Removing the Strong RSA Assumption from Arguments over the Integers

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European Research Council

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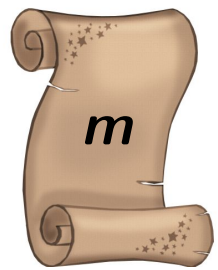
May 2, 2017

Commitment Schemes over Groups of Unknown Order



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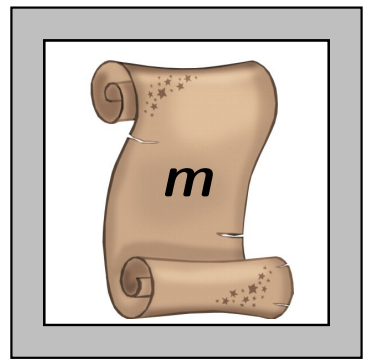

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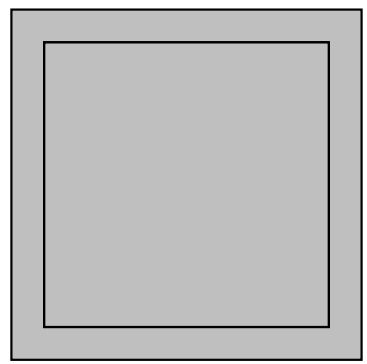

Hiding

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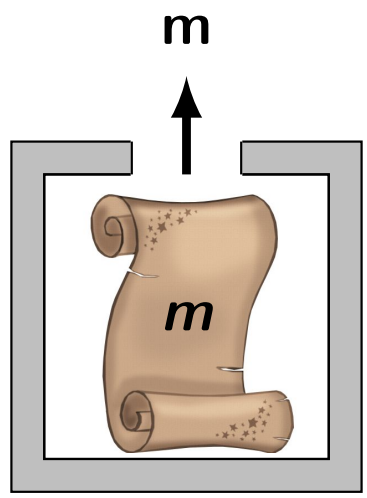

Binding

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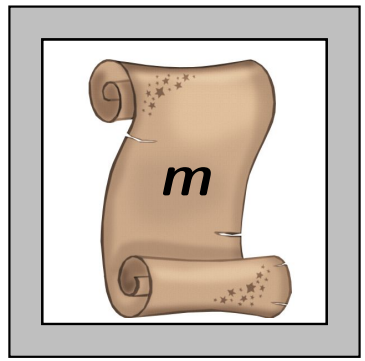

Fujisaki-Okamoto (1997):

$m \in \mathbb{G}$, $|\mathbb{G}|$ unknown

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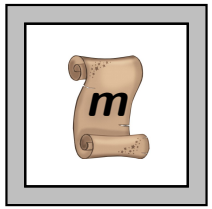

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Perfectly hiding, binding under Factorization

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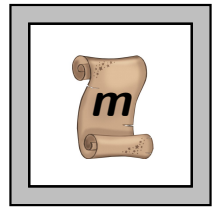

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Anonymous Credentials | MPC | E-Cash | E-Voting

Group Sig. | Range Proofs | Auctions | PPSS

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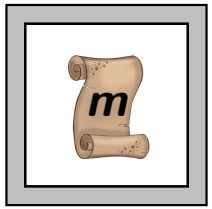

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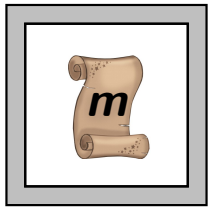

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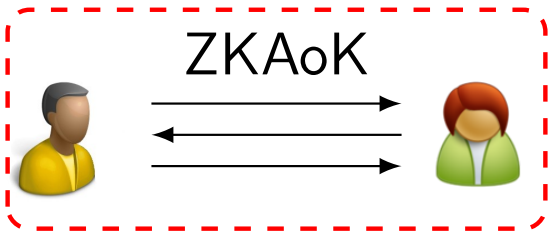

Strong-RSA

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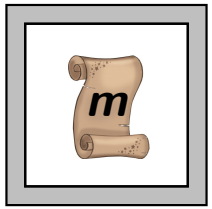

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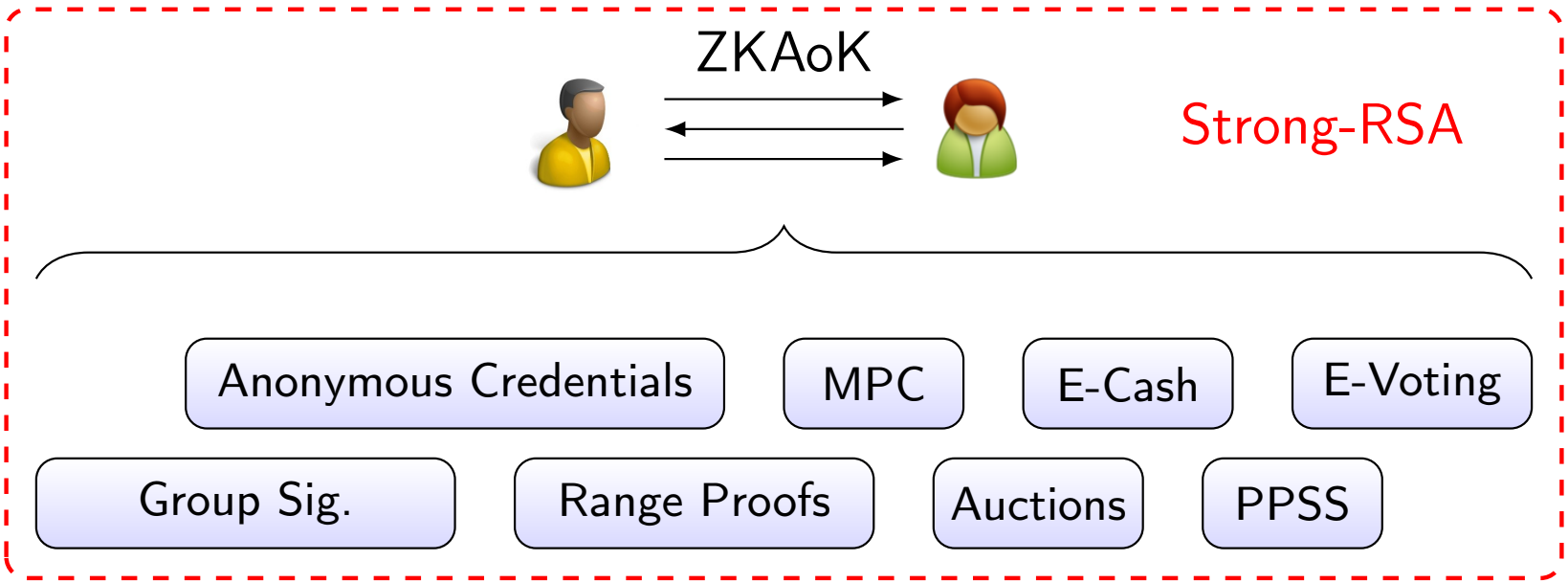

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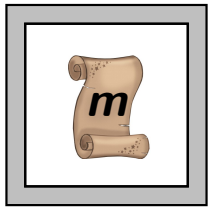

