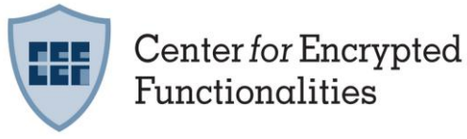
# Robust Transforming Combiners from iO to Functional Encryption


Prabhanjan Ananth
Aayush Jain
Amit Sahai

# Since 2013…

- Two-Round (Adaptive) Multi-Party Computation
- Instantiating Random Oracles
- Non-Interactive Multi-party Key Exchange
- Impossibility Results
- Theoretical Results (such as PPAD Hardness)
- Constant-Round Concurrent Zero Knowledge
- Separation Results for Circular Security
- Succinct Randomized Encodings
- Watermarking
- Patching

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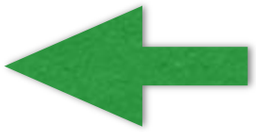
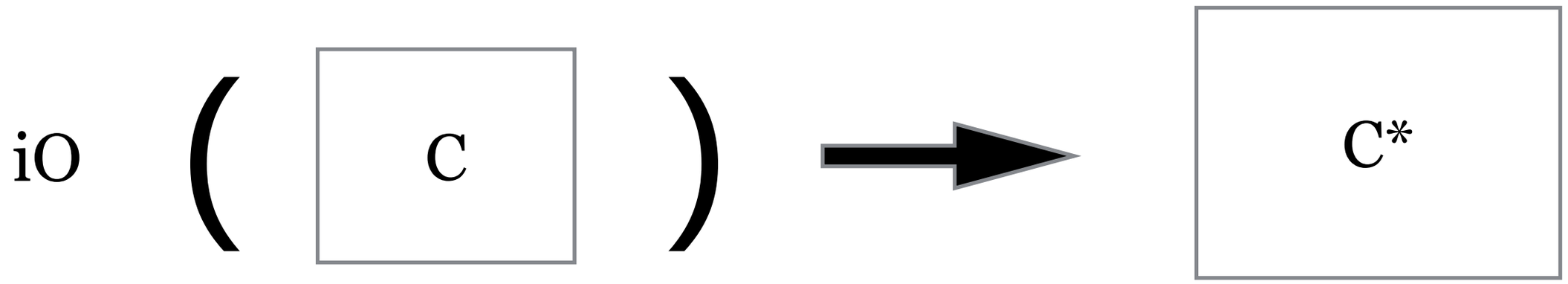


Indistinguishability Obfuscation (iO)/Functional Encryption

# What is iO?



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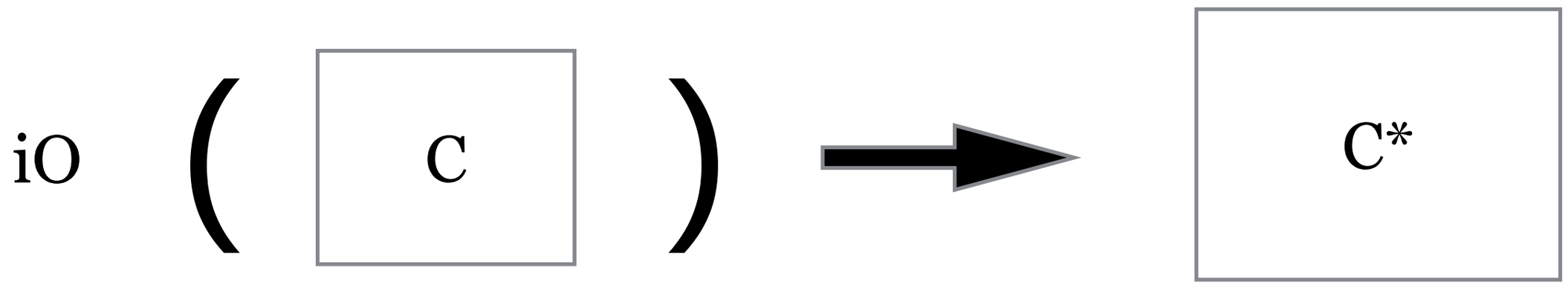



*Correctness*: for all x, $C^*(x) = C(x)$

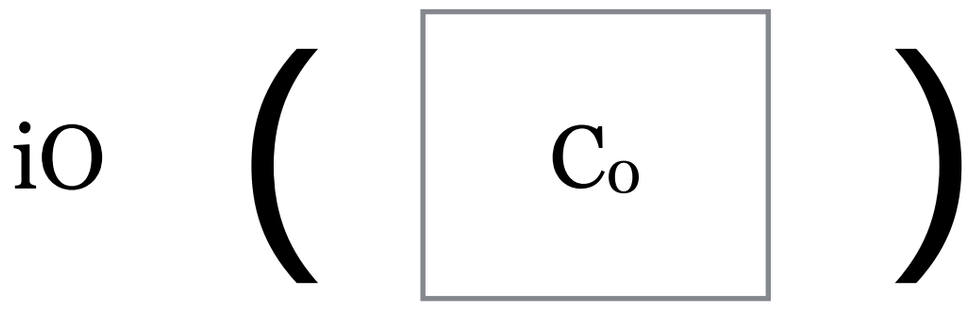
# What is iO?

$C_0$

≡

$C_1$

# What is iO?

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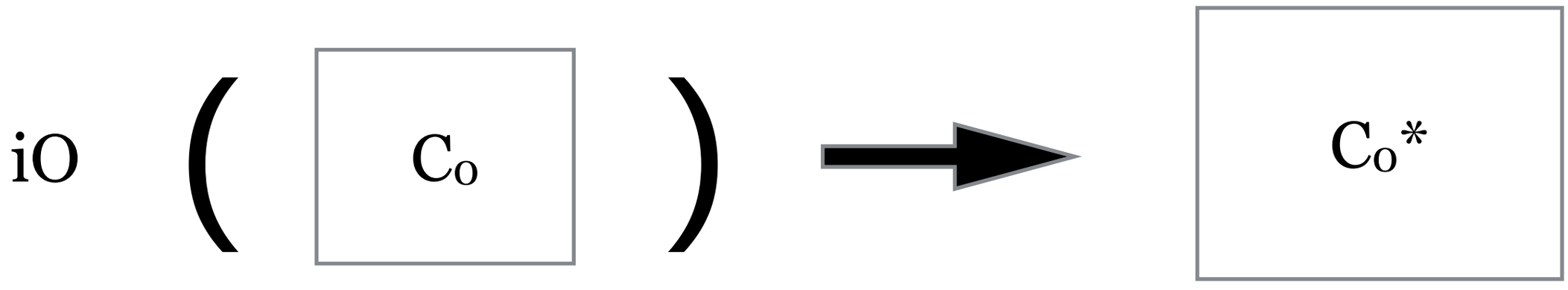


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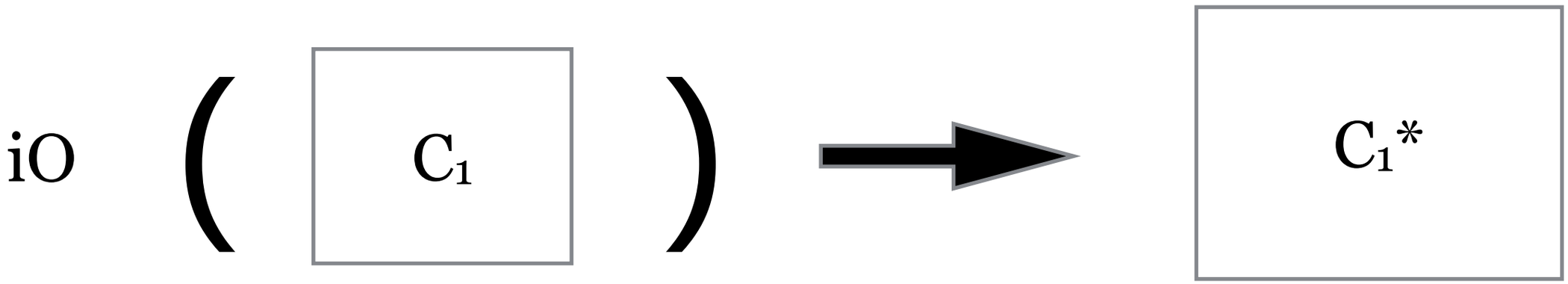

