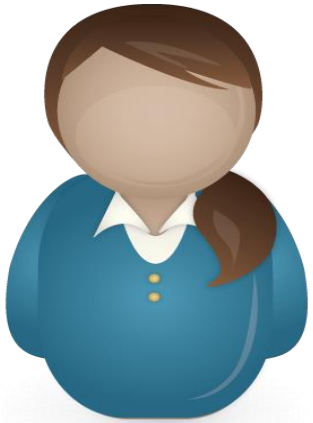

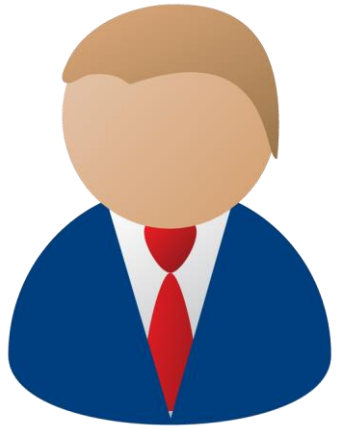

MSK

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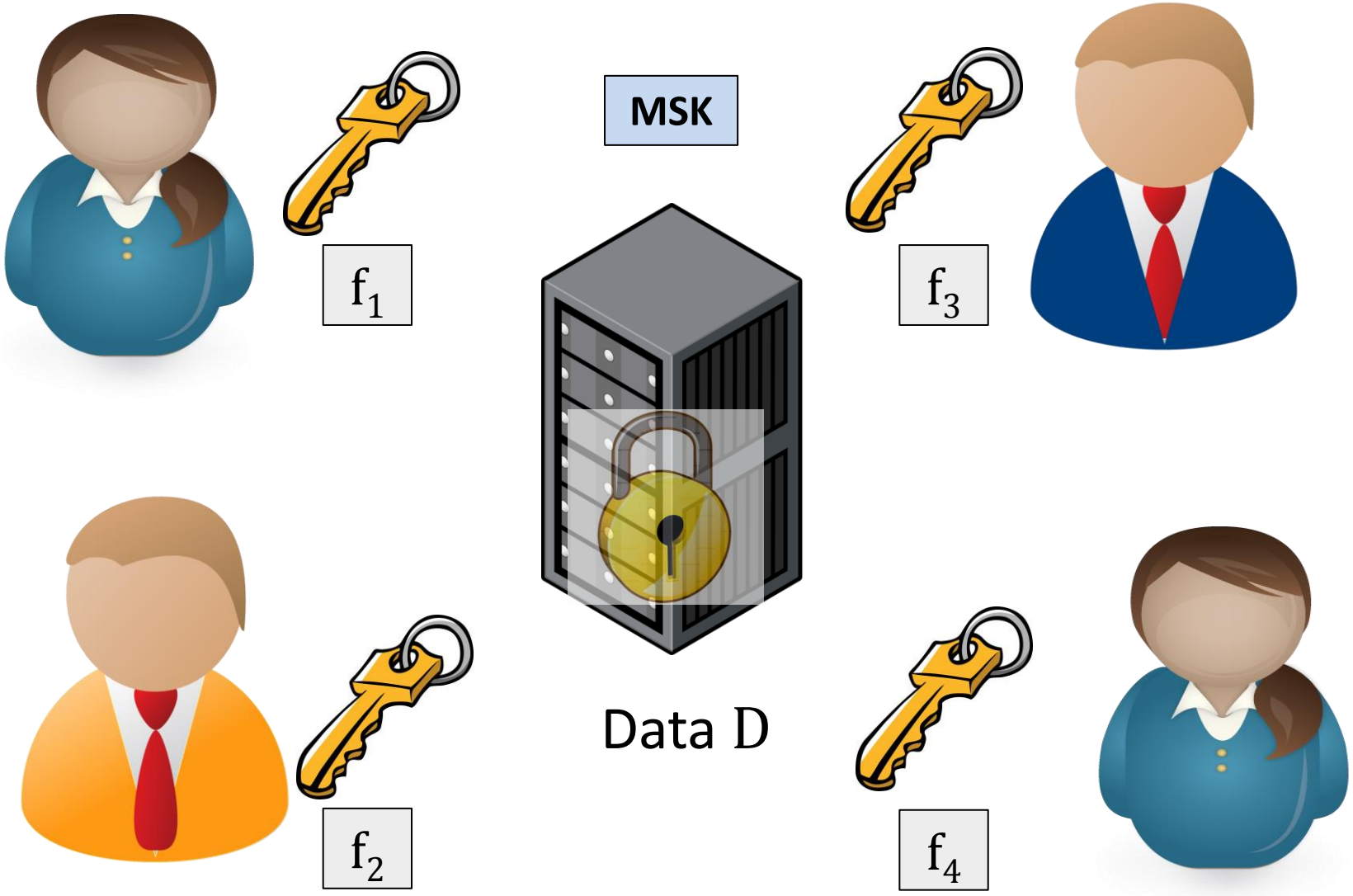

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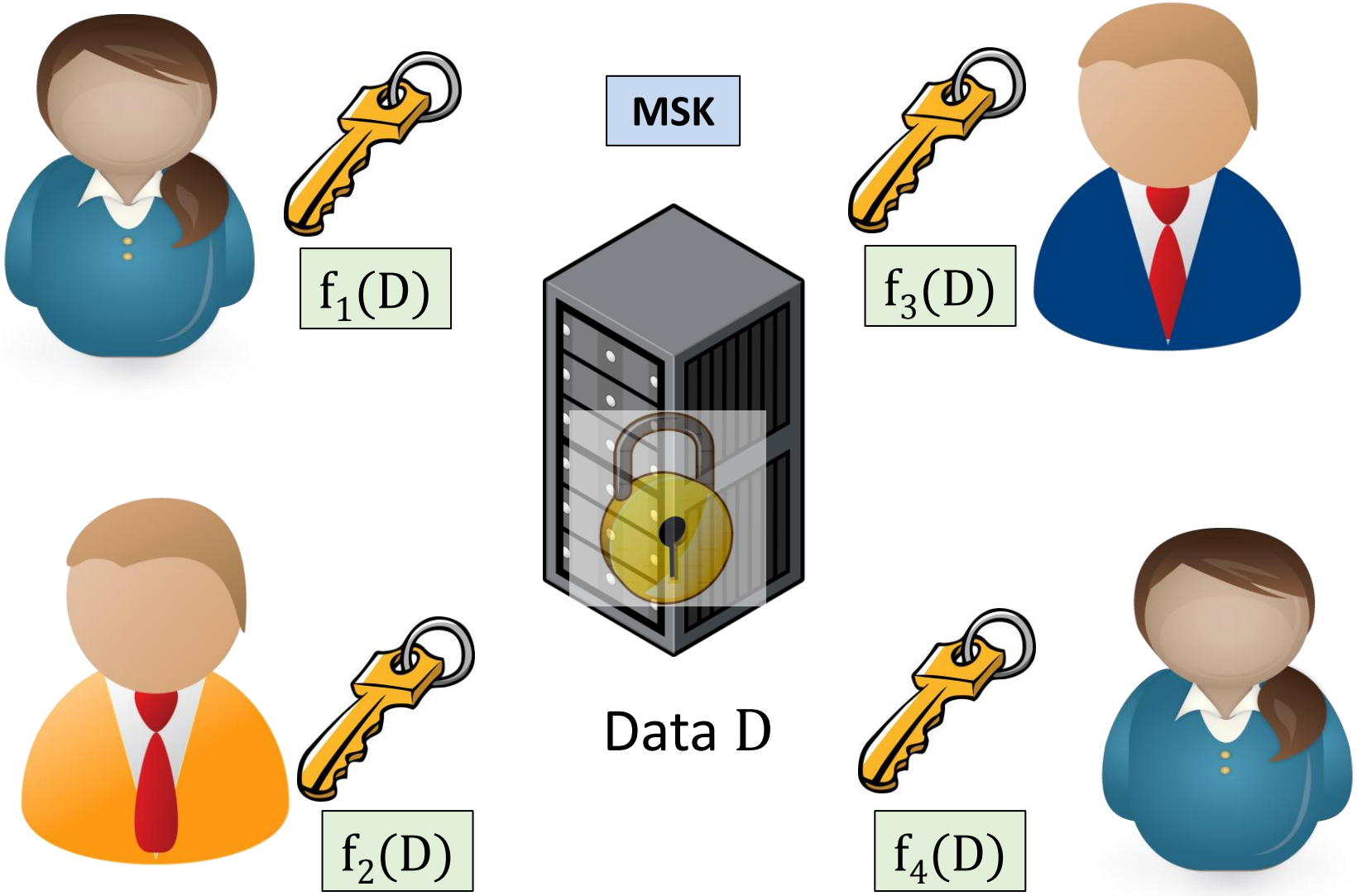