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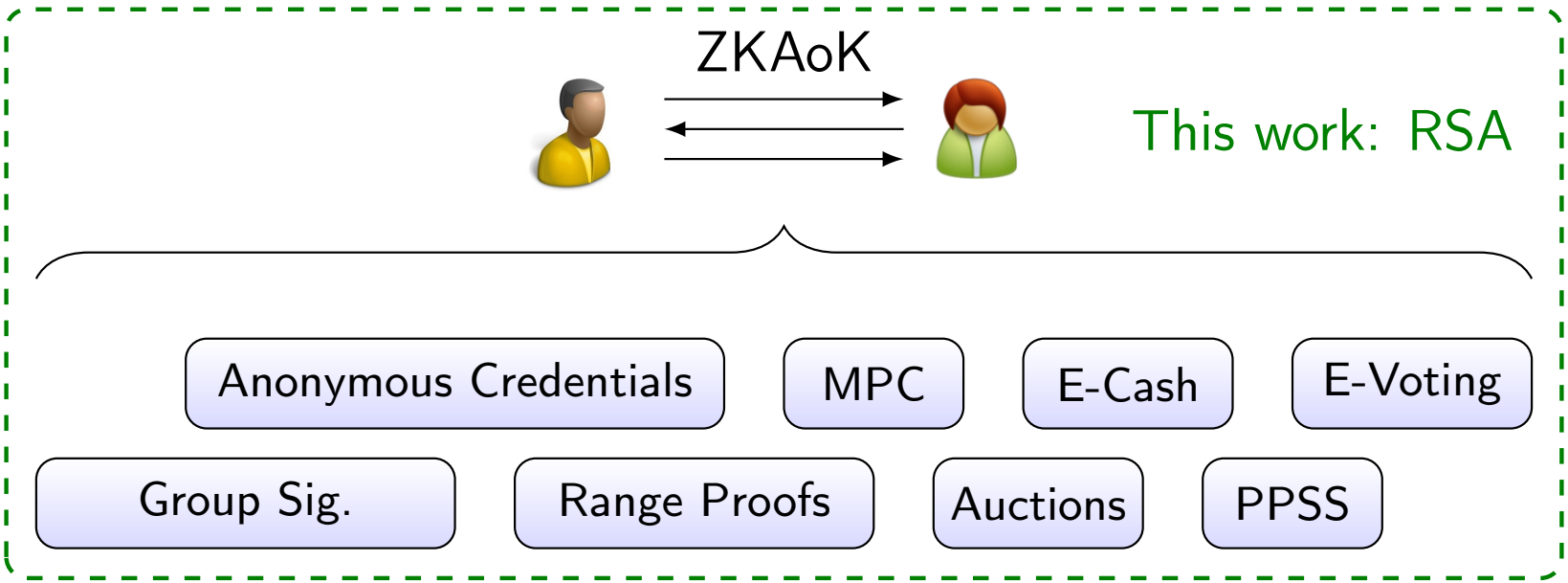
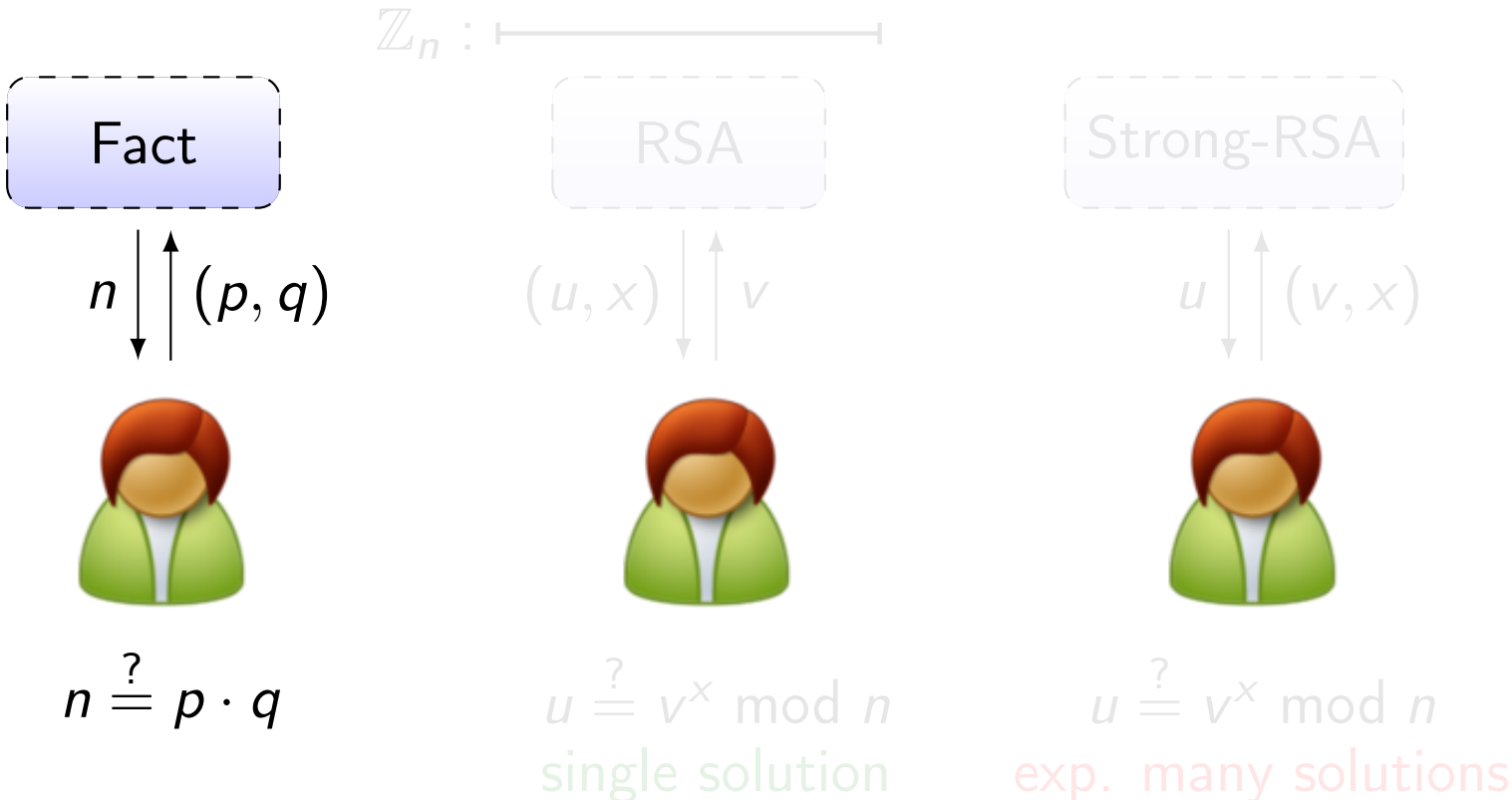


Preliminaries on RSA Groups

$\mathbb{Z}_n$, with $n = pq$, $p = 2p' + 1$, and $q = 2q' + 1$.

$$|\mathrm{QR}[n]| = \frac{(p-1)(q-1)}{4} = p'q'$$



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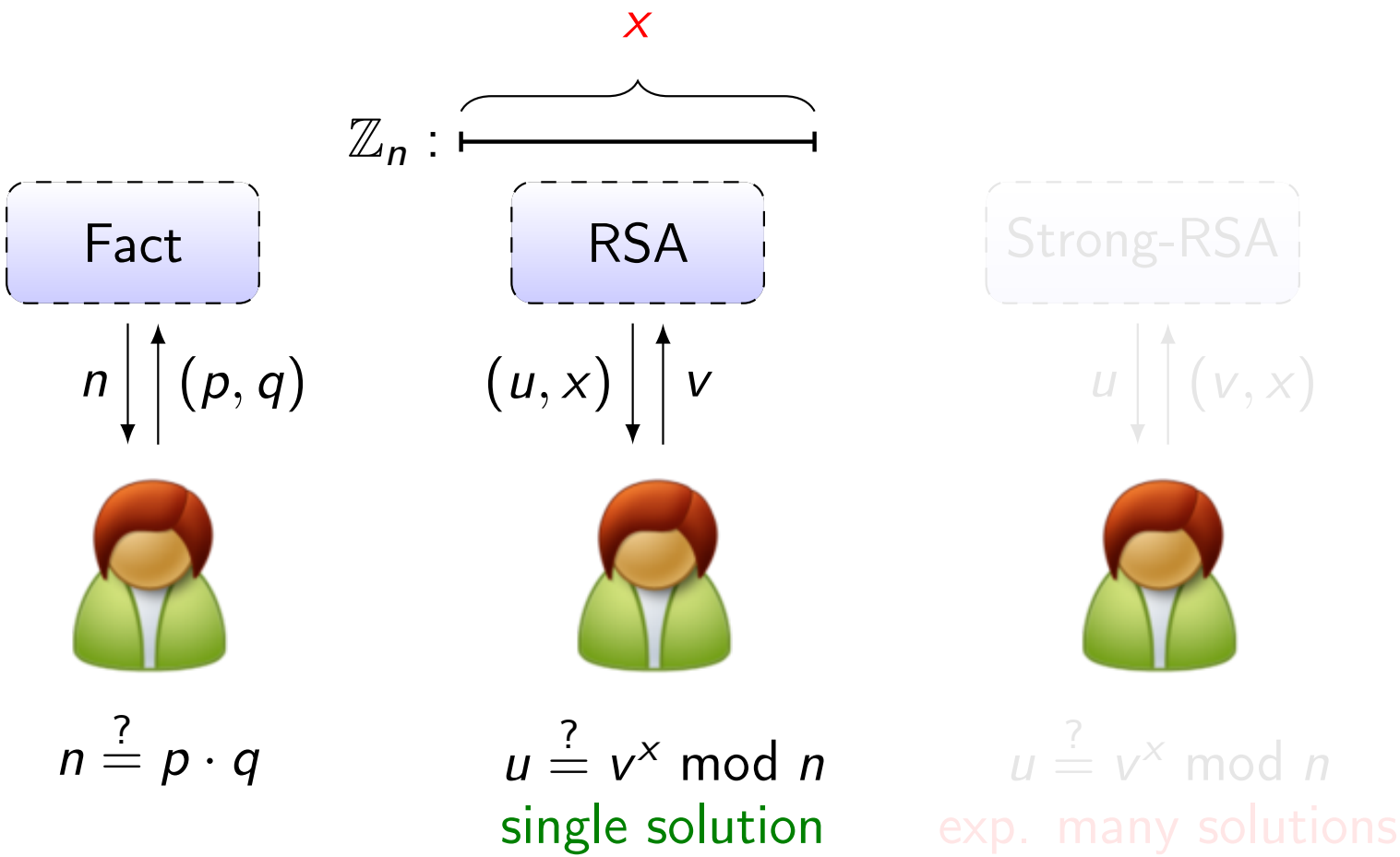

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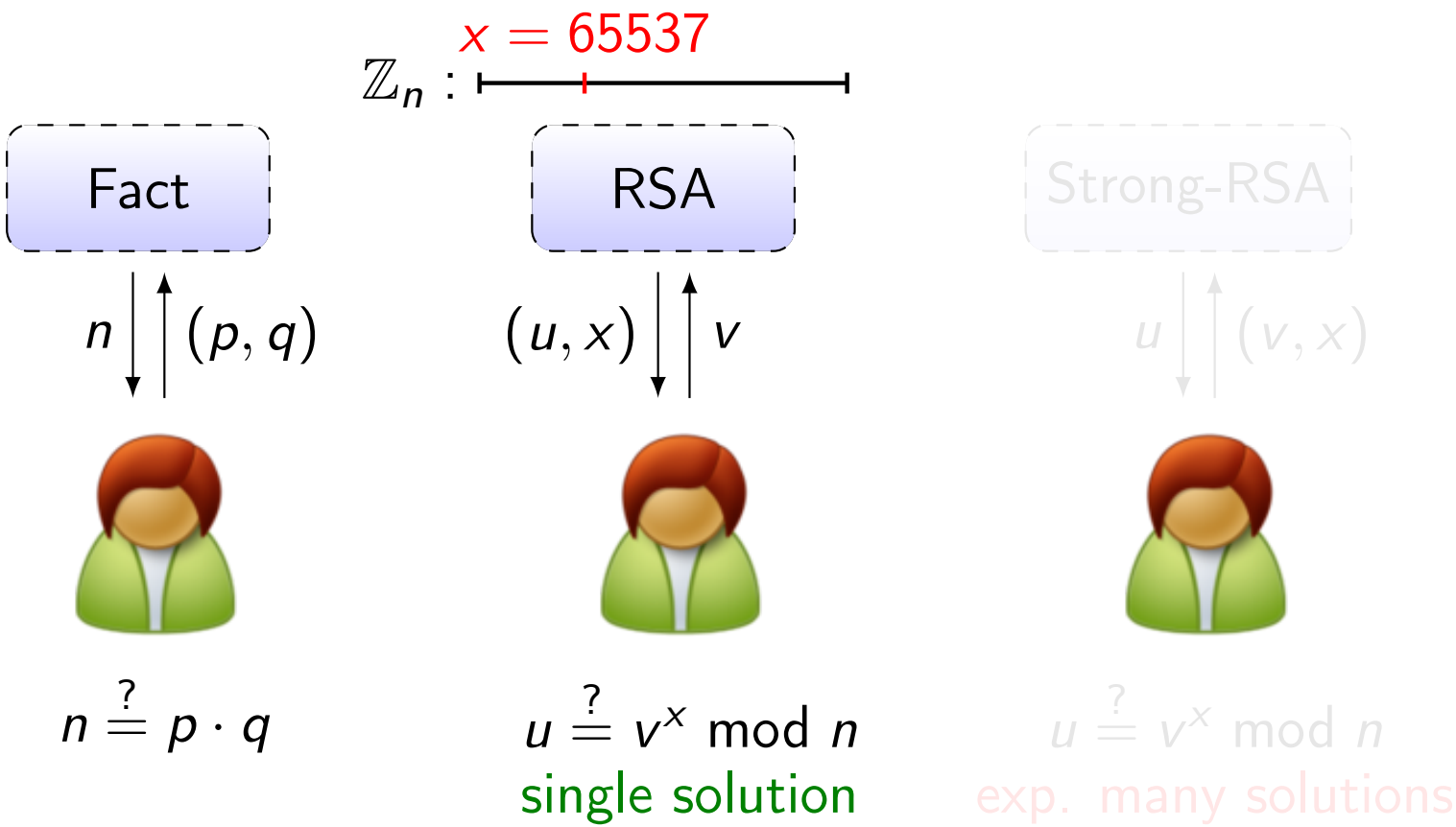