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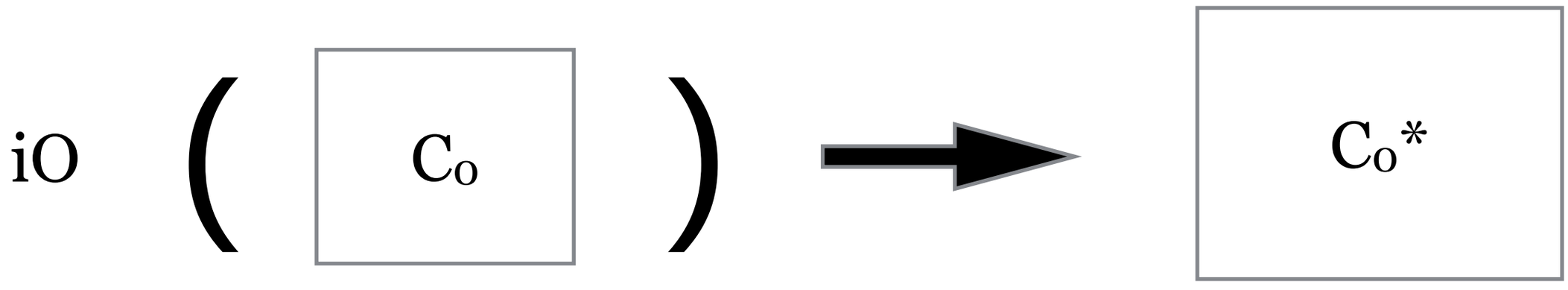
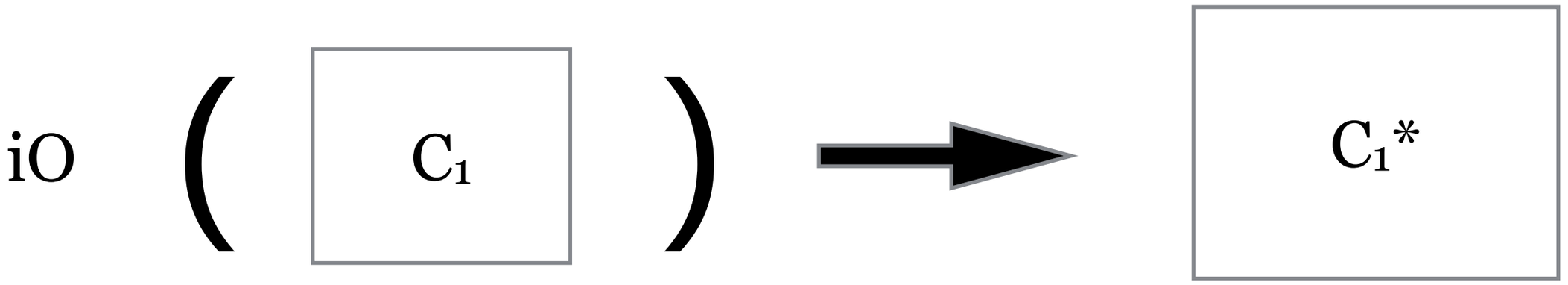



$\equiv$ *Security*: $\approx_c$



# Functional Encryption

[SW'05,GGHRSW13]

x

Fine Grained Access to Private Data

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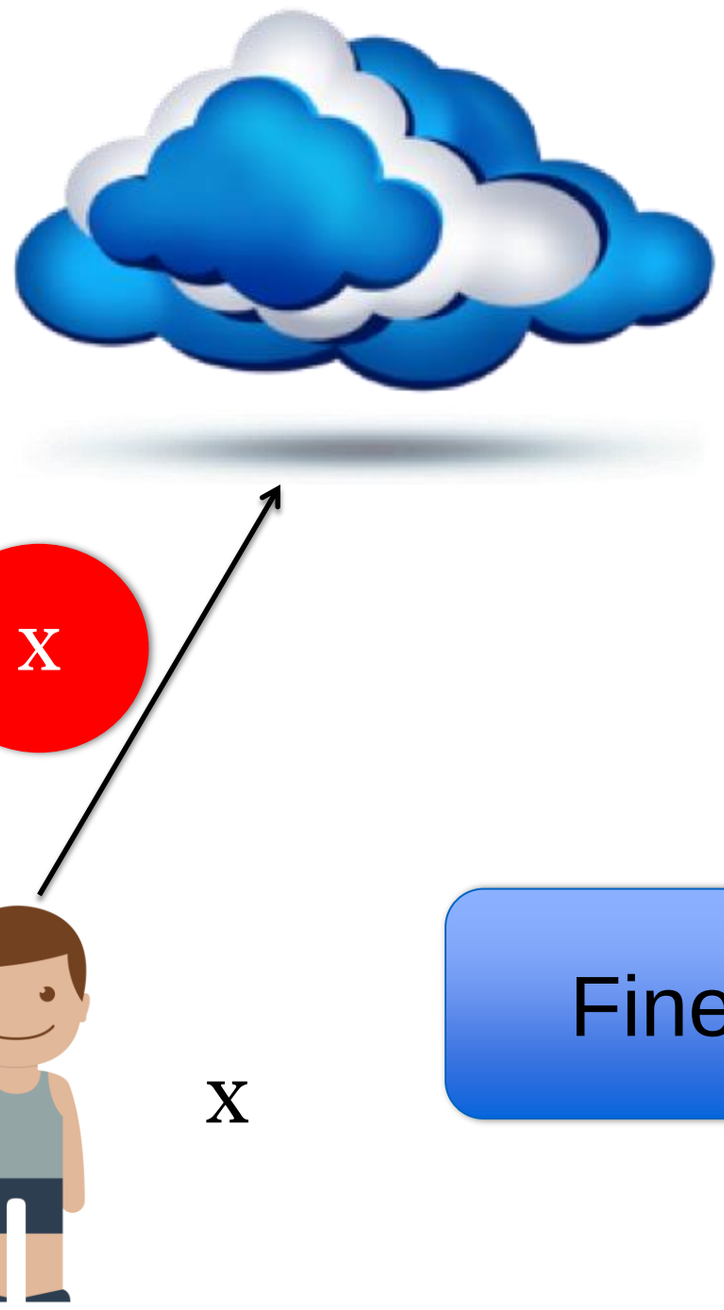


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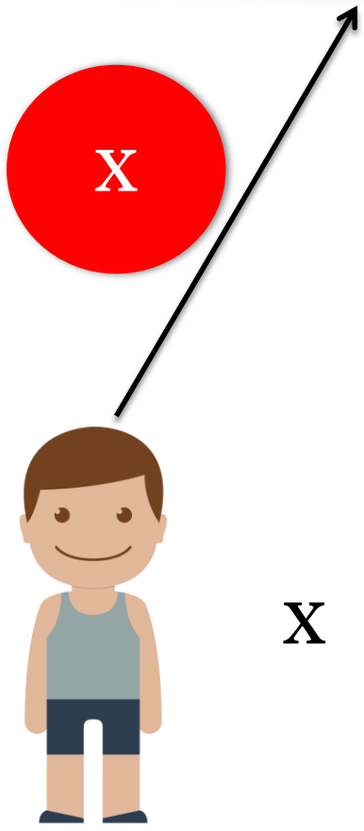