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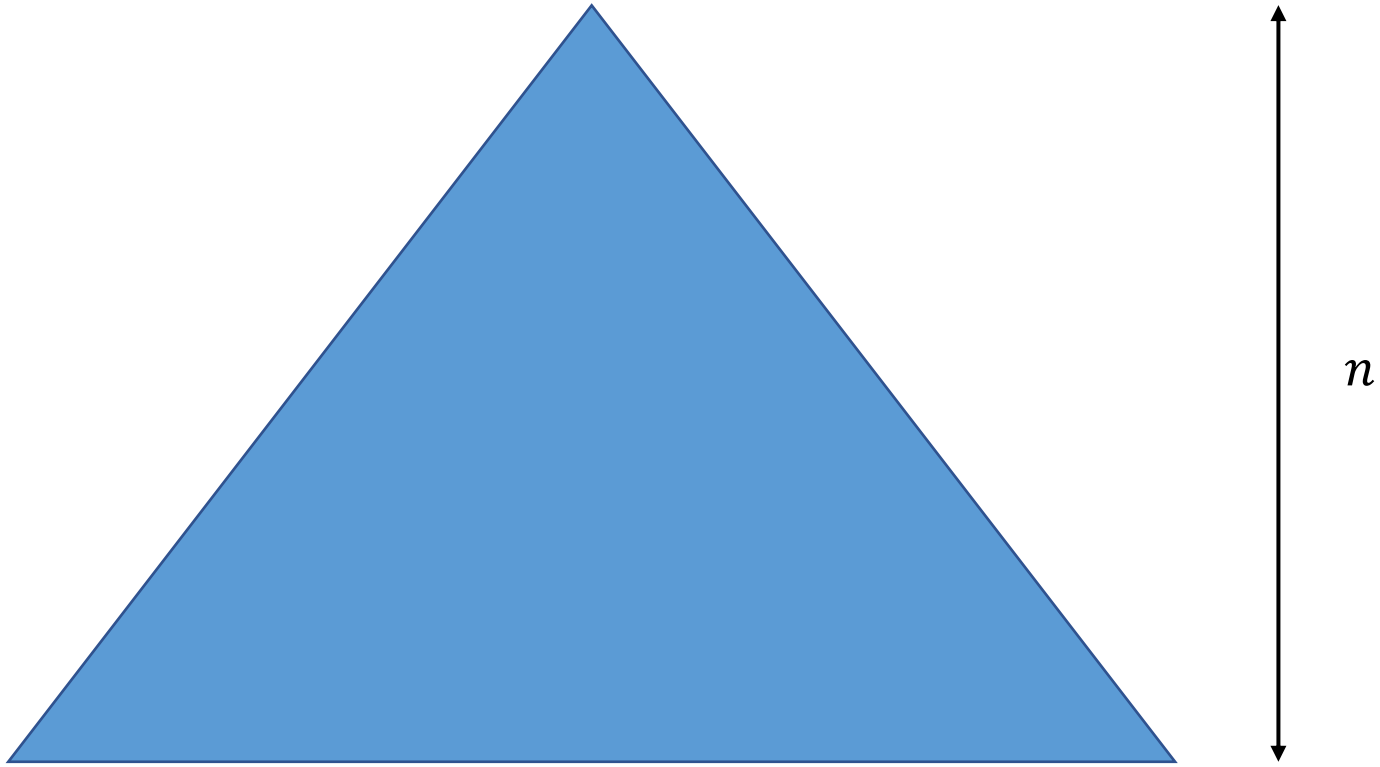

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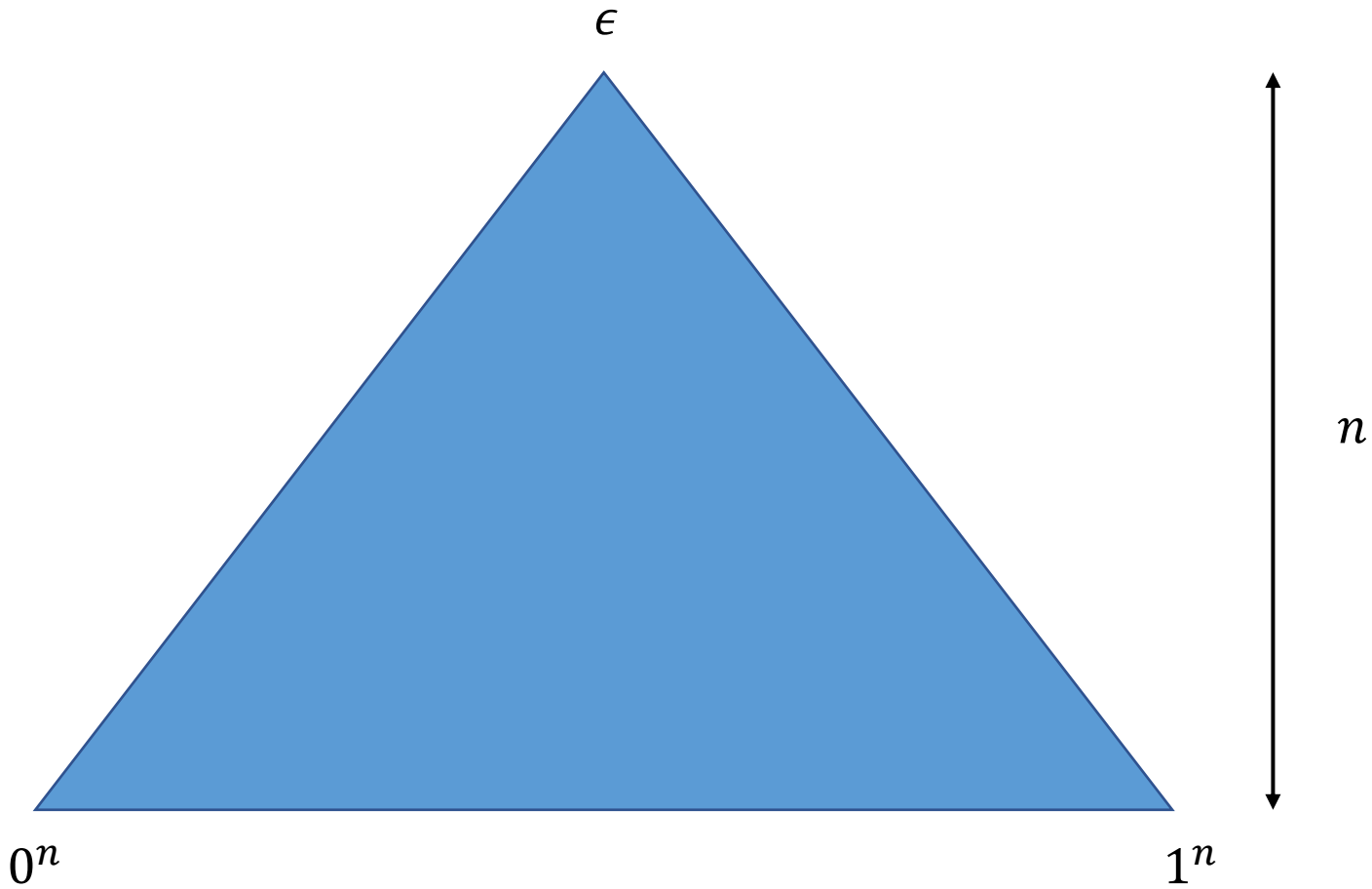

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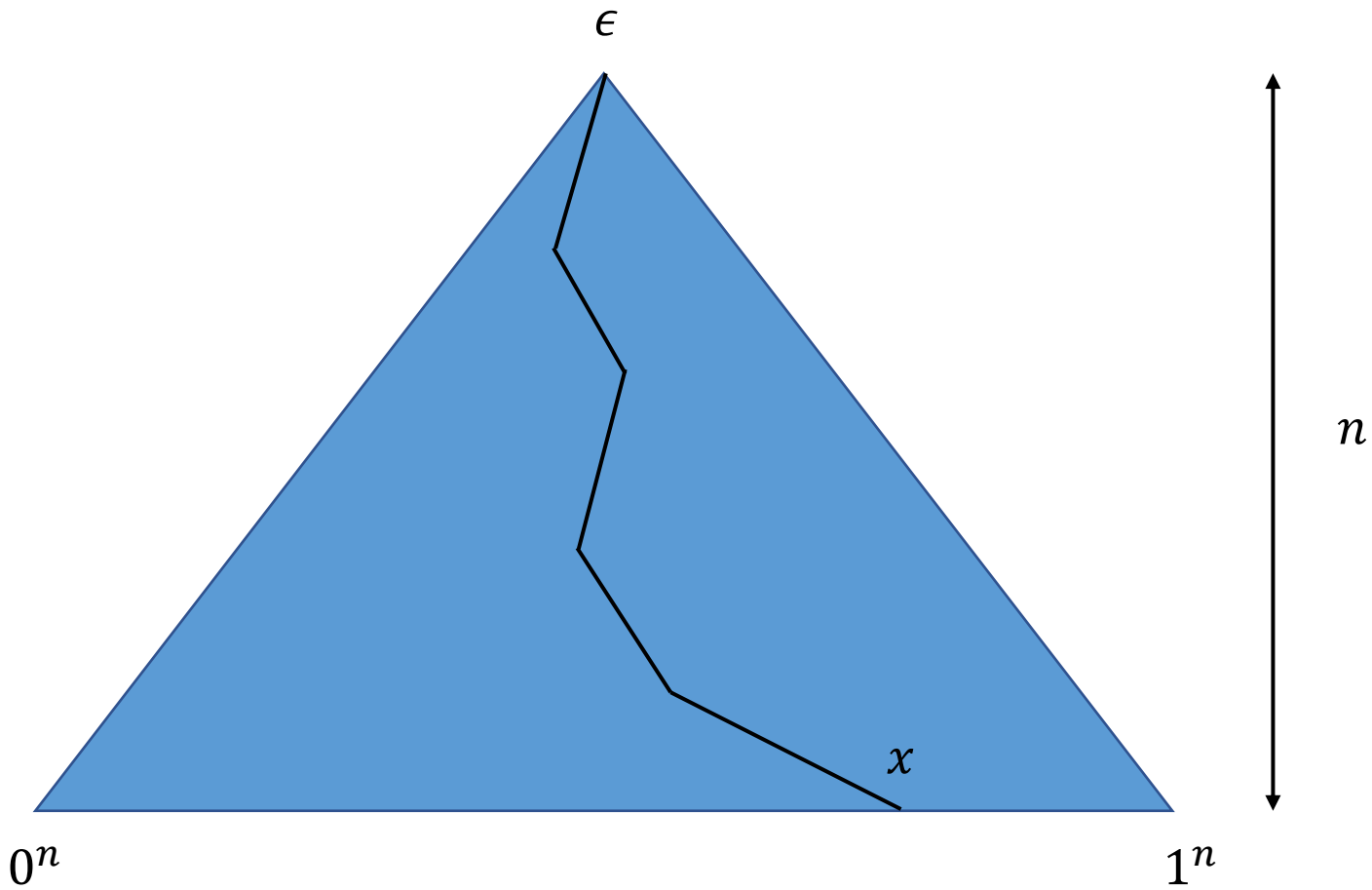