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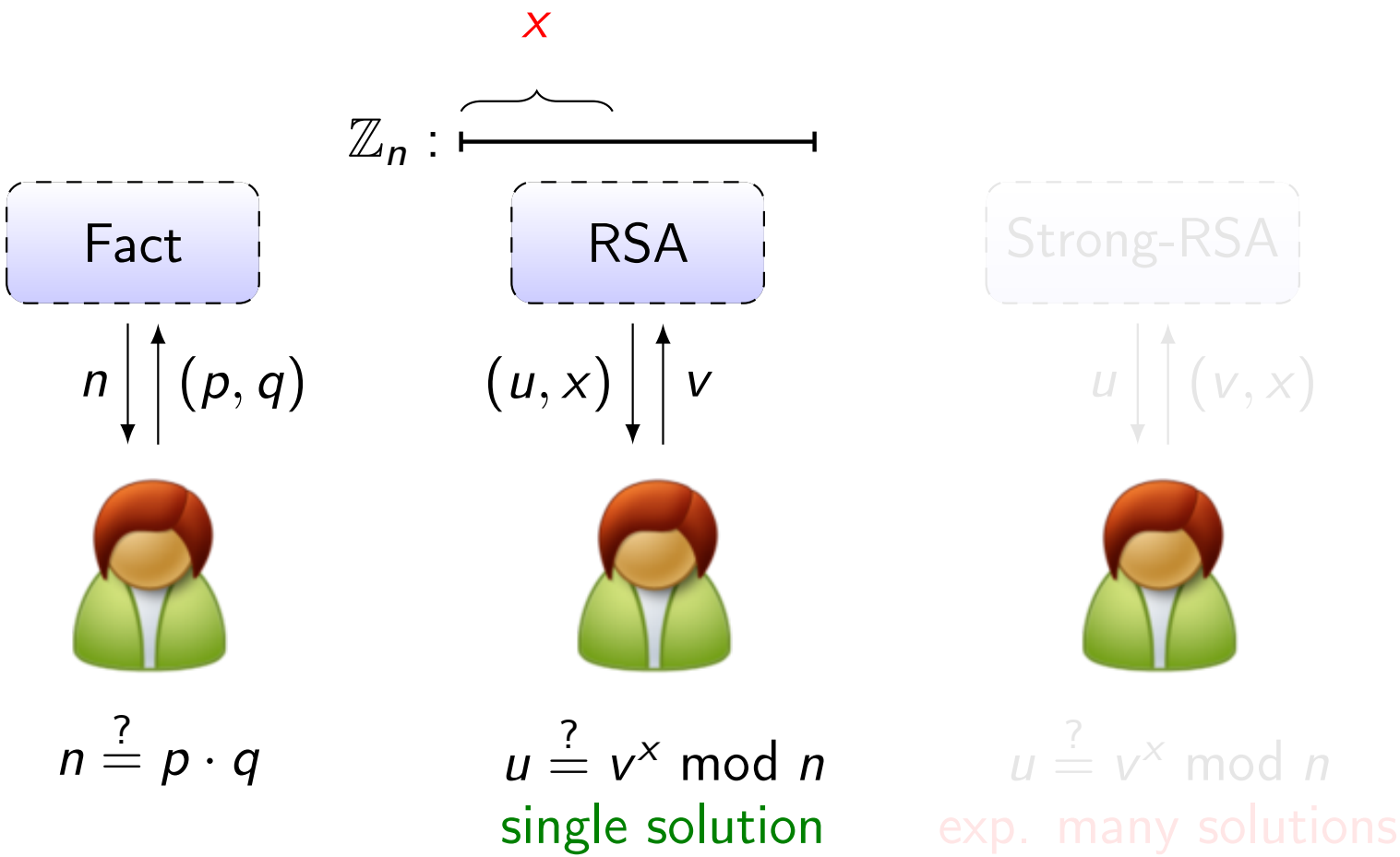

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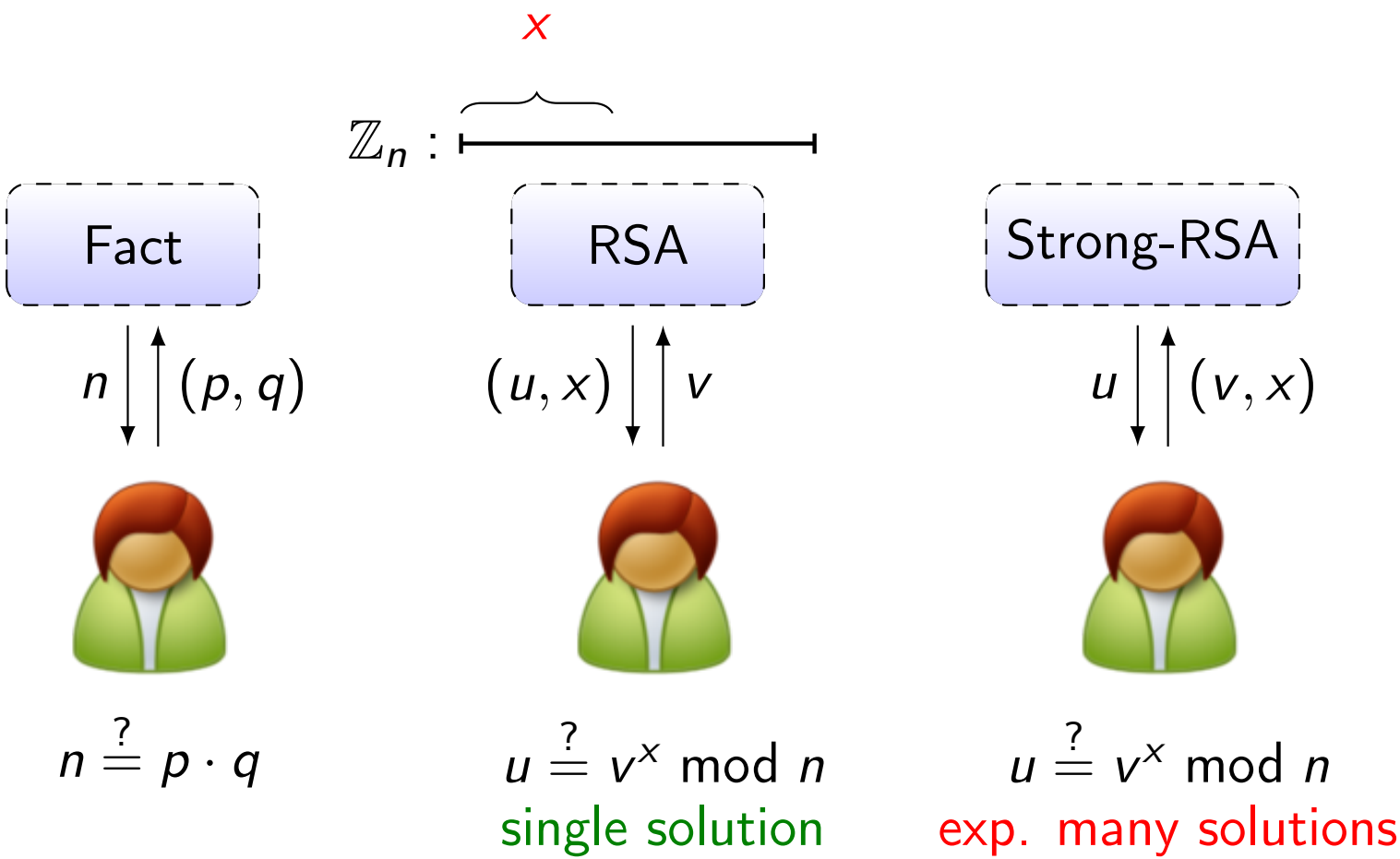
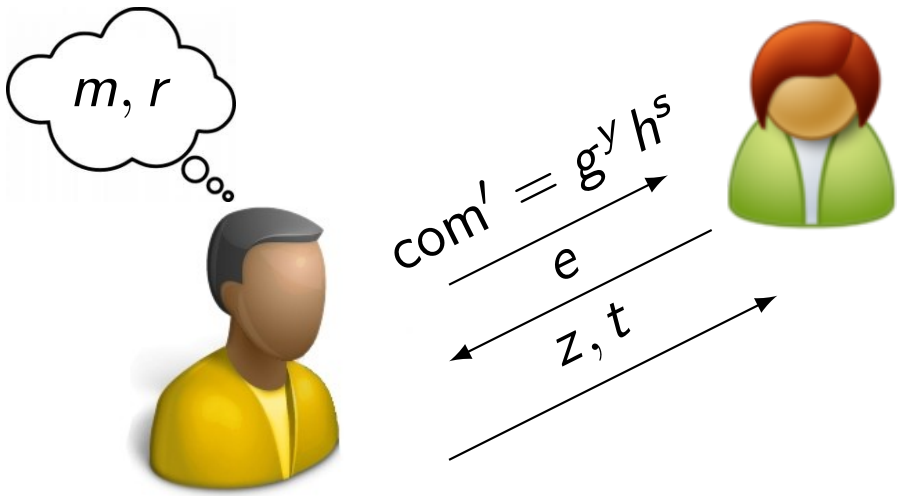


Zero-Knowledge Argument of Knowledge of an Opening

$n = p \cdot q$, $\langle g \rangle = \mathrm{QR}[n]$, $h^\alpha = g$

$$\boxed{\mathrm{com} = g^m h^r}$$



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V checks whether $\mathrm{com}^e \mathrm{com}' = g^z h^t$.