Fine Grained Access to Private Data

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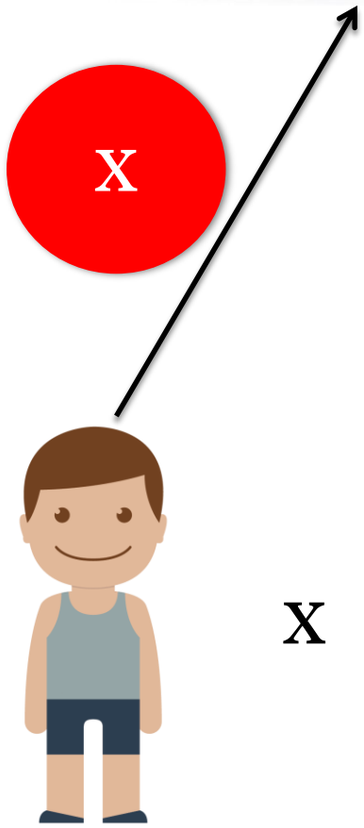
[SW'05,GGHRSW13]

MSK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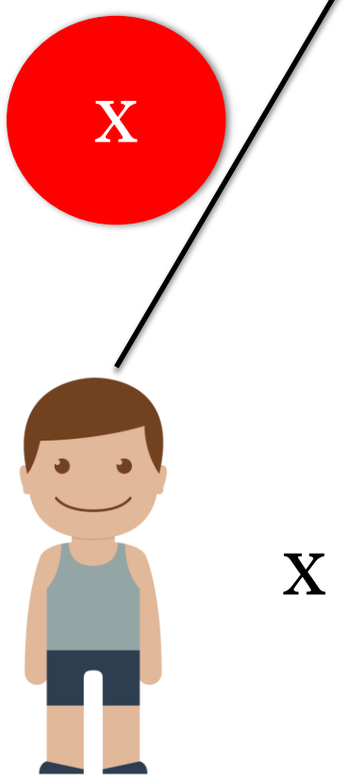


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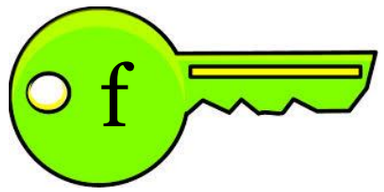


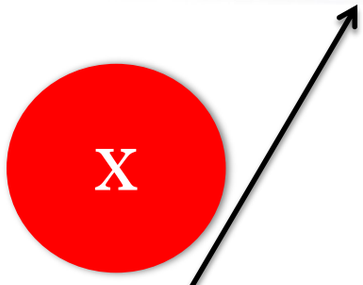


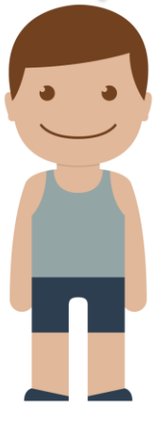

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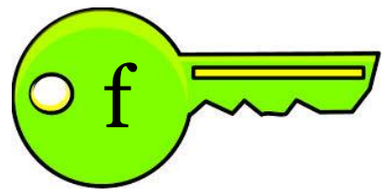


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$Dec(\ f\ ,\ x\ ) = f(x)$

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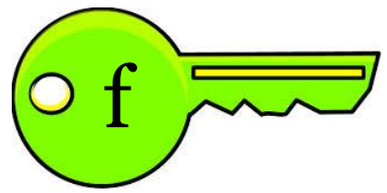
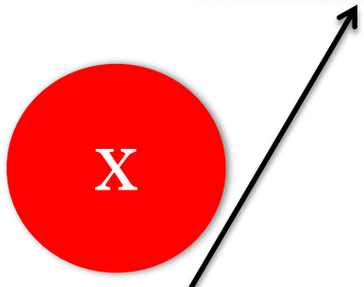
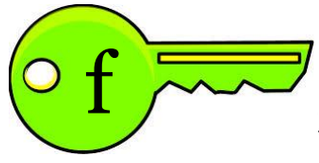
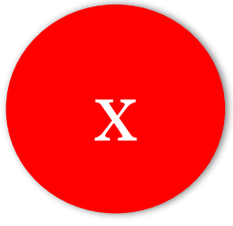
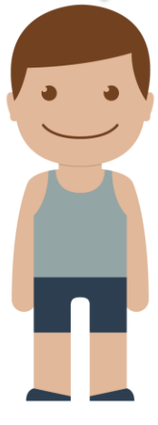



Dec( 

, 

 ) = f(x)



x

$SK_f$ should not allow adversary to compute anything other than f(x) !

# Known Constructions?

[GGHRSW’13, BGKPS’14, Zim’15, GLSW’15, AB’15, GMMSSZ’16, LV’16, L’16, AS’17, LT’17….]

Are all candidates of iO broken?

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**NO!**

We have several unbroken iO candidates,
including with proofs of security in various models.

# Our Goal

Find a iO candidate that is secure
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**iO combiner**

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**Robust iO combiner:**

*In fact we only require the secure candidate to be correct*
*All other candidates can violate correctness*
**[AJNSY16, FHNS16]**

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**Correctness:** y = C(x)

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**Security:** *If $C_0$ is equivalent to $C_1$,*

RCiO.Obf( **P** , $C_0$) $\approx_c$ RCiO.Obf( **P** , $C_1$)

# Implications

Robust iO combiners imply **universal iO [AJNSY’16]**

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**Universal iO:**

A scheme P is a universal iO scheme

if *iO exists* then *P is a secure iO scheme*

# Previous Work

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- **AJNSY16** gave candidate construction of a robust combiner from DDH/LWE.
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## Questions?

- Can we achieve some applications of iO if the secure candidate is polynomially secure?
- Can we weaken the assumptions to rely on only one-way functions?

# This Work

**Theorem 1 (Combiner -> Robust Combiner):**
*Given:*

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Previously, as observed in AJNSY'16 and BV'15, this result required sub-exponential DDH/LWE and the underlying candidate to be sub-exponentially secure

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**Theorem 2:** *Given:*

- *N correct iO Candidates (with one secure)*

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We show:
There exists a transforming robust combiner from iO to Functional Encryption.
This also yields any primitive implied by FE (such as NIKE. [GPSZ17])

# Technical Overview

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Secure candidate is unchanged as it is correct.

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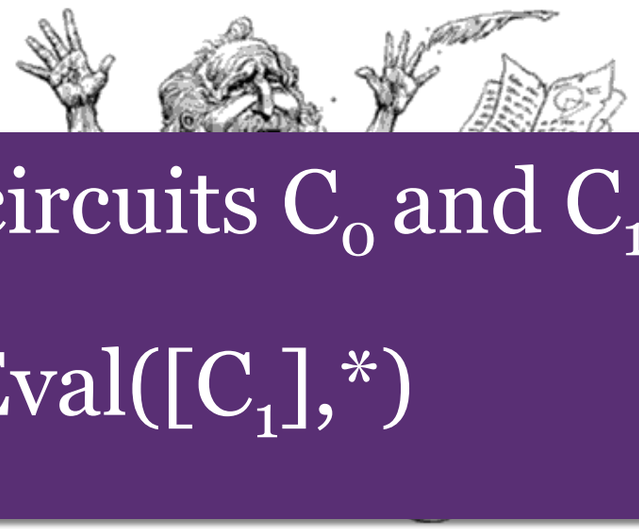

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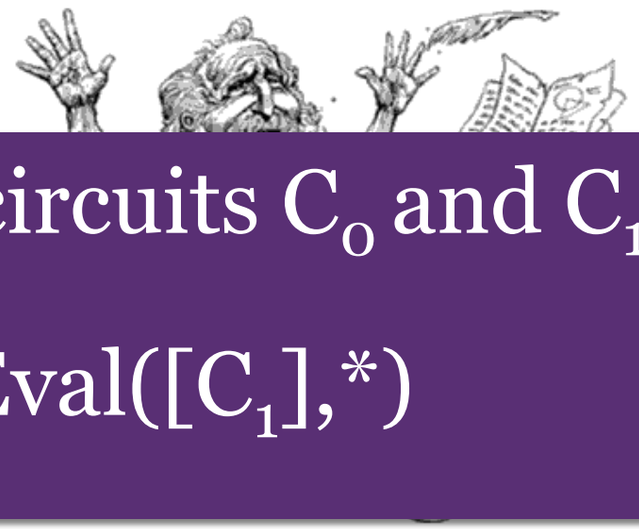

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- Such garbled circuits can be constructed from one-way functions.