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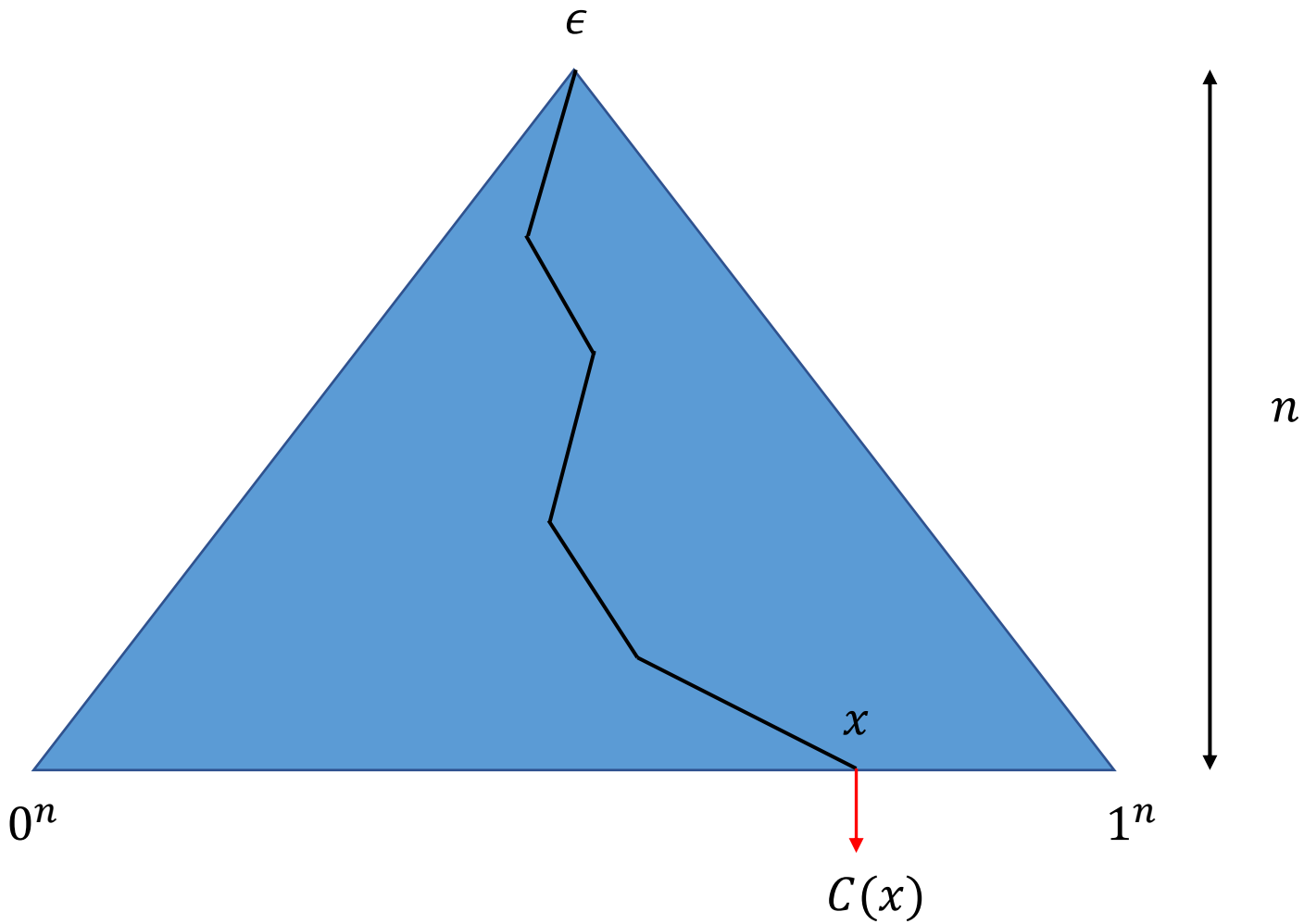

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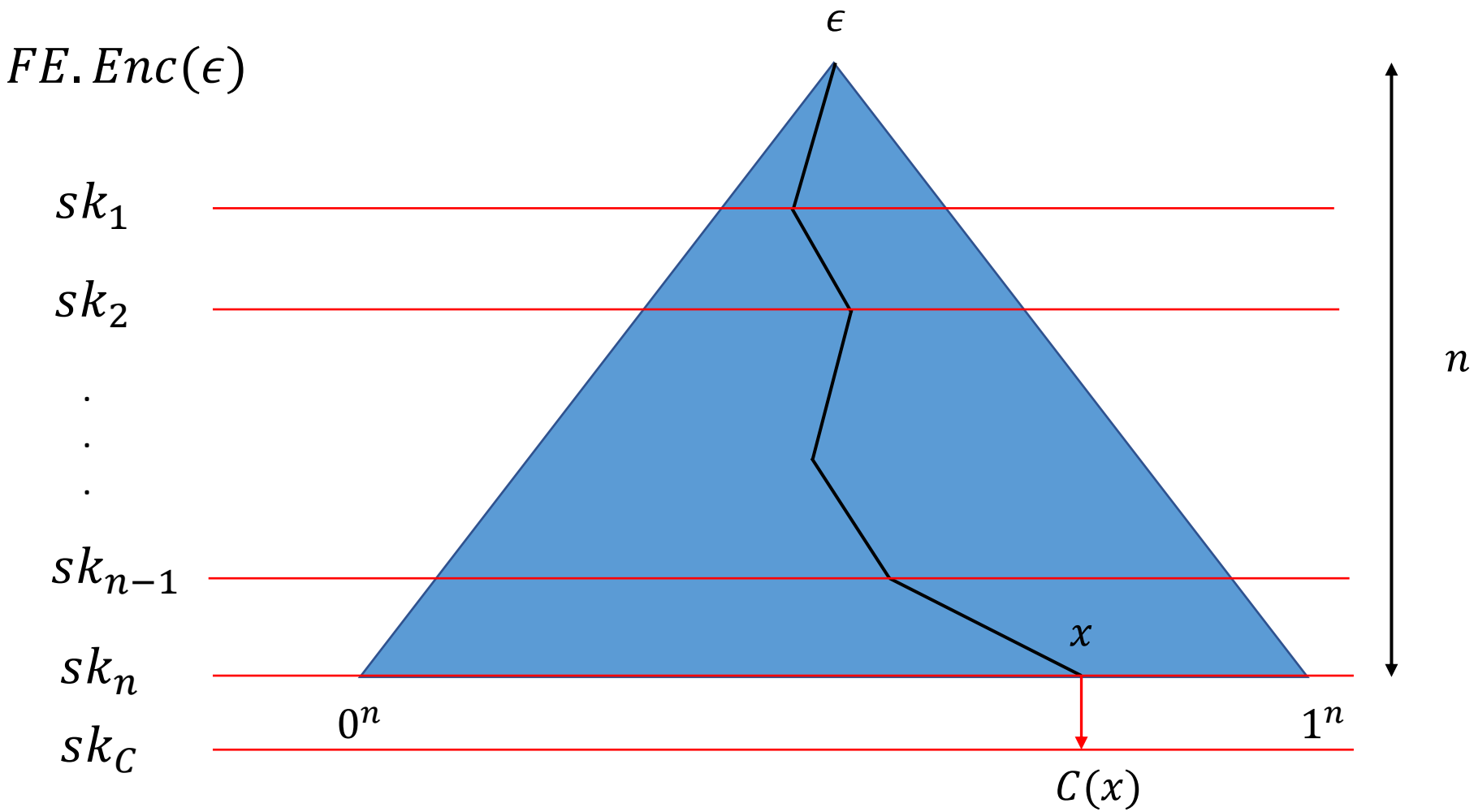