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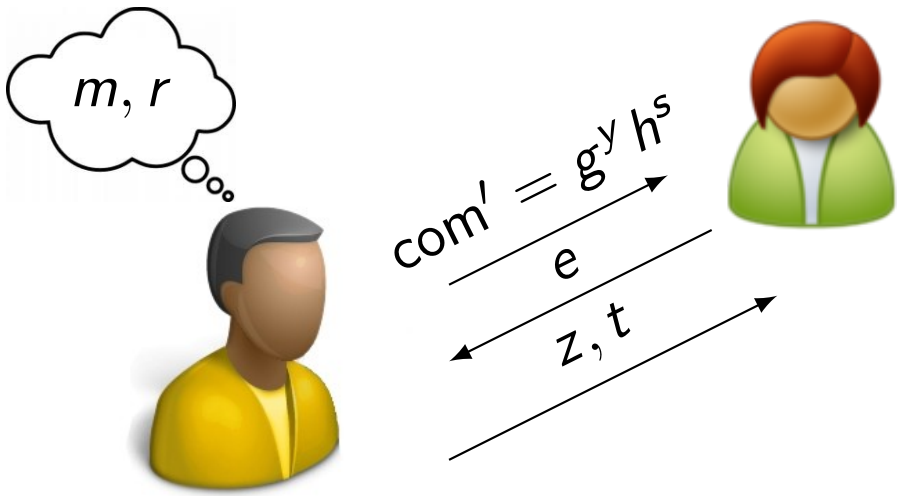

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Soundness. With rewinding, extract $(m, r) = \left(\frac{z_0 - z_1}{e_0 - e_1}, \frac{t_0 - t_1}{e_0 - e_1} \right)$

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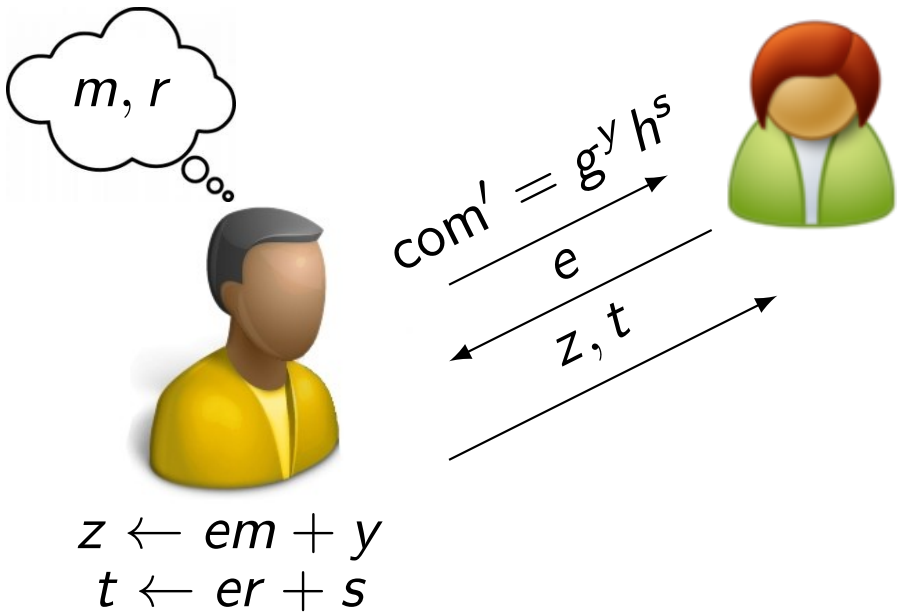

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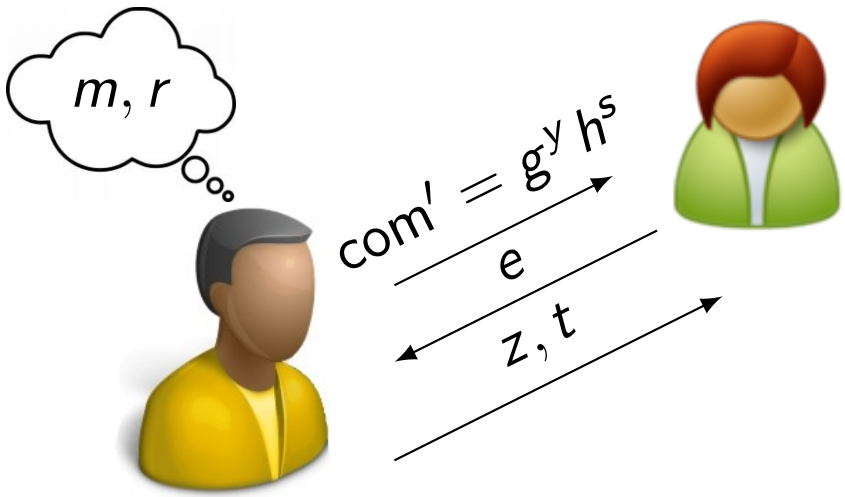
Requires inversions over the exponents of $\mathbb{G}$!

Soundness Argument

$\boxed{\text{com} = g^m h^r}$ $g = h^\alpha$



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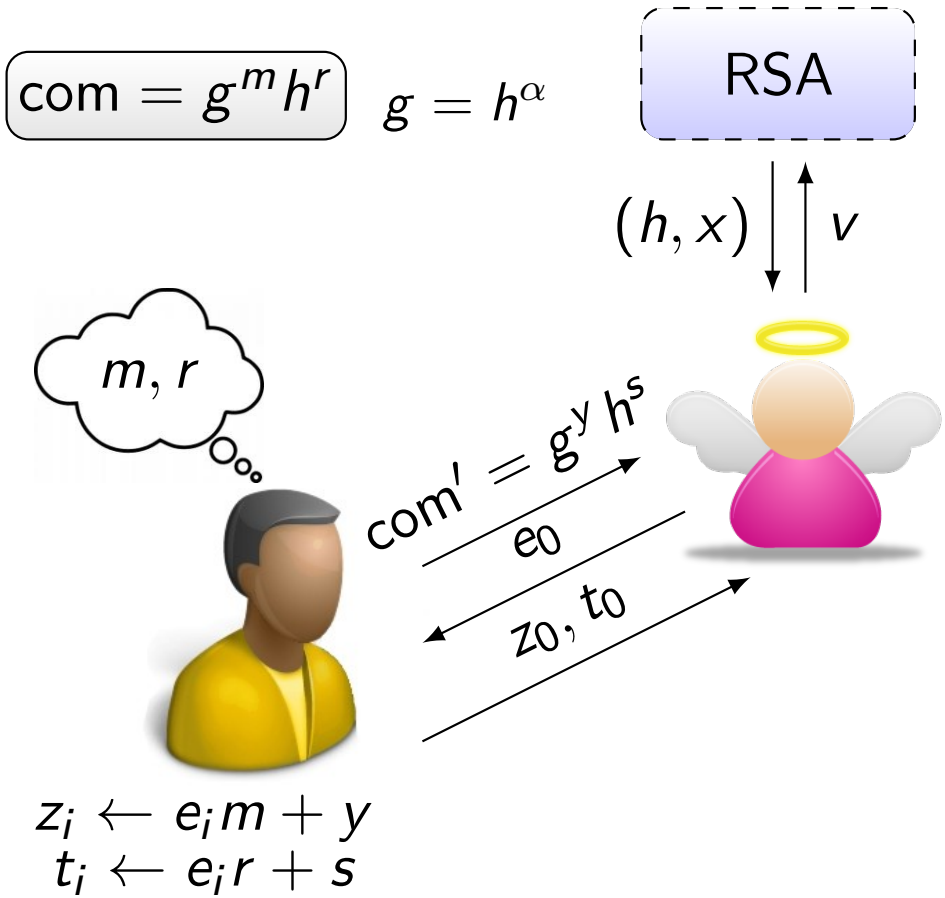