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1. Use the modified obfuscator to obfuscate Eval([C],*)
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Perform BPP Amplification to
get almost correctness

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How to do this?

**Use MPC Techniques!**

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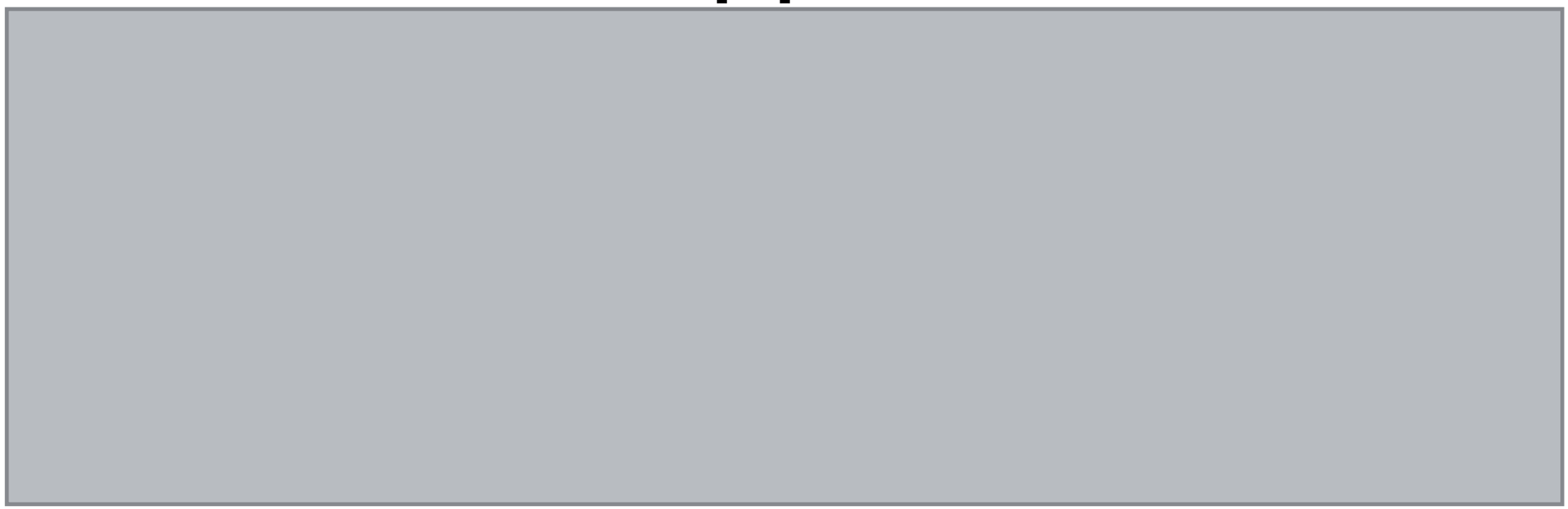
Can we weaken assumptions by relying on interactive MPC?

# Our Approach



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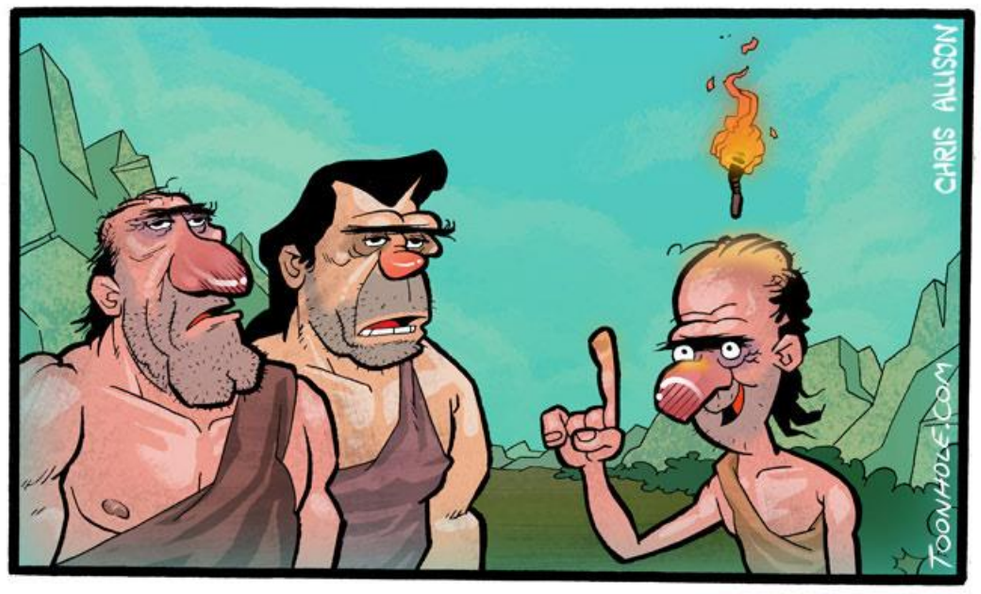


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- Run the MPC protocol for $U(C_1+...+C_N, x)$ to learn $C(x)$


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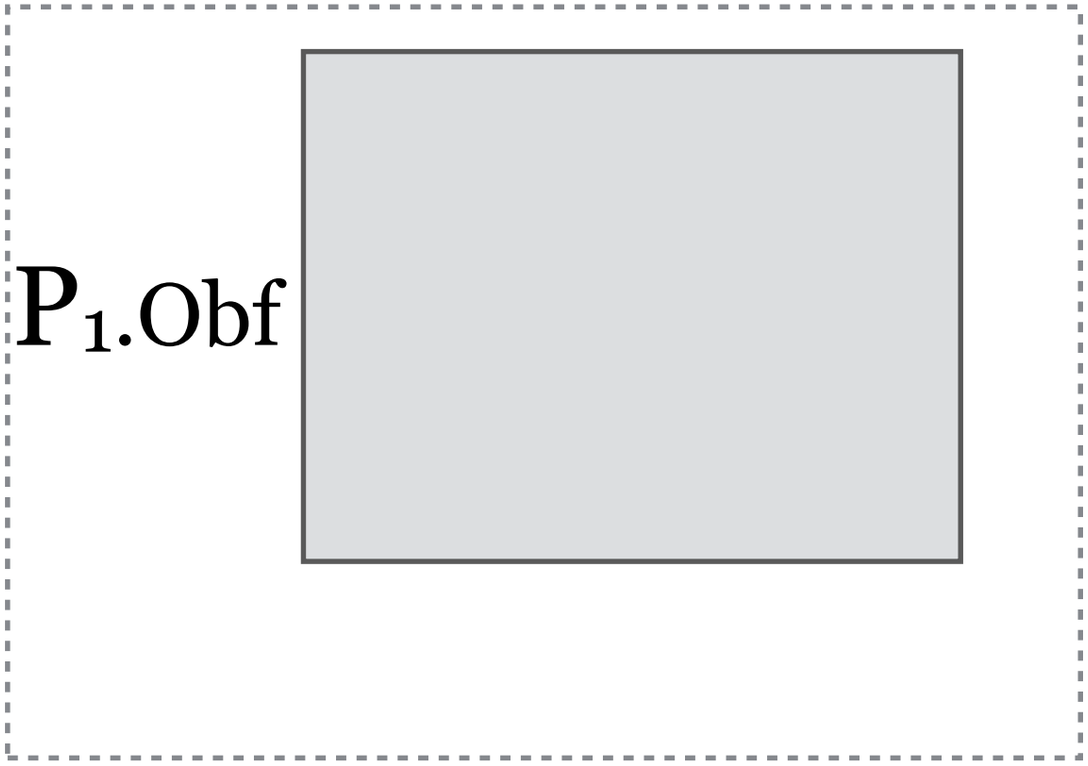