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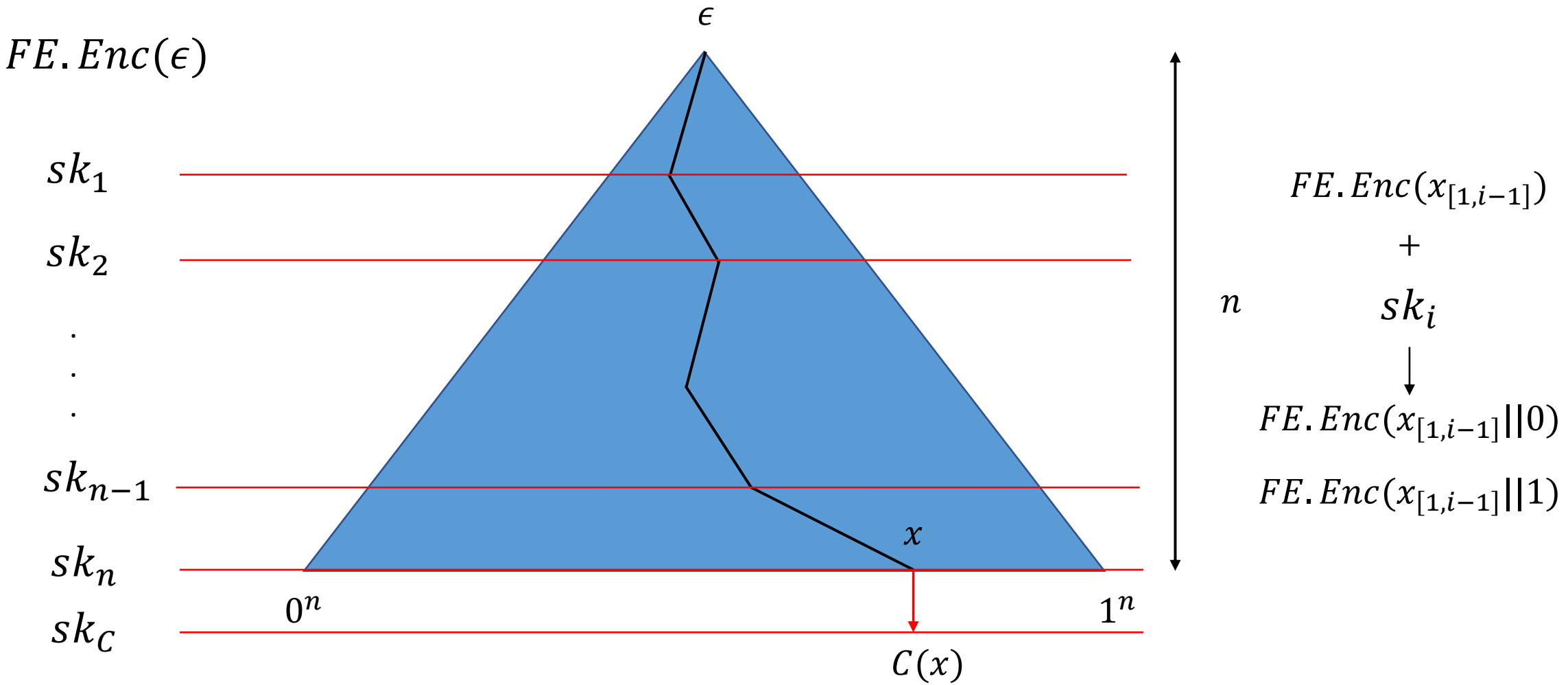

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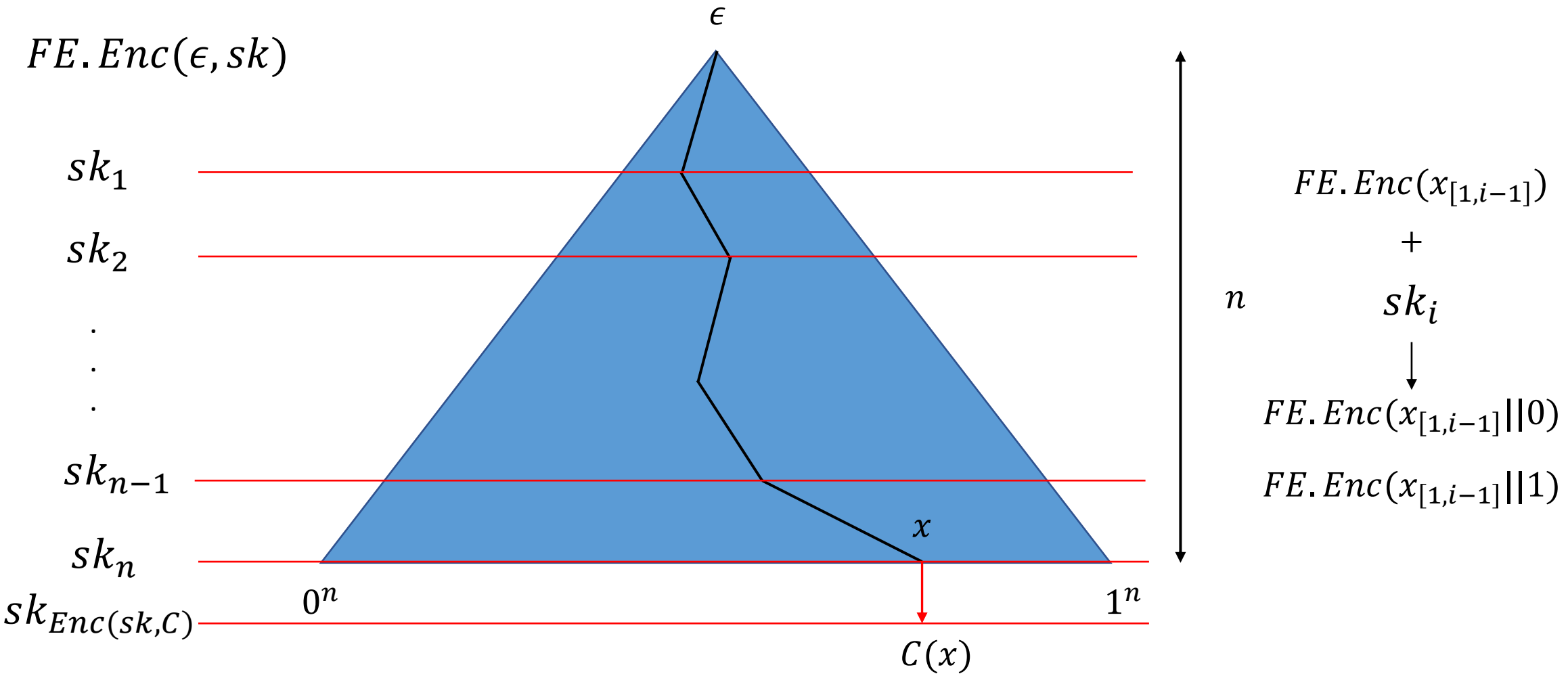

Sub-Exponential Barrier

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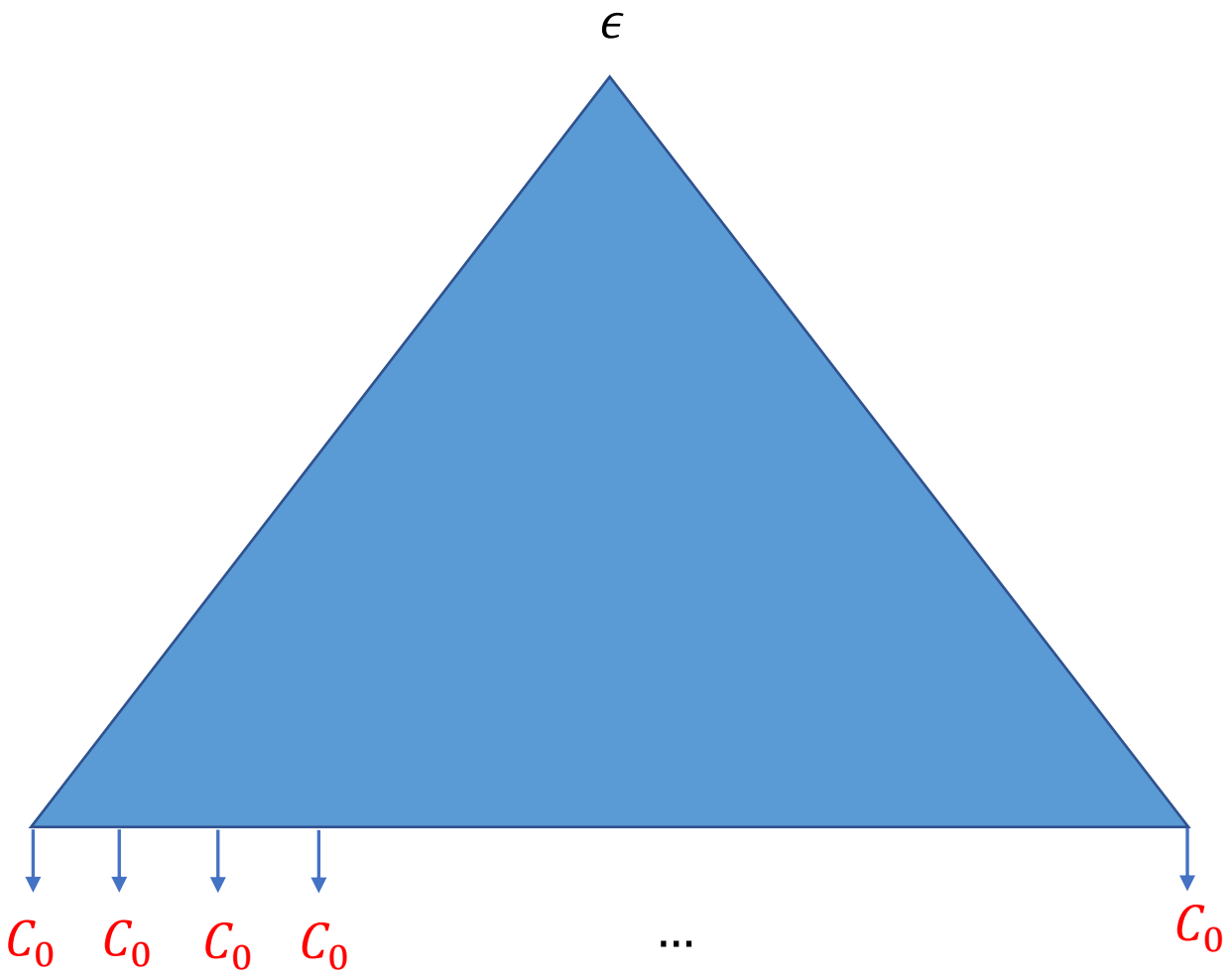