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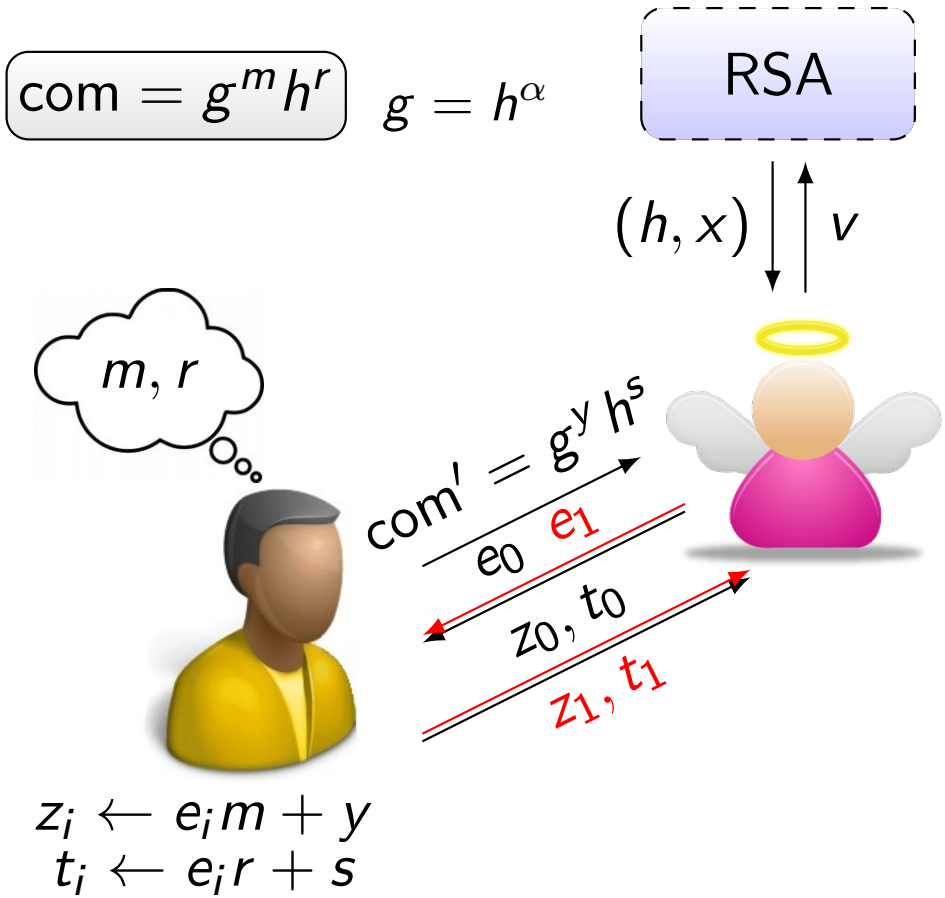
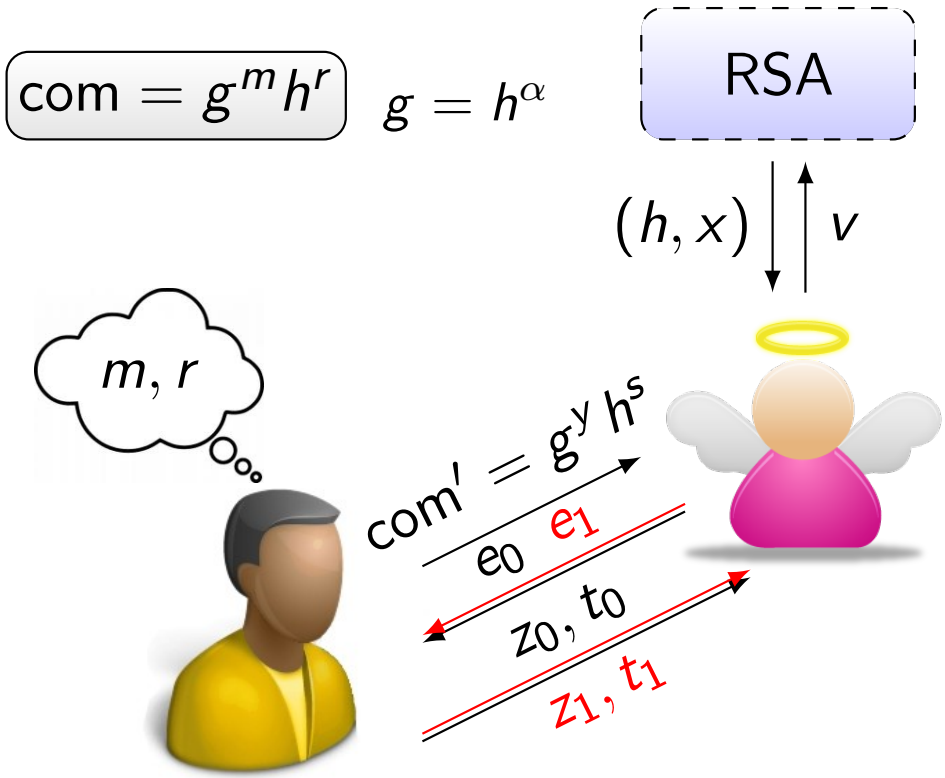


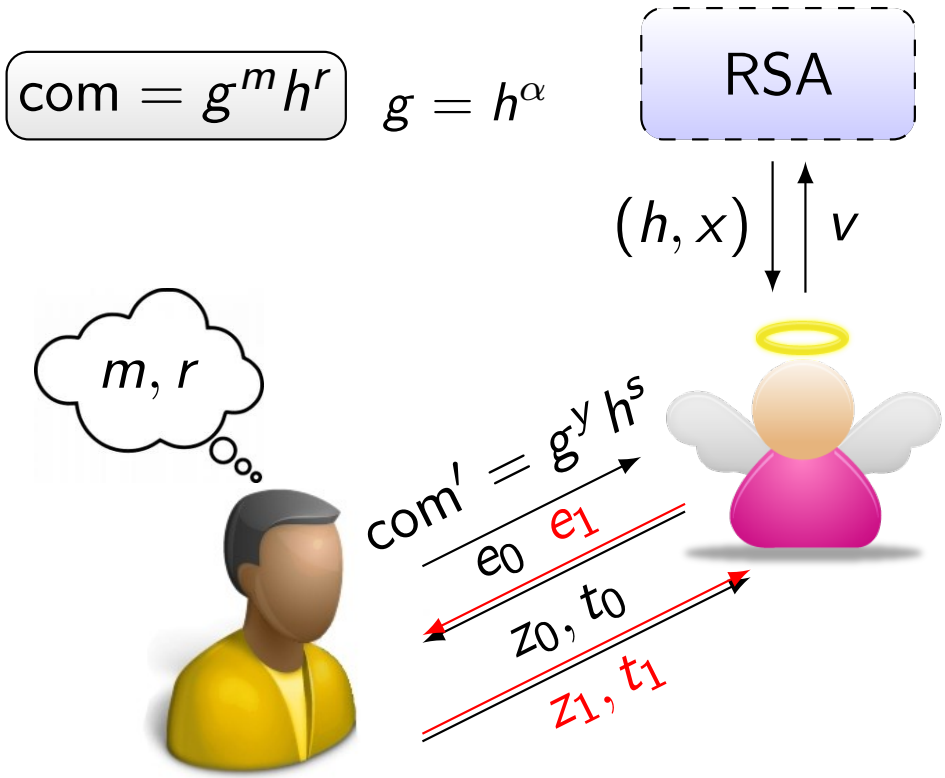
Rewind P w/ (e_0, e_1);
with pr. ε^2,
$\mathrm{com}^{e_0 - e_1} = g^{z_0 - z_1} h^{t_0 - t_1}$

Soundness Argument



Rewind P w/ (e_0, e_1); with pr. ε^2, $\text{com}^e = g^z h^t$, but we cannot divide by e!

Soundness Argument



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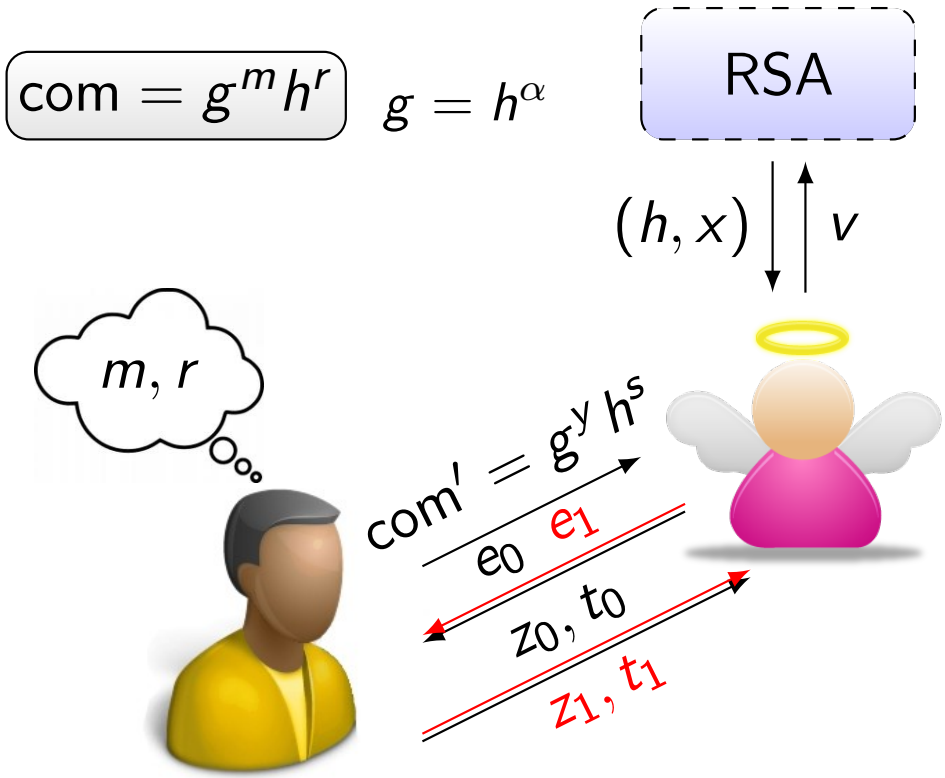
Case 1.

$e \mid z$ and $e \mid t$

Soundness Argument



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Case 1.

$e \mid z$ and $e \mid t$

$$\text{com} = \pm g^{z/e} h^{t/e}$$

Soundness Argument

$\boxed{\text{com} = g^m h^r}$ $g = h^\alpha$

RSA

(h, x) v

Rewind P w/ (e_0, e_1);
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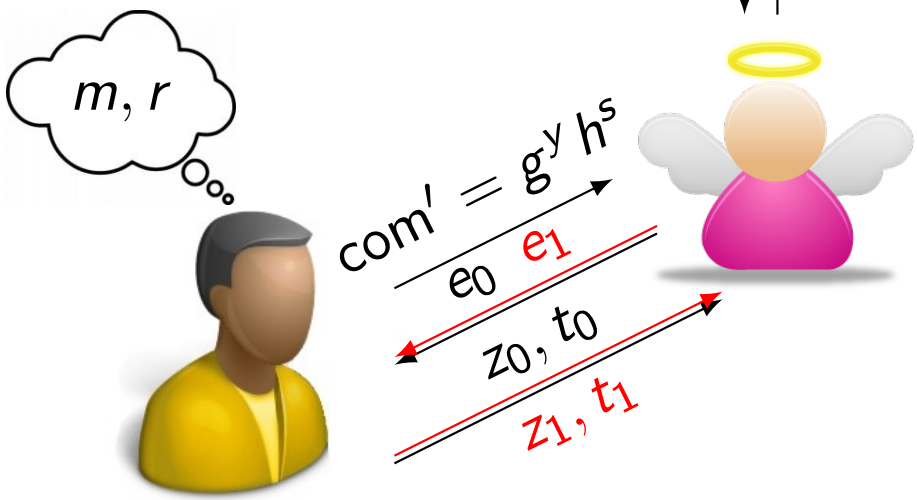