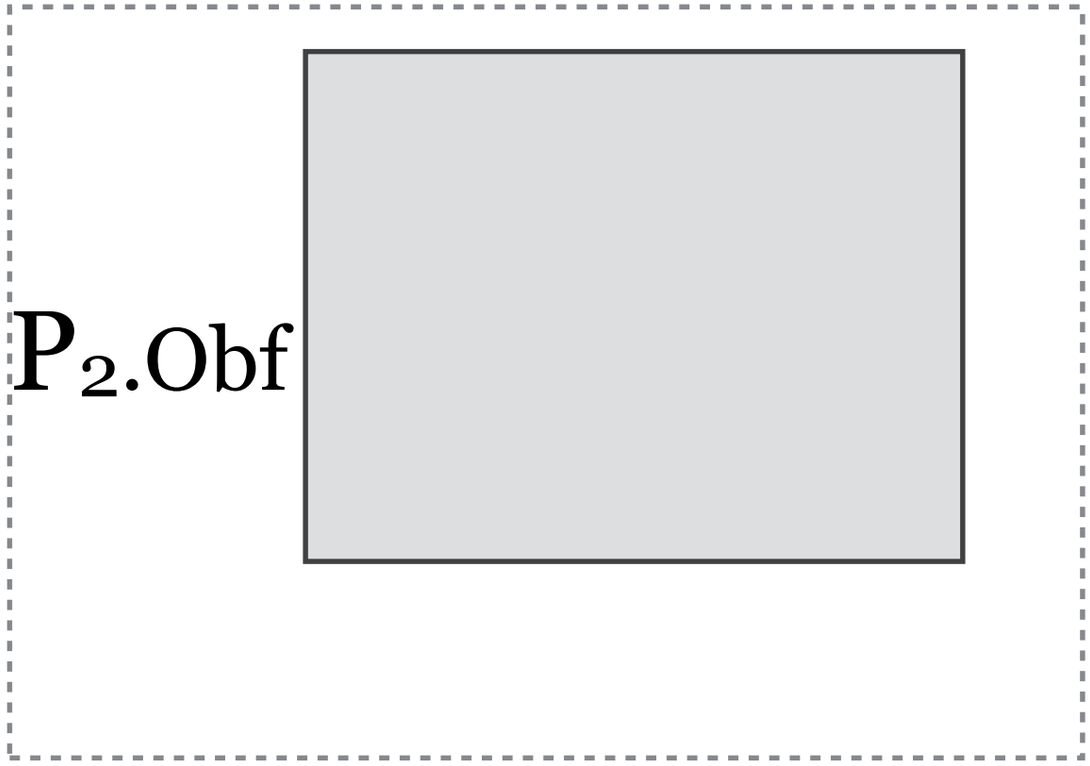

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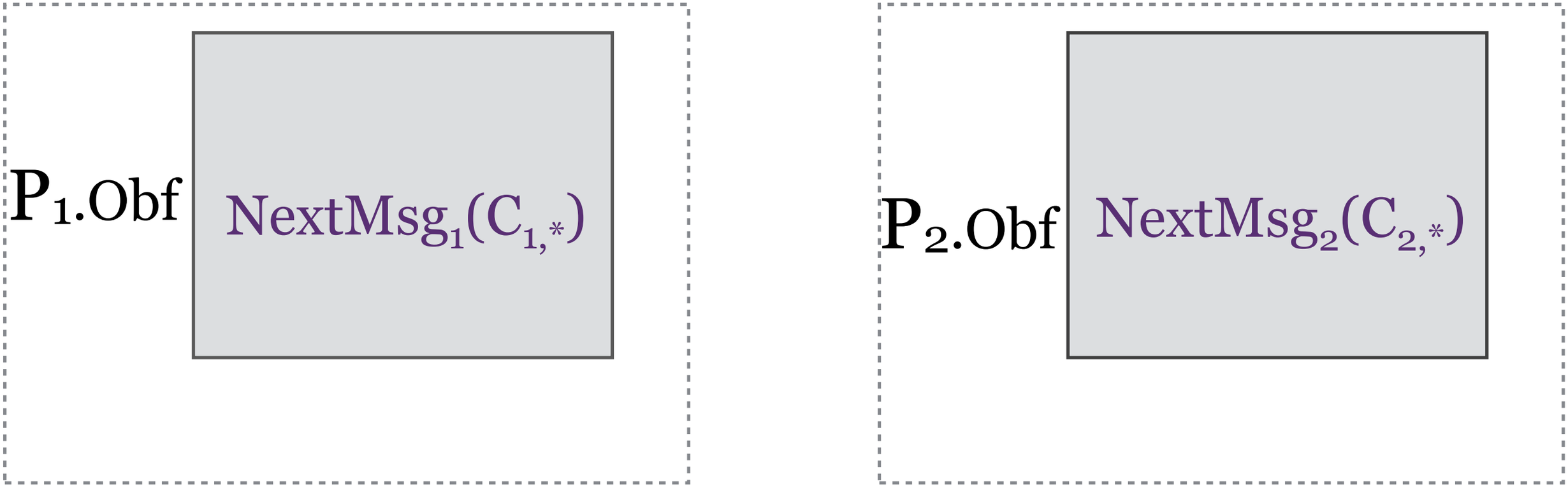

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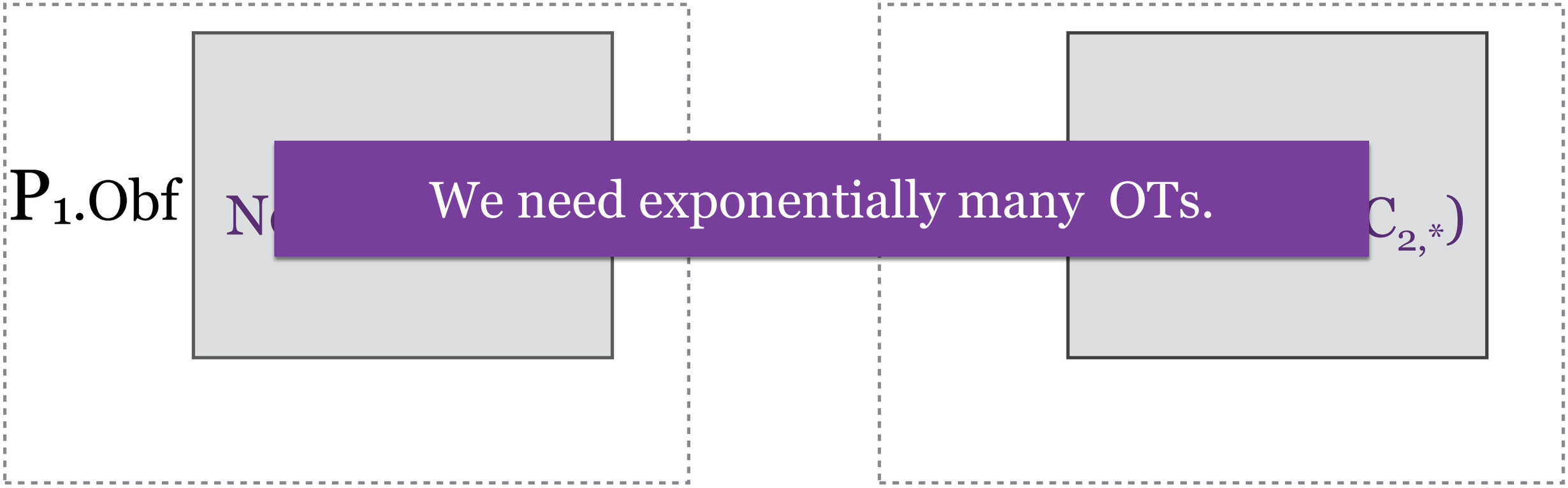