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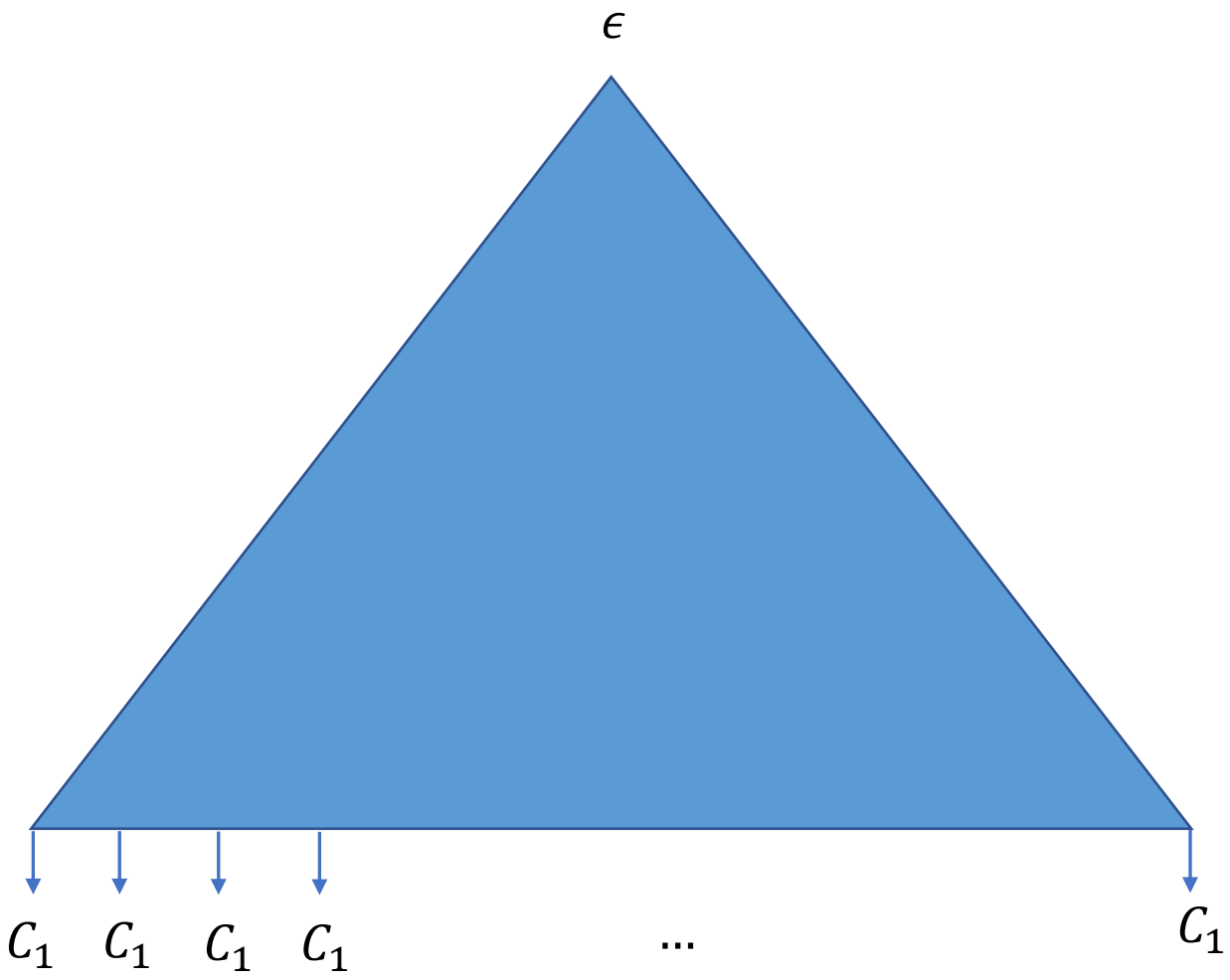

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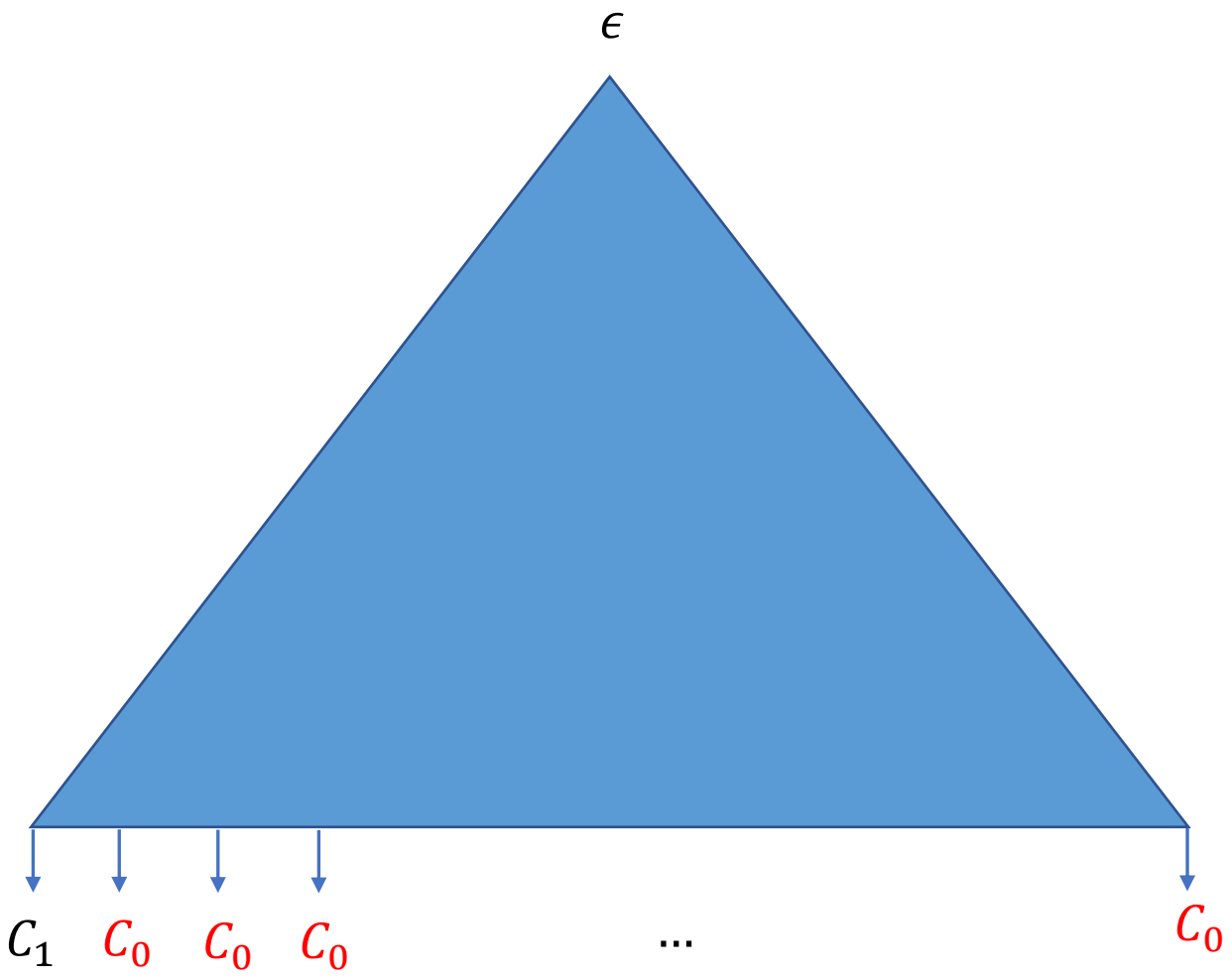