$z_i \leftarrow e_i m + y$
$t_i \leftarrow e_i r + s$

$z = z_0 - z_1, e = e_0 - e_1,$
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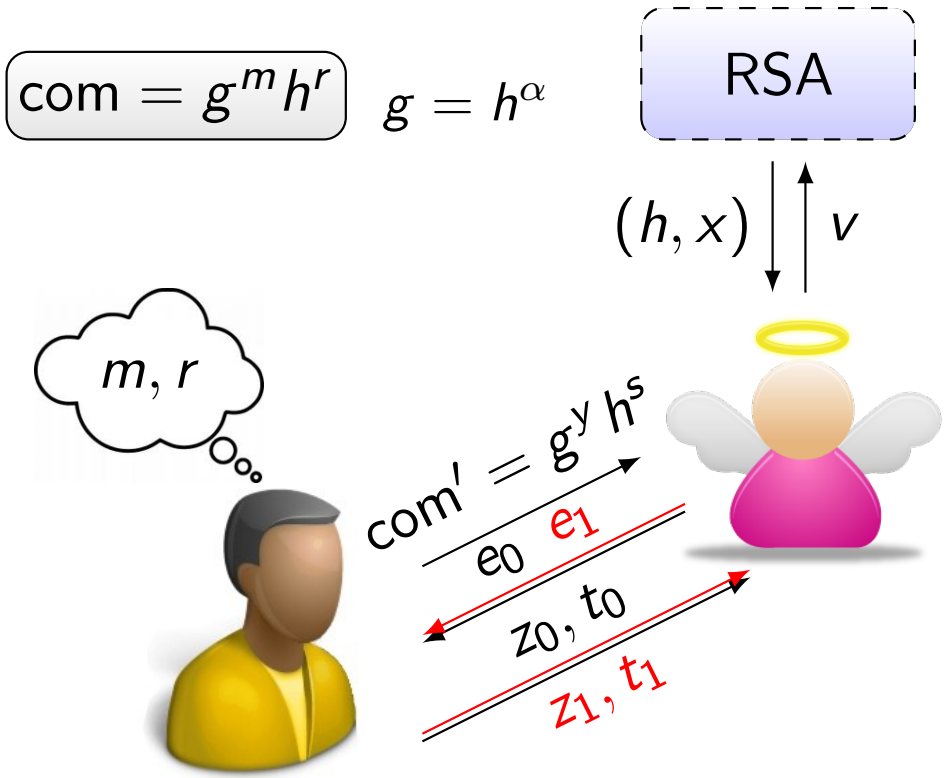
Case 2.

$e \nmid z$ or $e \nmid t$

Soundness Argument



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$e \nmid z$ or $e \nmid t$

[DF02]: With probability $1/2$, $e \nmid \alpha z + t$.

Soundness Argument

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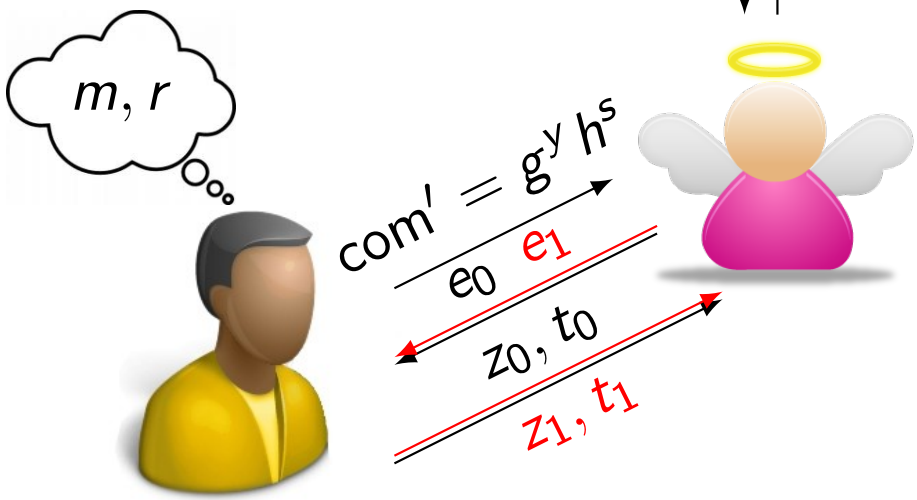

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Case 2.

Shamir's gcd trick:
$e / \gcd(e, \alpha z + t) = \pi$
can find v such that
$v^\pi = \pm h$

Soundness Argument

$\boxed{\text{com} = g^m h^r}$ $g = h^\alpha$

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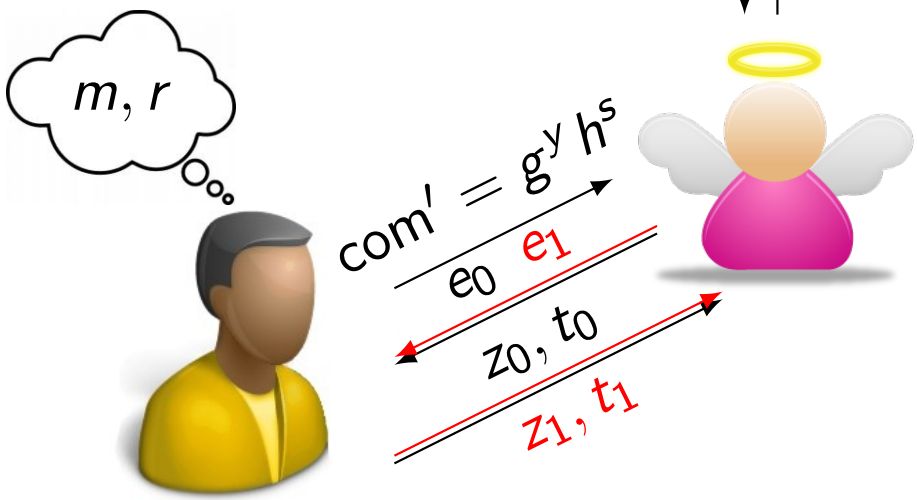

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Case 2.

Solves a Strong RSA challenge w/ π

Soundness Argument

$\boxed{\text{com} = g^m h^r}$ $g = h^\alpha$

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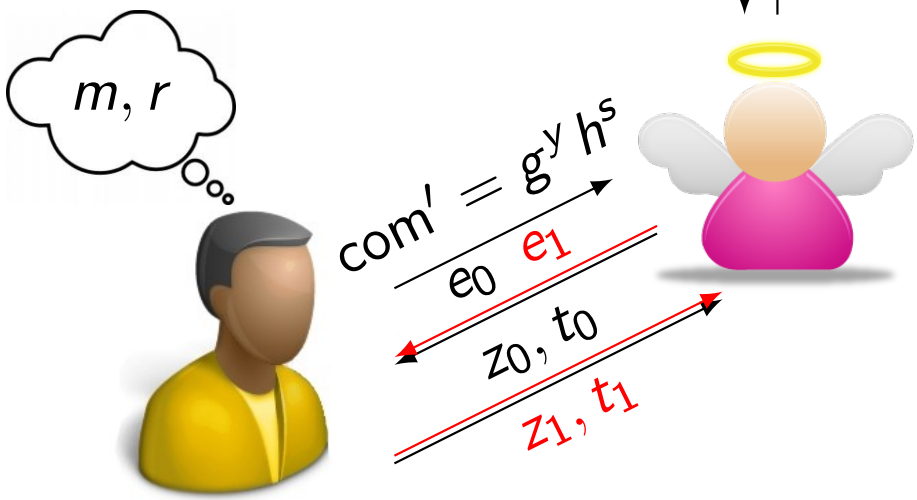

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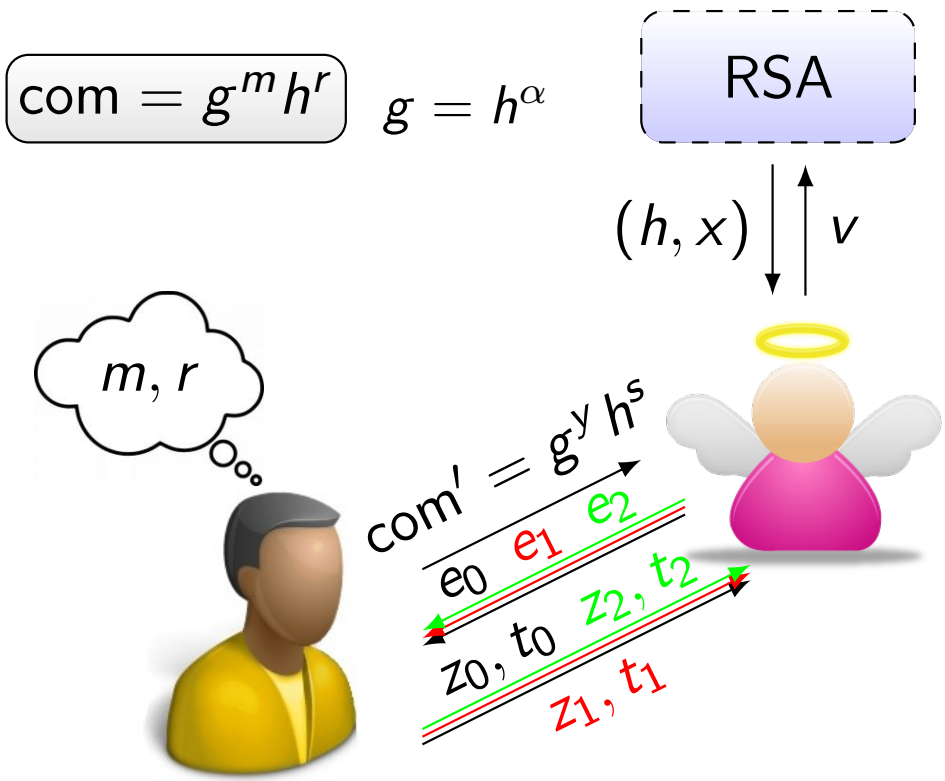
Case 2.

Core observation:
π can't be too large.