# (Random) OT

$P_1$

$P_2$

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$P_1$

$P_2$

$(r_0, r_1)$

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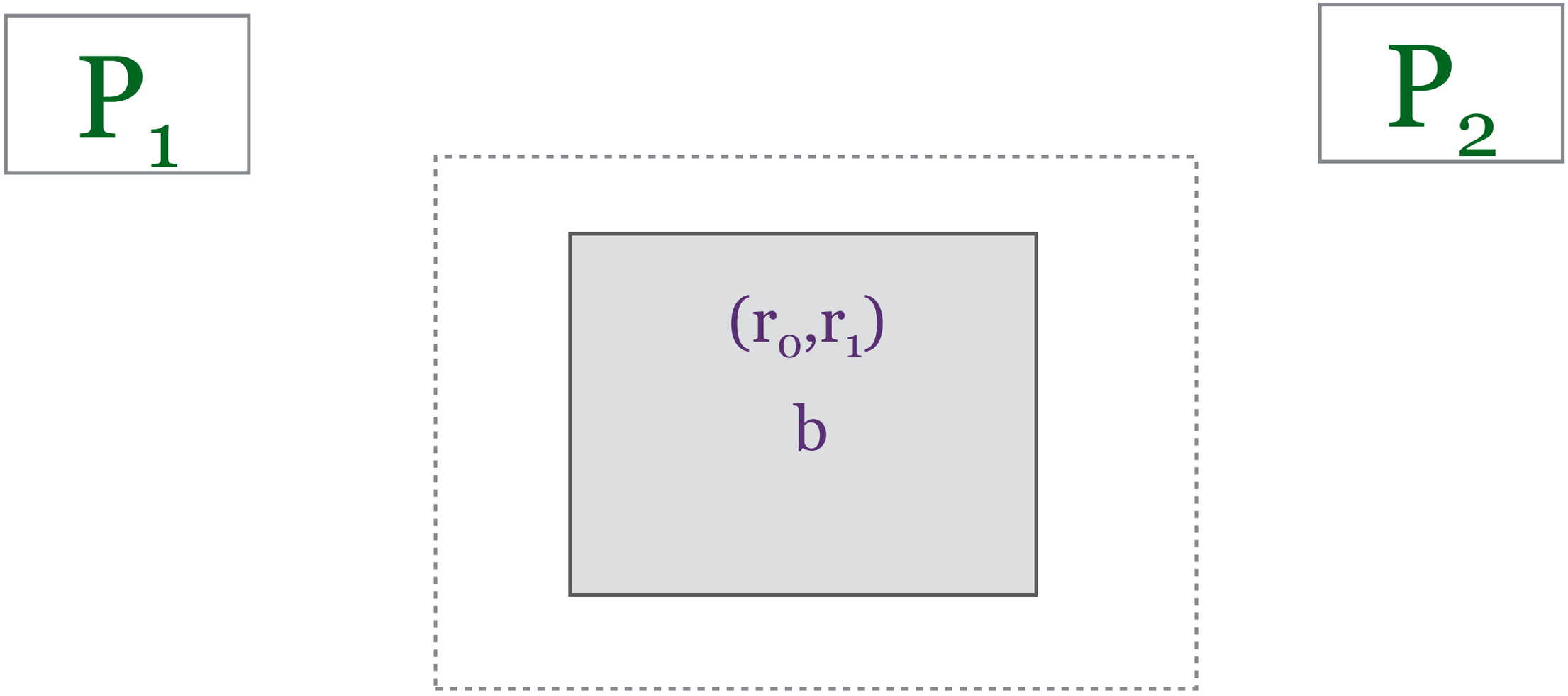

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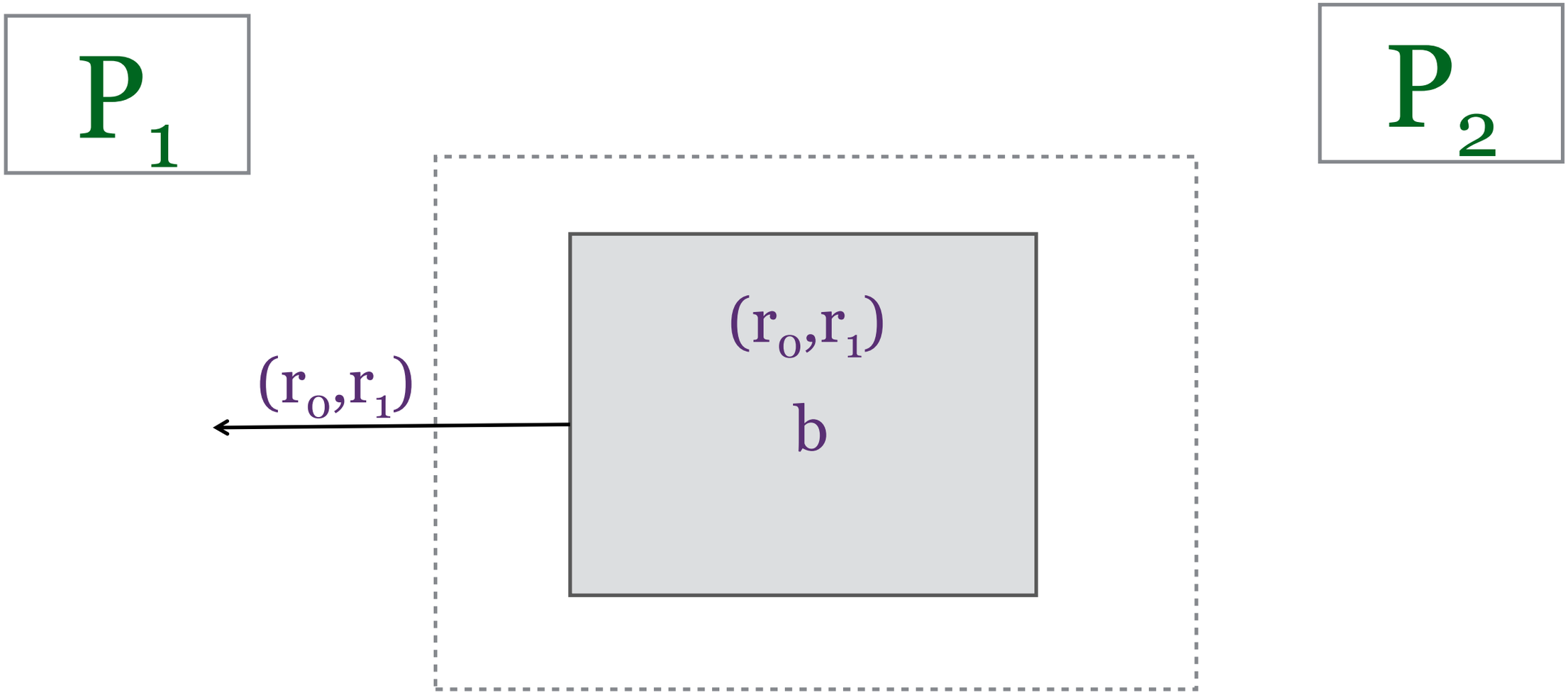