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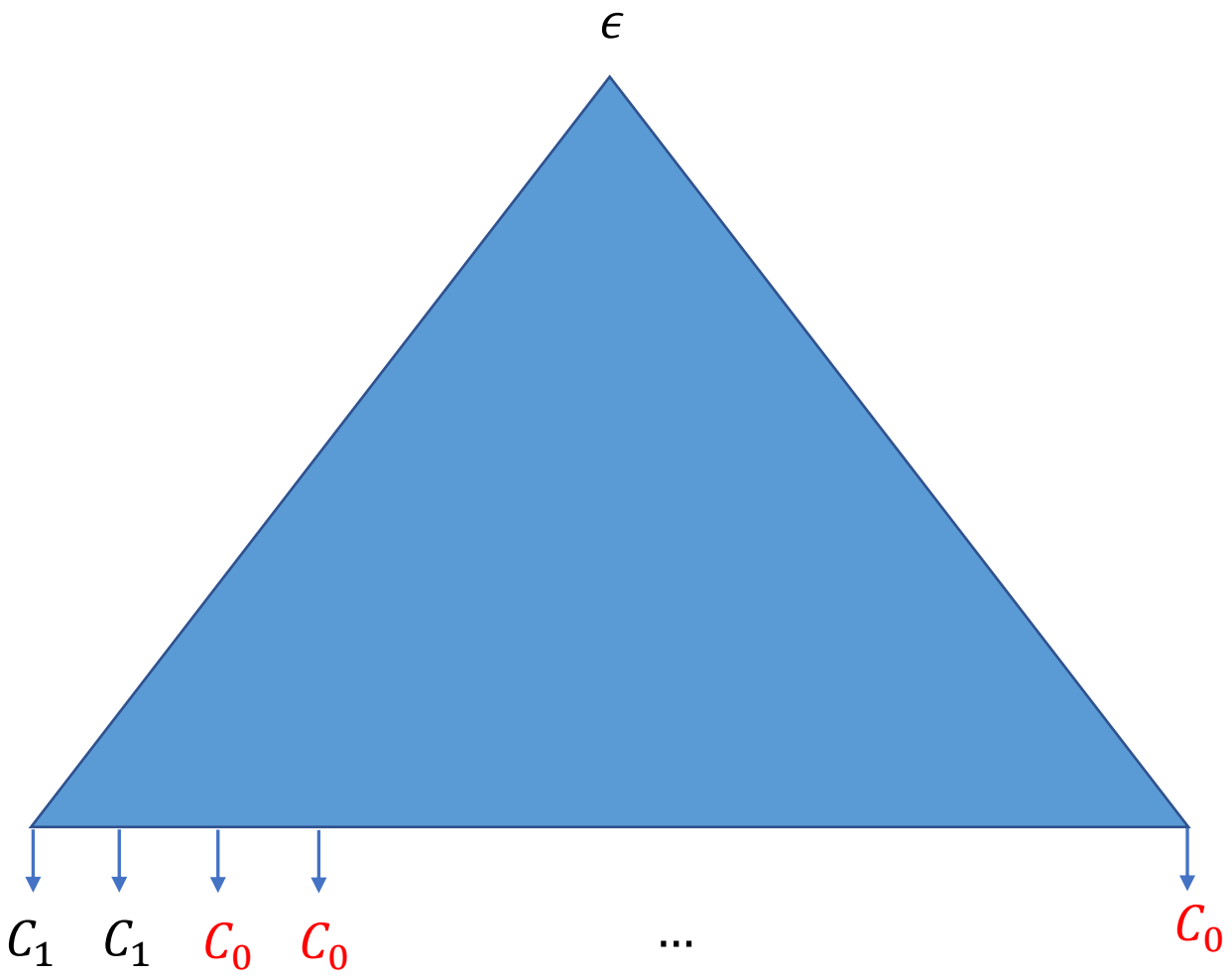

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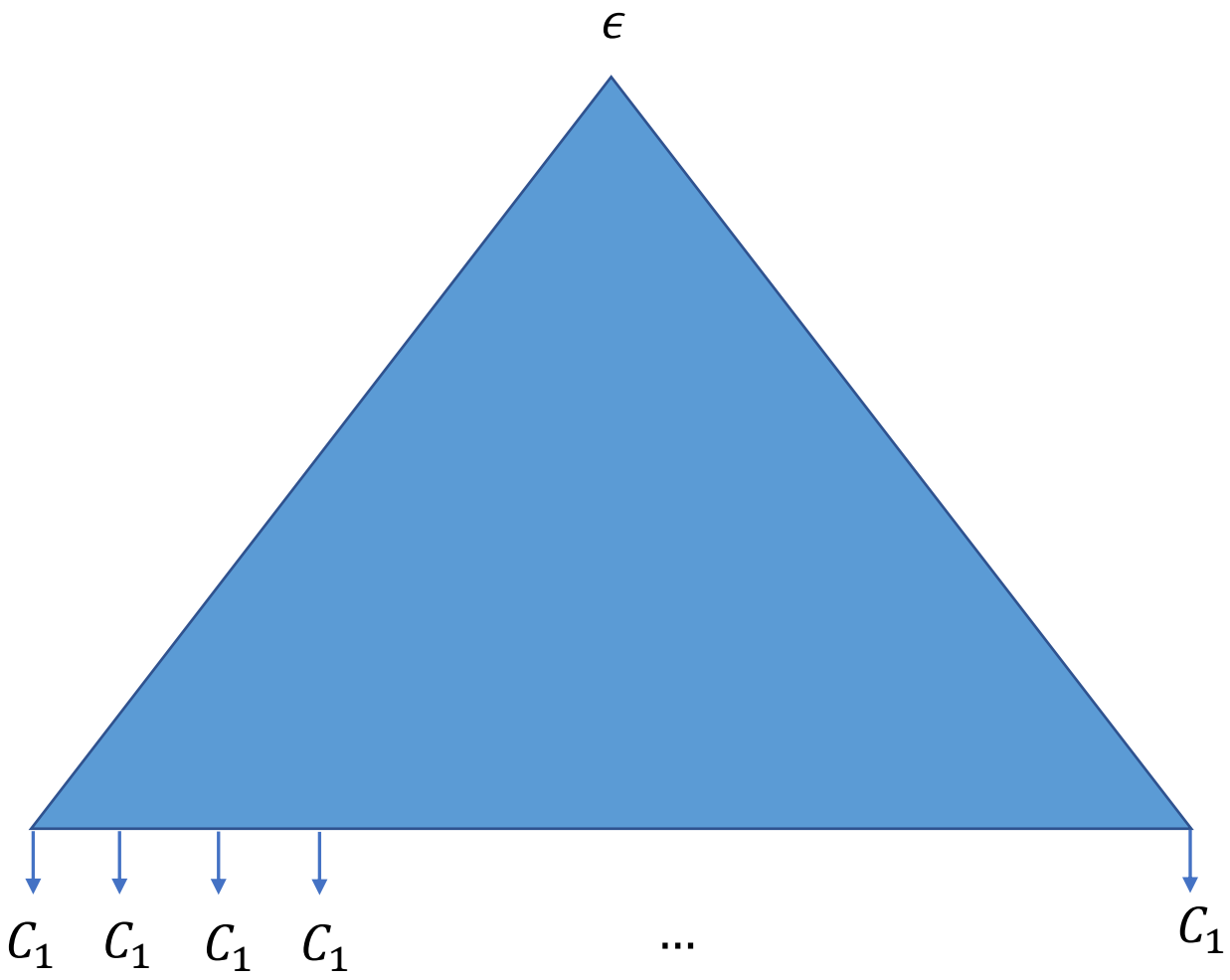

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Insight: Typical circuits encountered in iO proofs have similar structure.

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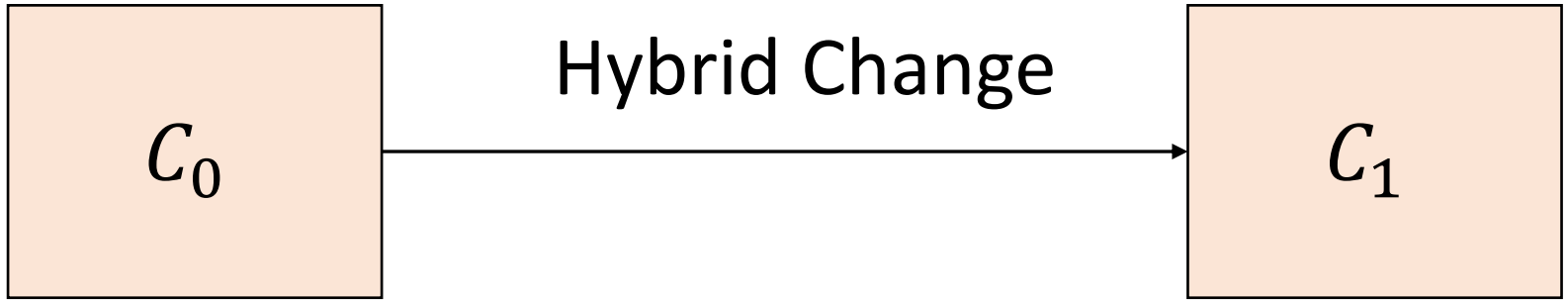