Soundness Argument

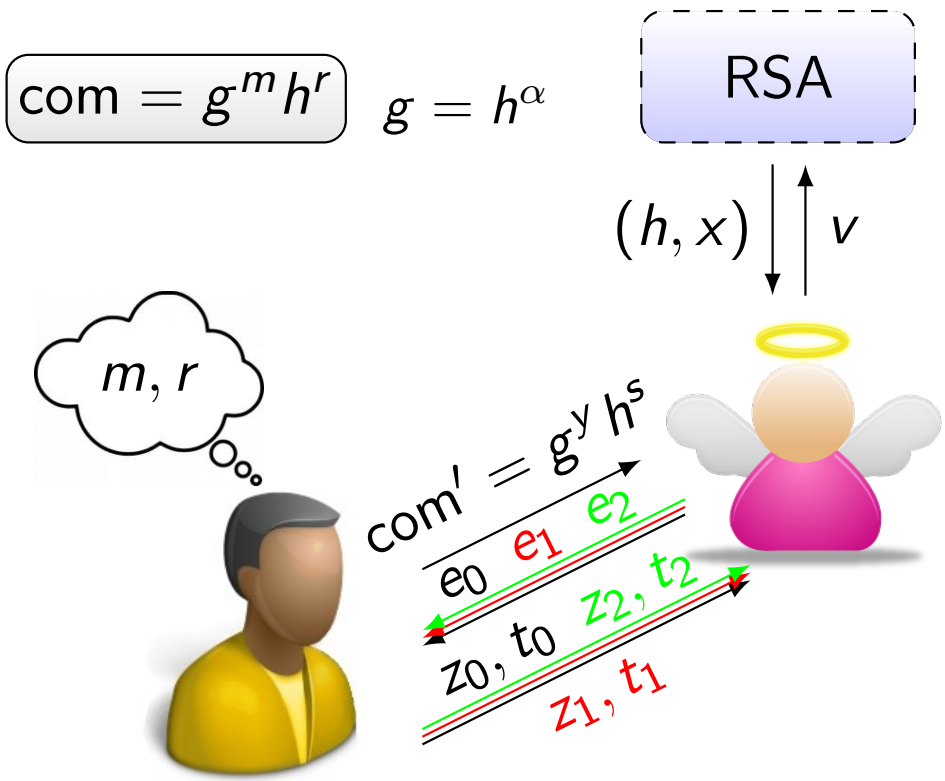


Rewind P w/ (e_0, e_1, e_2);
with pr. ε^3,
$\text{com}^e = g^z h^t$,
$\text{com}^{e'} = g^{z'} h^{t'}$
$\rightarrow g^a = h^b$

Case 2.

Suppose $\pi > 8/\varepsilon$

Soundness Argument



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Suppose $\pi > 8/\varepsilon$
$g^a = h^b$ factors n unless
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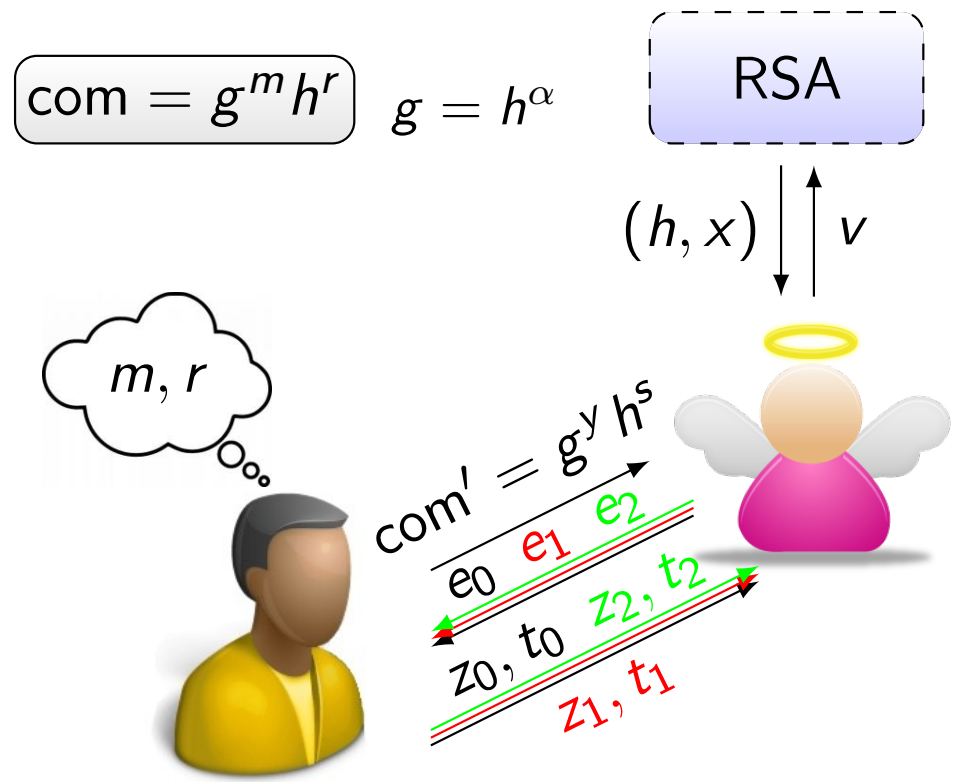
Soundness Argument



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π' divides e', e' is random
$\Pr[\pi = \pi'] \leq \Pr[\pi \text{ divides } e'] = O(\varepsilon)$

Rewind P w/ (e_0, e_1, e_2);
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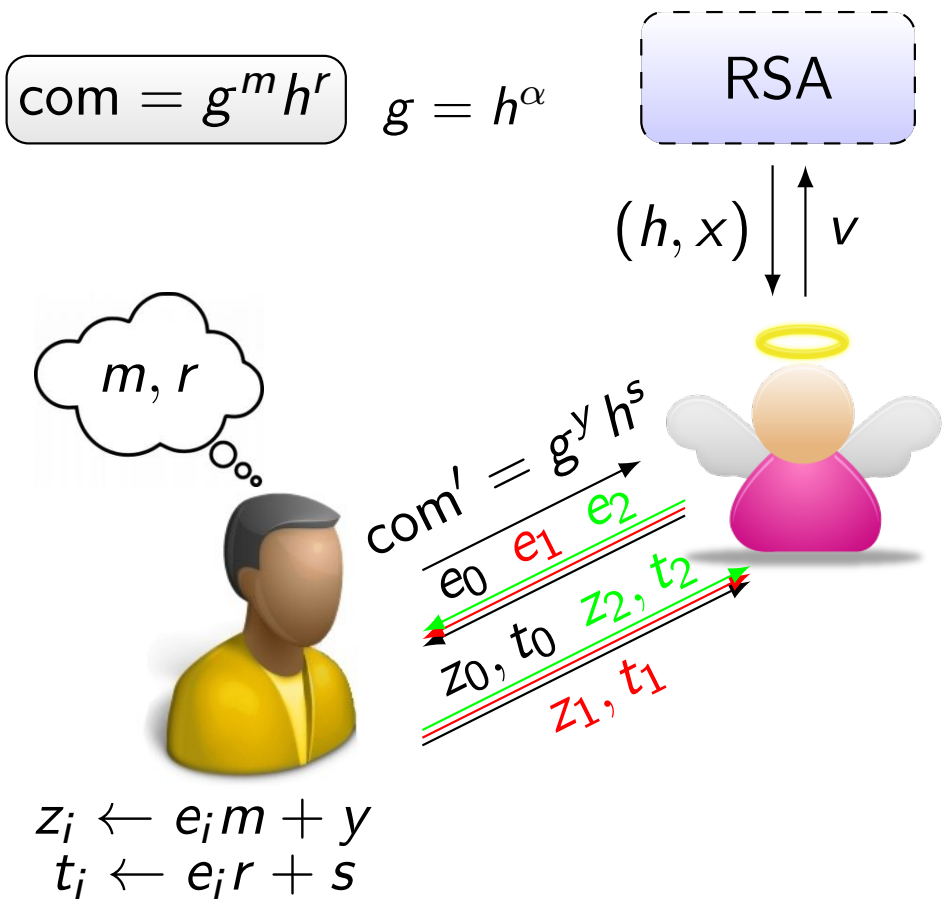

Factors n with $\frac{1}{\text{poly}}$ probability

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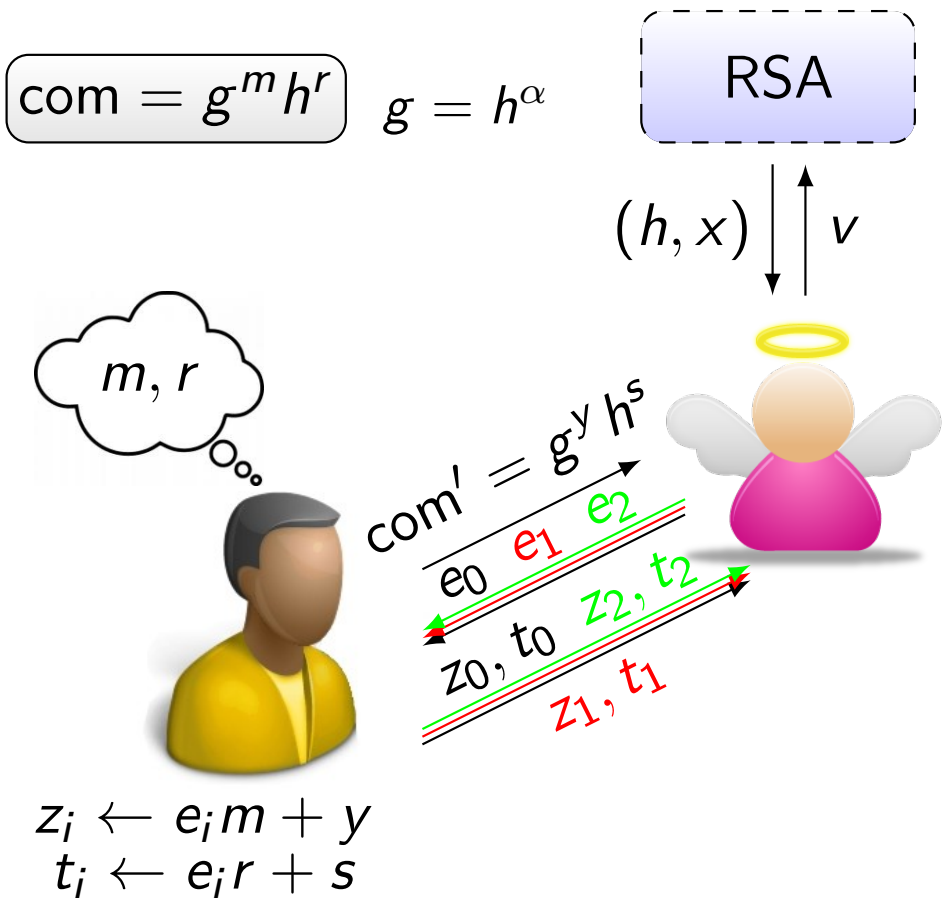
$a = b = 0 \Rightarrow \pi = \pi'$