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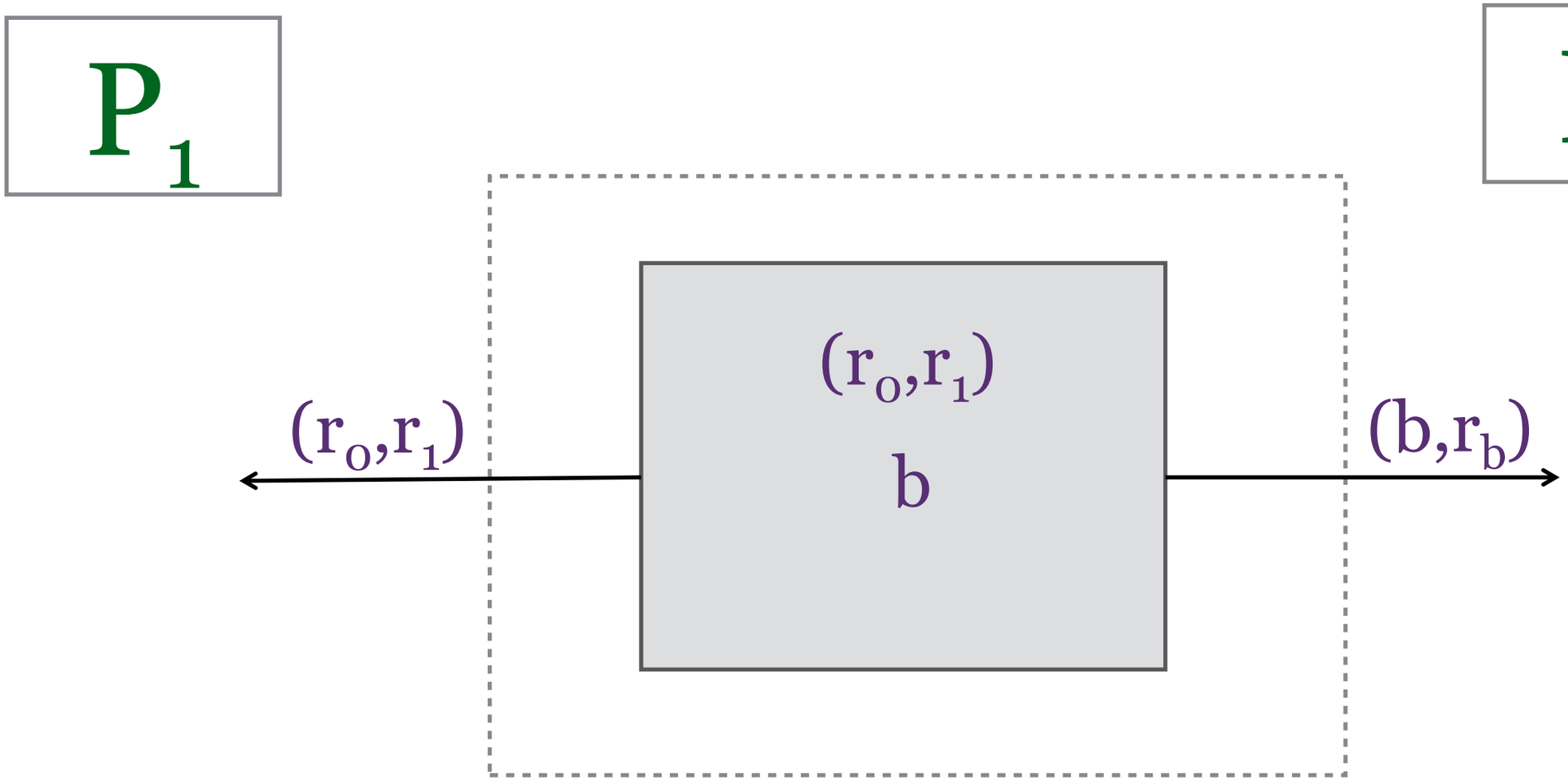

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- Use PRF keys to generate OTs on the fly.

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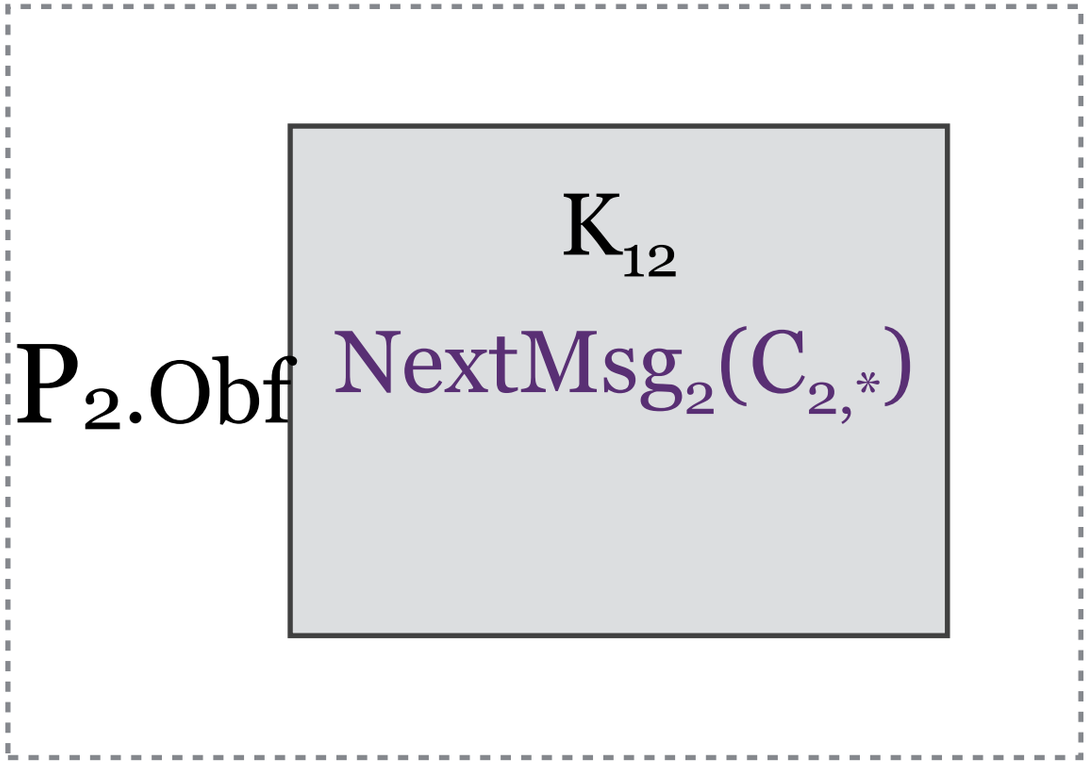