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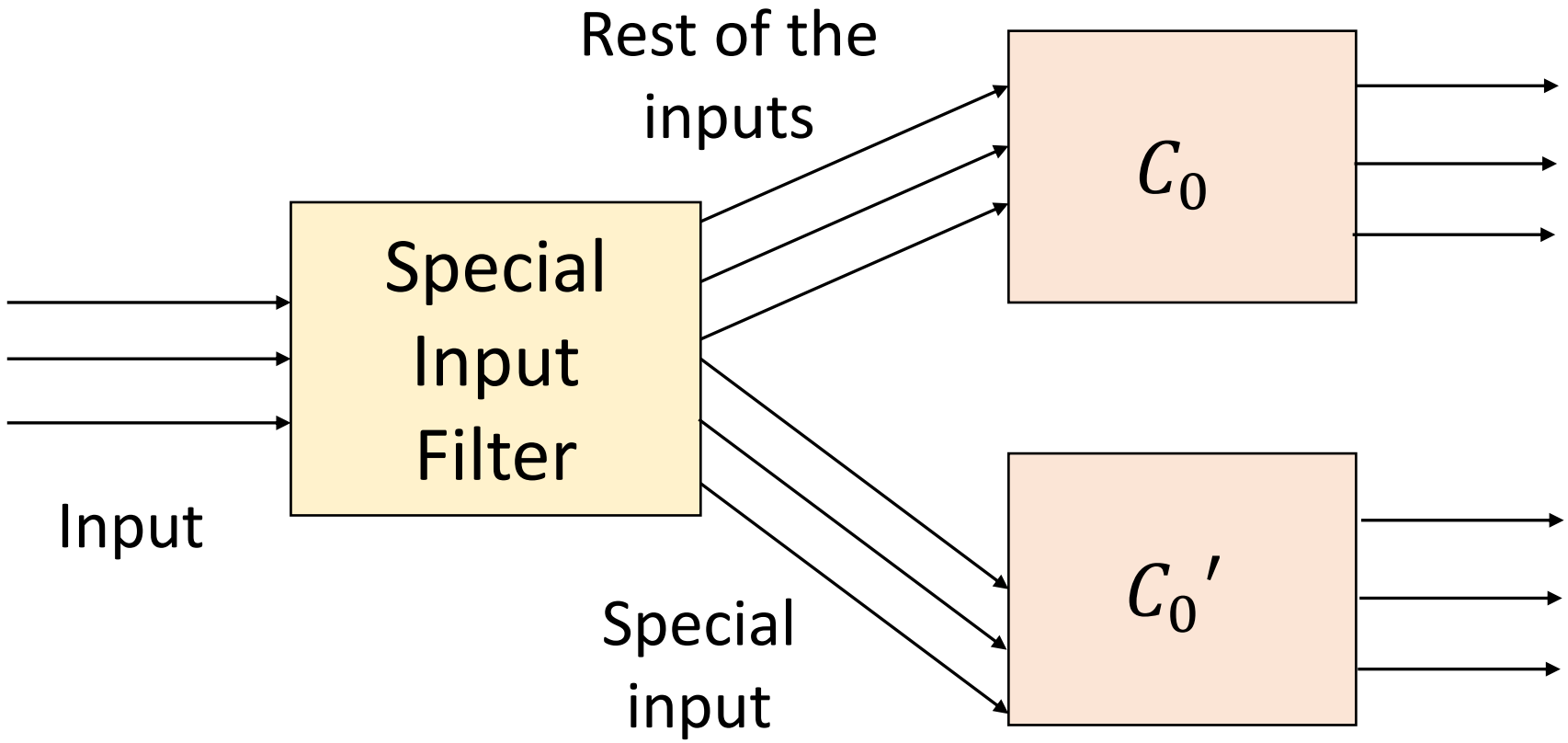
Structure of C_1



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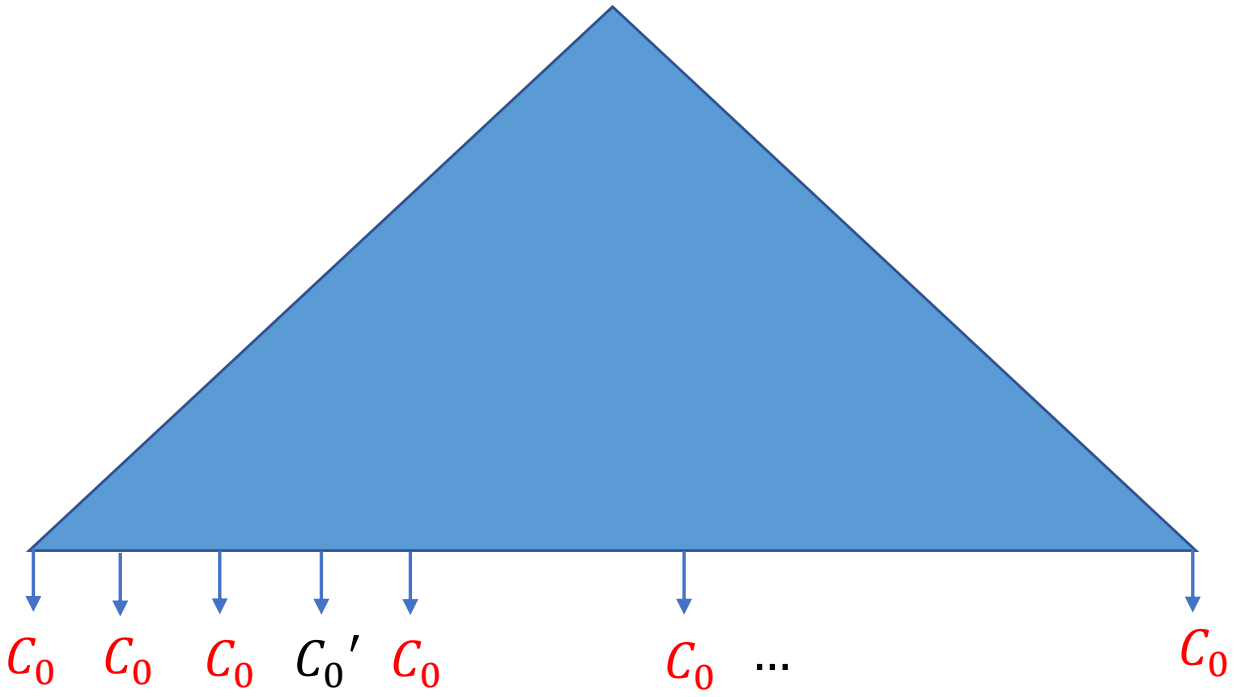

Non-Interactive Key Exchange

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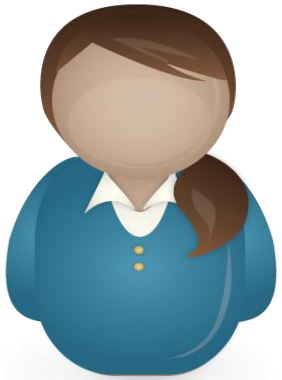
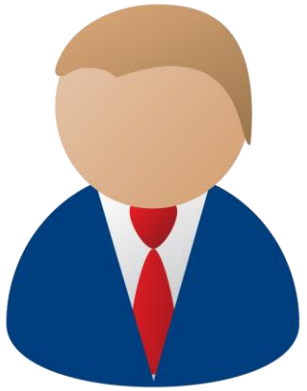
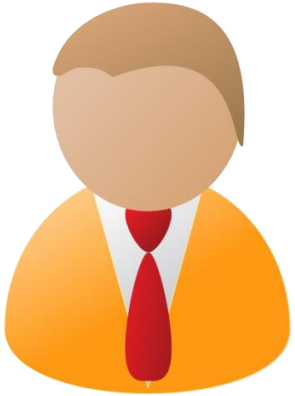
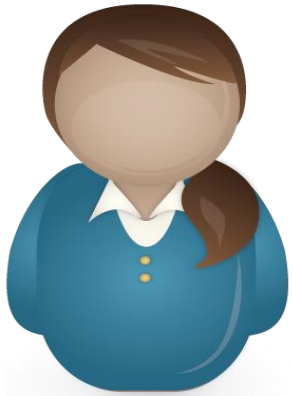