Soundness Argument

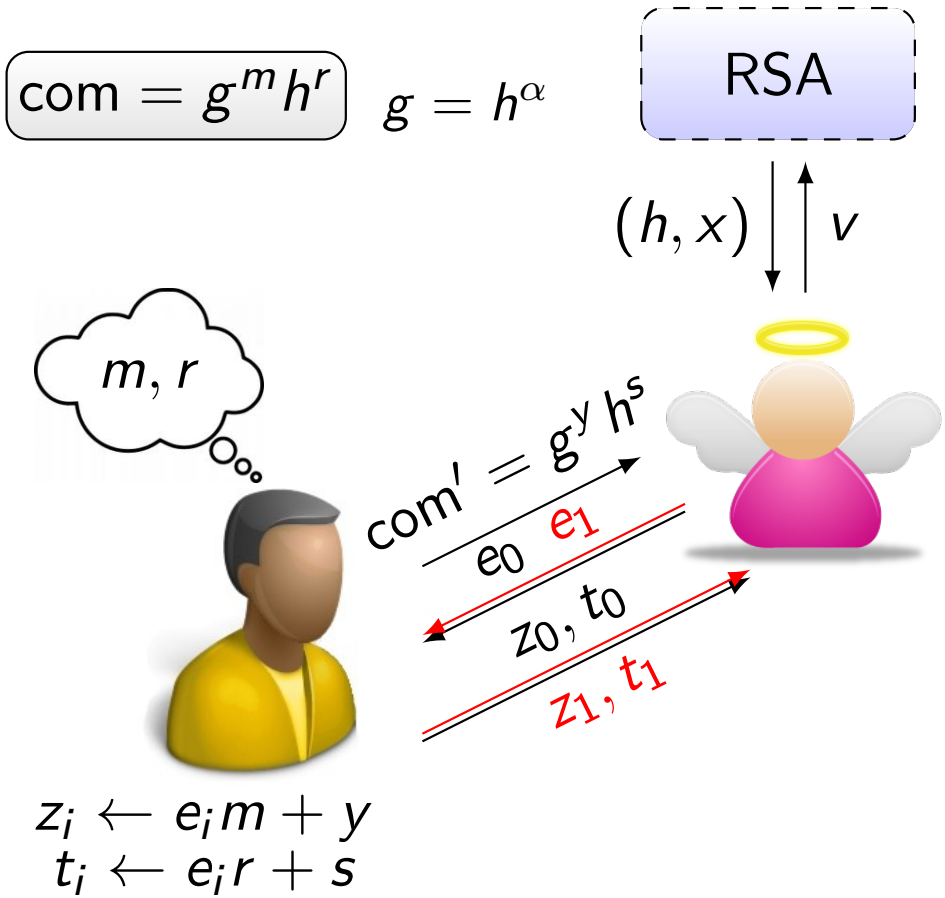


Rewind P w/ (e_0, e_1, e_2);
with pr. ε^3,
$\text{com}^e = g^z h^t$,
$\text{com}^{e'} = g^{z'} h^{t'}$
$\rightarrow g^a = h^b$

Case 2.

Suppose $\pi > 8/\varepsilon$
$g^a = h^b$ factors n

Soundness Argument

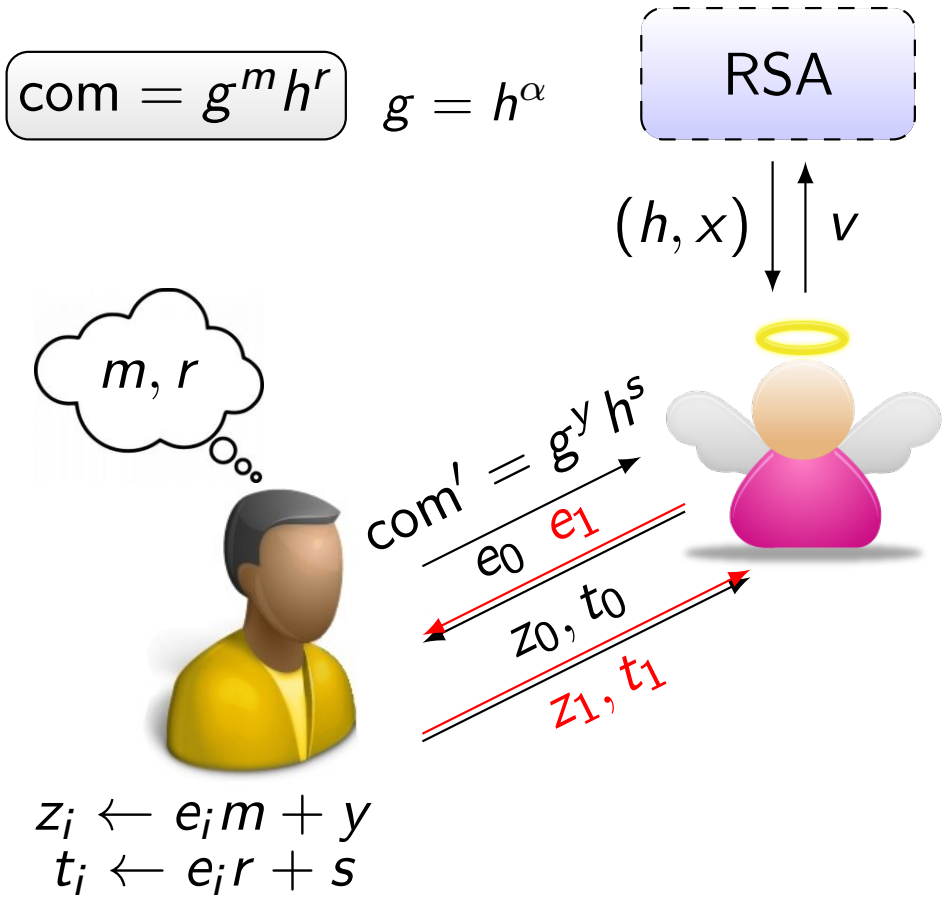


Rewind P w/ (e_0, e_1); with pr. ε^2, $com^e = g^z h^t$, but we cannot divide by e!

Case 2.

$\pi \leq 8/\varepsilon$

Soundness Argument



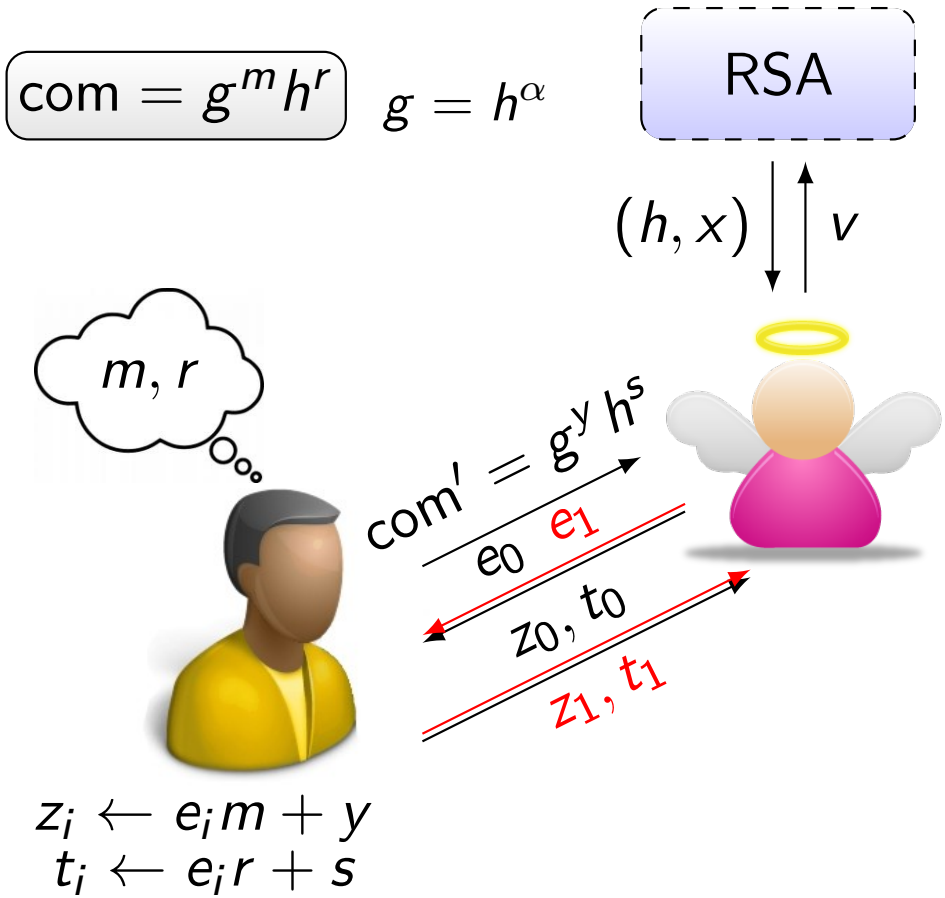
Rewind P w/ (e_0, e_1); with pr. ε^2, $com^e = g^z h^t$, but we cannot divide by e!

Case 2.

$\pi \leq 8/\varepsilon$

A random small RSA challenge is equal to π with $O(\varepsilon)$ probability

Soundness Argument



Sim gets (m, r) or solves RSA with $O(\varepsilon^3)$ proba

Rewind P w/ (e_0, e_1); with pr. ε^2, $\text{com}^e = g^z h^t$, but we cannot divide by e!

Case 2.

$\pi \leq 8/\varepsilon$

A random small RSA challenge is equal to π with $O(\varepsilon)$ probability

Applications, Other Contributions, and Open Problems

Applications.

- Relations between committed values (e.g. [CM99])
- Range proofs ([Lip03])

Other Contributions.

- Can convert an FO commitment (integers) into a Gennaro commitment (modulo a small prime)
- Allows integer ZK proofs with efficient verification

Open Problems.

- Can we build short algebraic RSA-based signatures?

Thank you for your attention

Questions?