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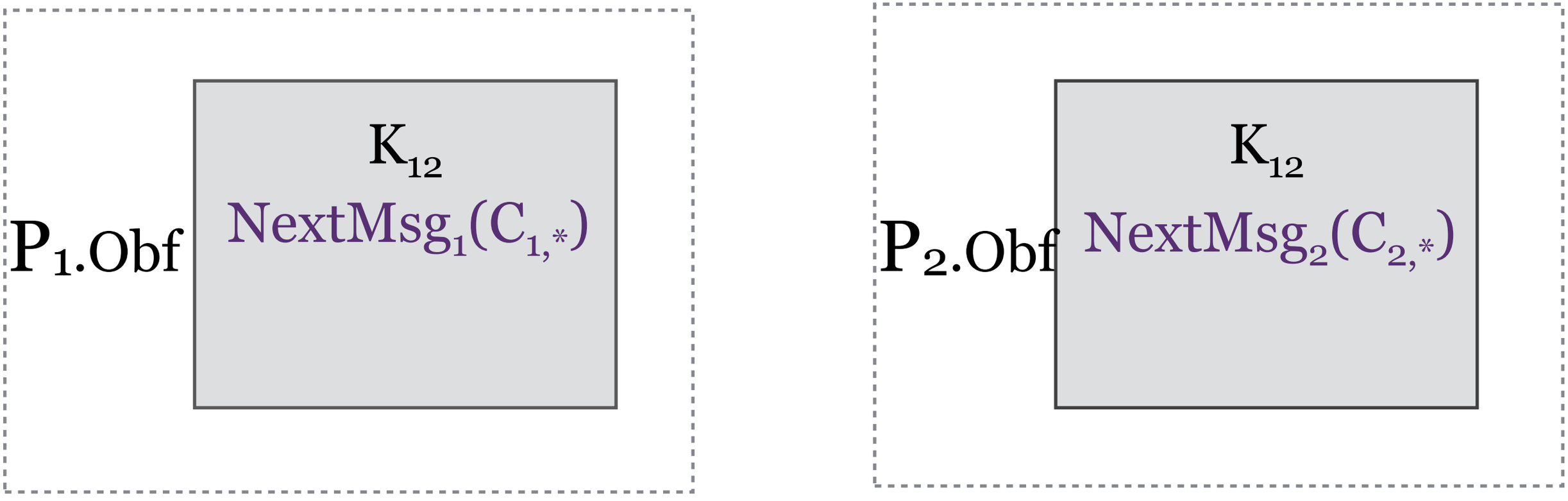

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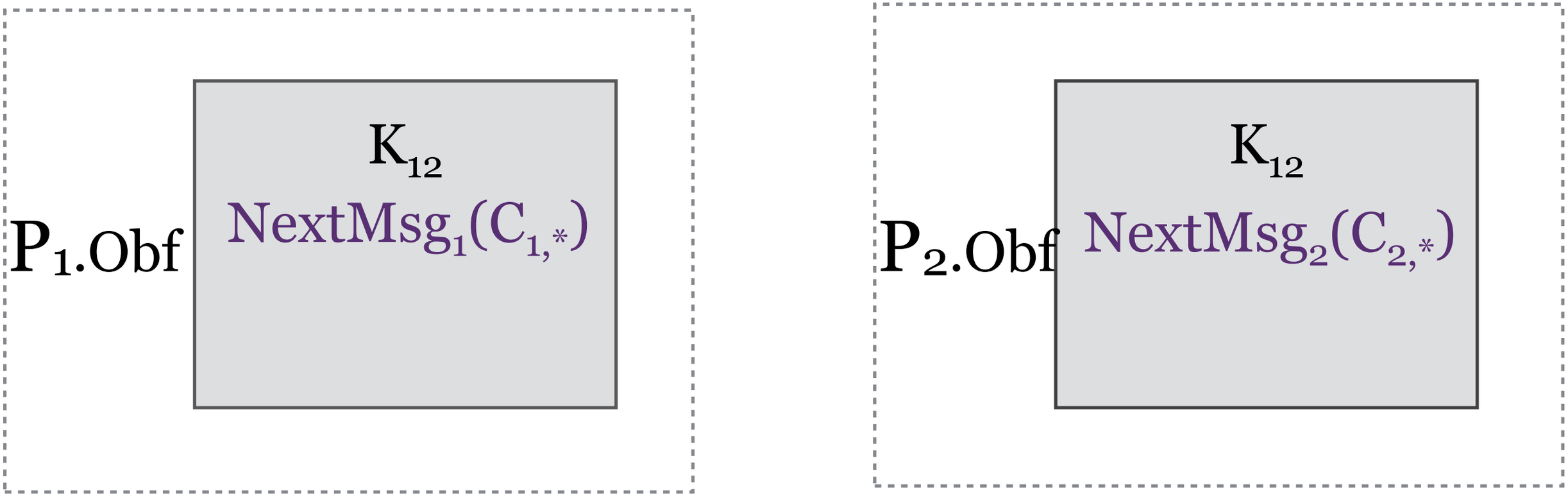

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$NextMsg_1(C_{1,*})$

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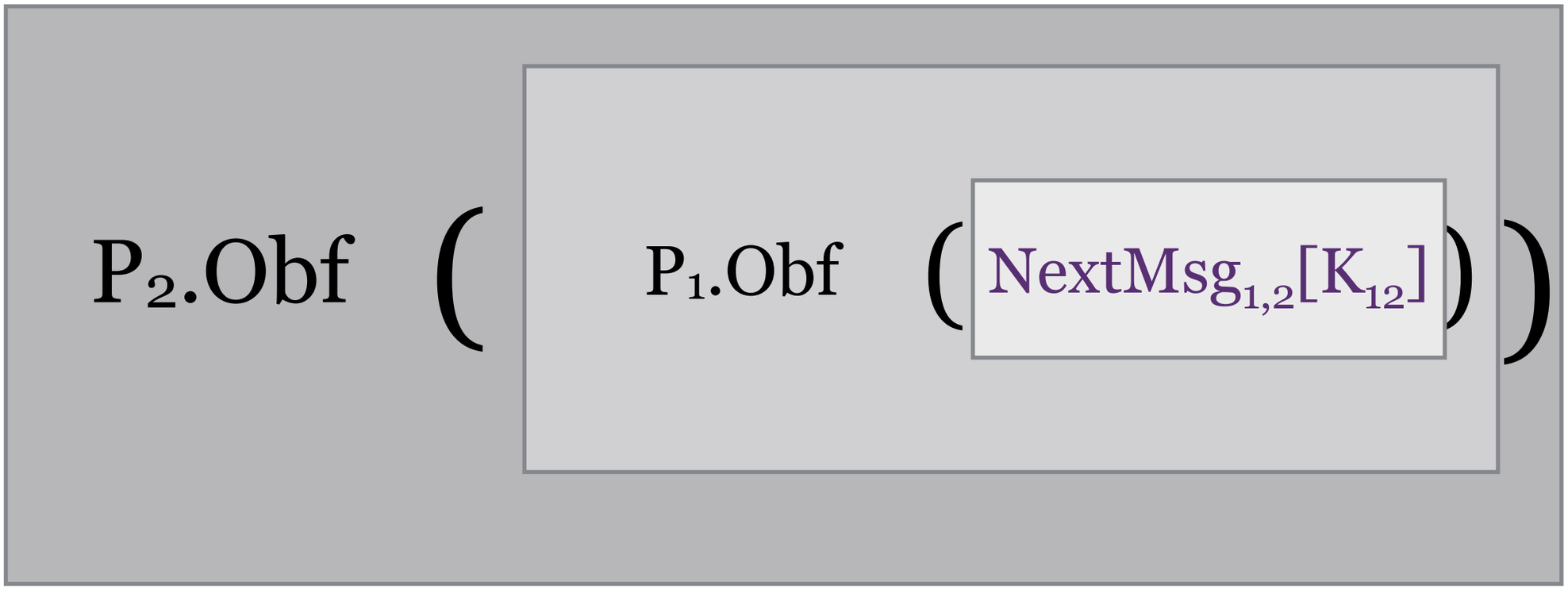

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# Open Questions

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2. **Combiner for poly–hard Functional Encryption from OWF/DDH**