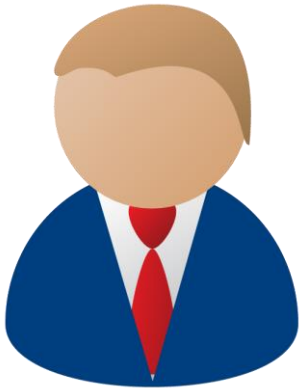


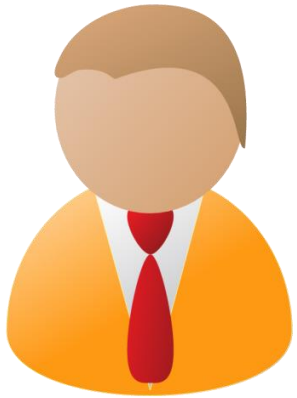


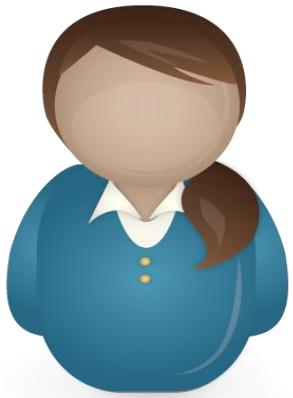


Non-Interactive Key Exchange

Public Bulletin Board







Non-Interactive Key Exchange

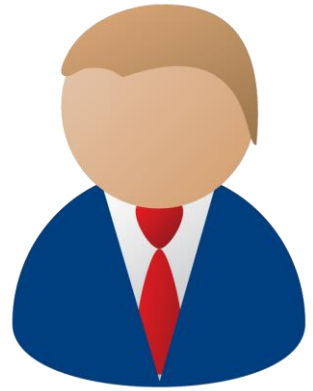
sk_1

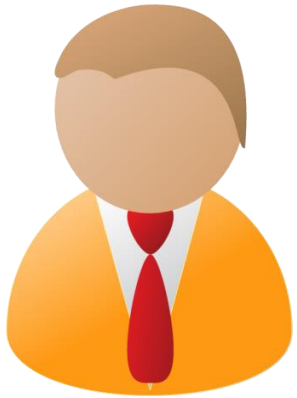
Public Bulletin Board

pk_1 pk_2

pk_4 pk_3

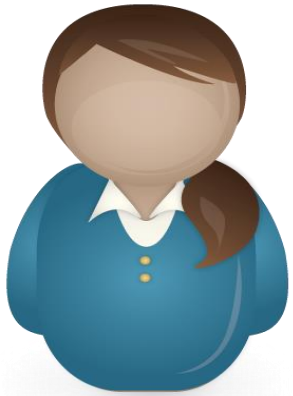
sk_2





sk_4

sk_3



Non-Interactive Key Exchange

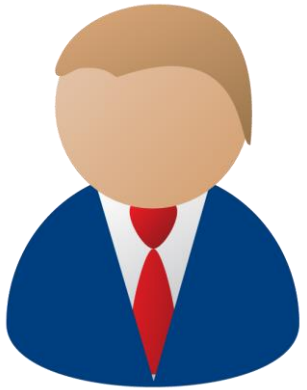
sk_1

K

Public Bulletin Board

pk_1 pk_2

pk_4 pk_3



sk_2

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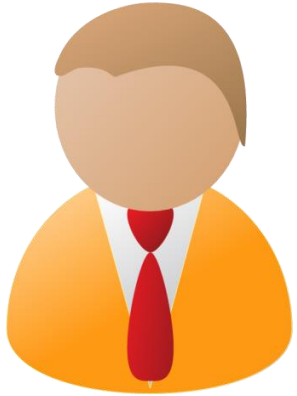

sk_4

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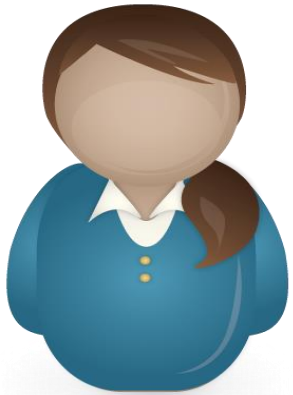


sk_3

K

Boneh-Zhandy NIKE from iO

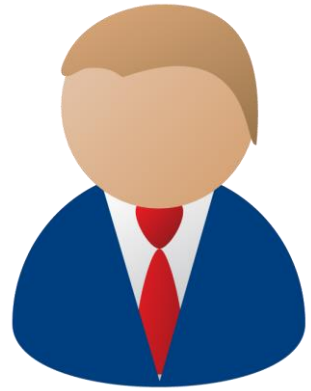
sk_1

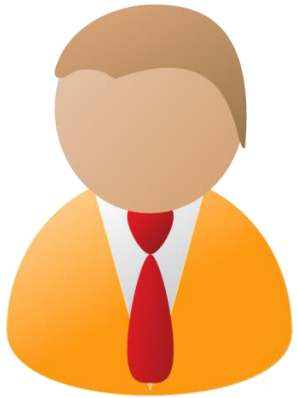
Public Bulletin Board

pk_1 pk_2

pk_4 pk_3

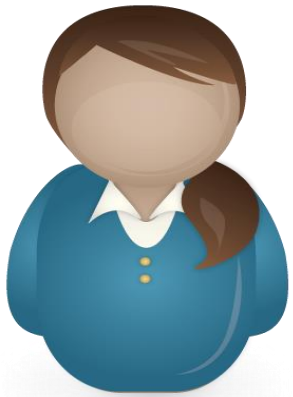
sk_2





sk_4

sk_3



Boneh-Zhandy NIKE from iO

sk_1

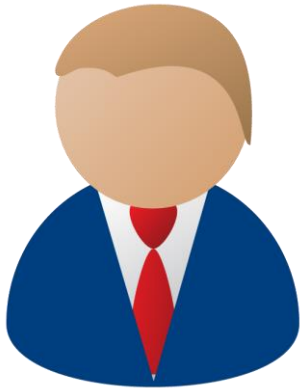
$K = PRF_S(pk_1|| \ \ldots||pk_4)$

Public Bulletin Board

pk_1 pk_2

pk_4 pk_3

sk_2



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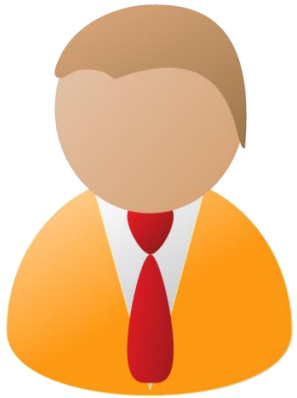
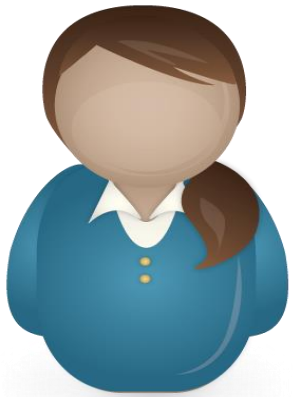


sk_4

$K = PRF_S(pk_1|| \ \ldots||pk_4)$

sk_3



$K = PRF_S(pk_1|| \ \ldots||pk_4)$

Introduce trusted Party

Program P

Input: pk_1, pk_2, pk_3, pk_4

Constants: A PRF key S

- Compute $K = PRF_S(pk_1 || \dots || pk_4)$.
- Output $\{PKE.Enc_{pk_i}(K)\}_i$

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Proof of Security: At a high level

Program P

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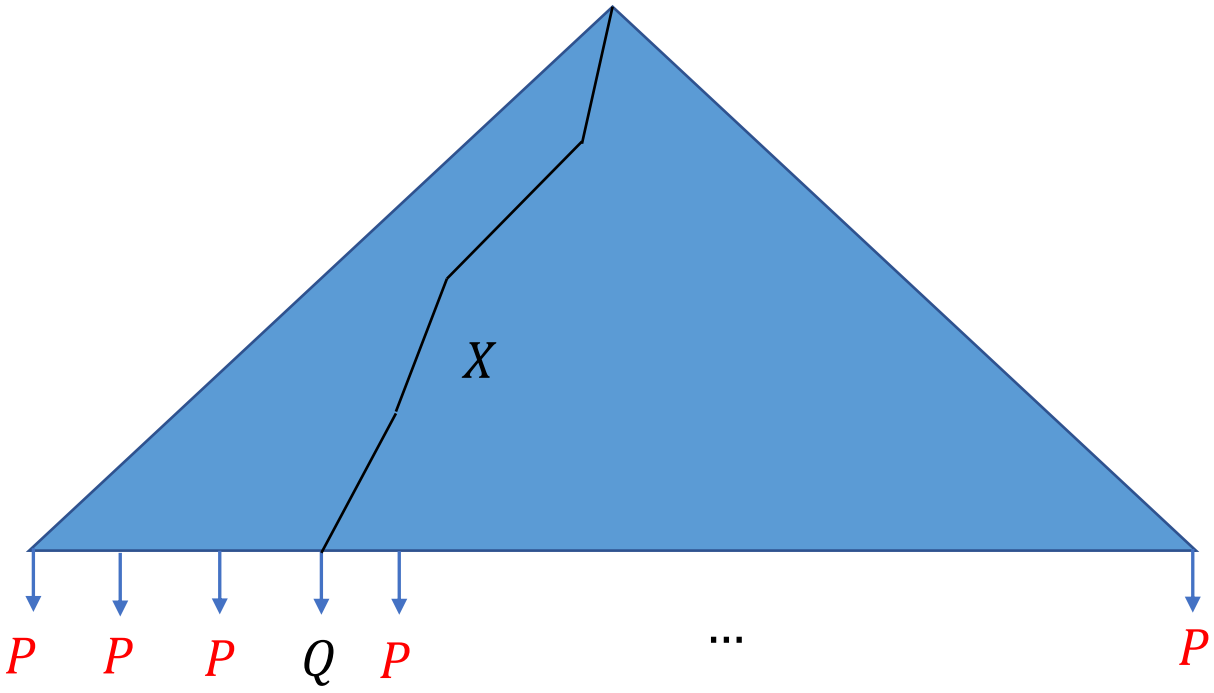
Constants: A PRF key S
Compute
$K = PRF_S(pk_1 || \ldots || pk_4)$.
Output $\{PKE.Enc_{pk_i}(K)\}_i$

Program P'

Input: pk_1, pk_2, pk_3, pk_4
Constants: A PRF key S, X, Z
If($\boldsymbol{pk_1} || \ldots || \boldsymbol{pk_4}) = \boldsymbol{X}$) then
output $\boldsymbol{Z}$
Else, compute
$K = PRF_S(pk_1 || \ldots || pk_4)$.
Output $\{PKE.Enc_{pk_i}(K)\}_i$

Applying our technique



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- Not all applications can be obtained! Ex. NIZK (Sahai-Waters)
- Follow-up work by Liu-Zhandry provides a simple interface for constructing applications from polynomial hardness assumptions.

Thank you!