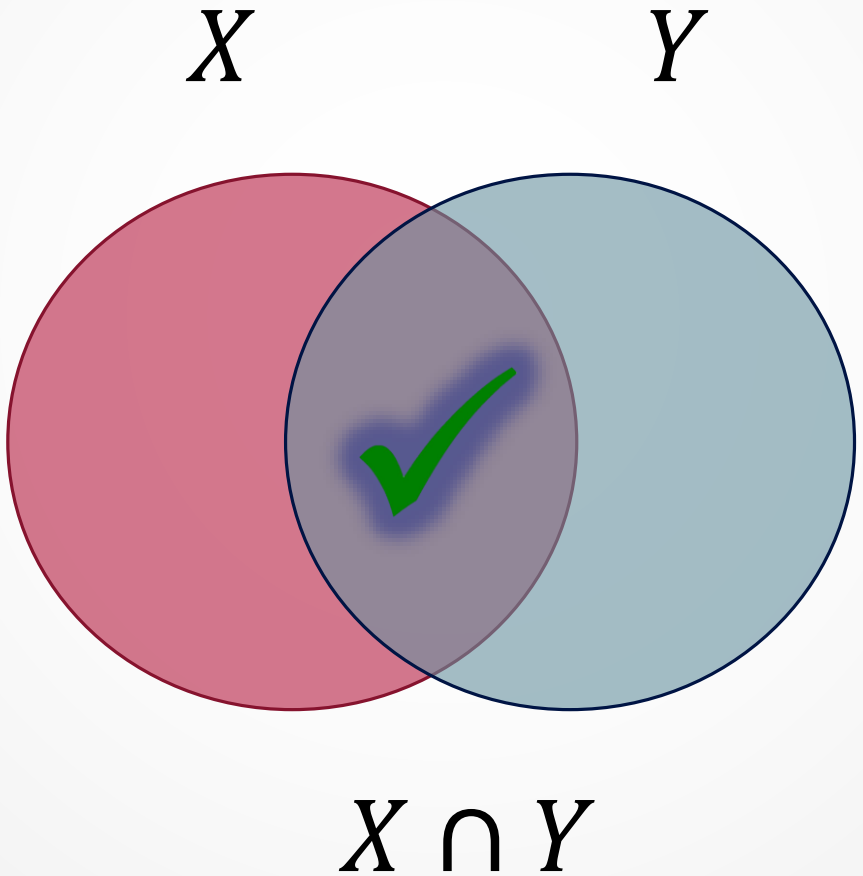
# Improved Private Set Intersection against Malicious Adversaries

**Peter Rindal**
Mike Rosulek

Oregon State University OSU

# Private Set Intersection (PSI)

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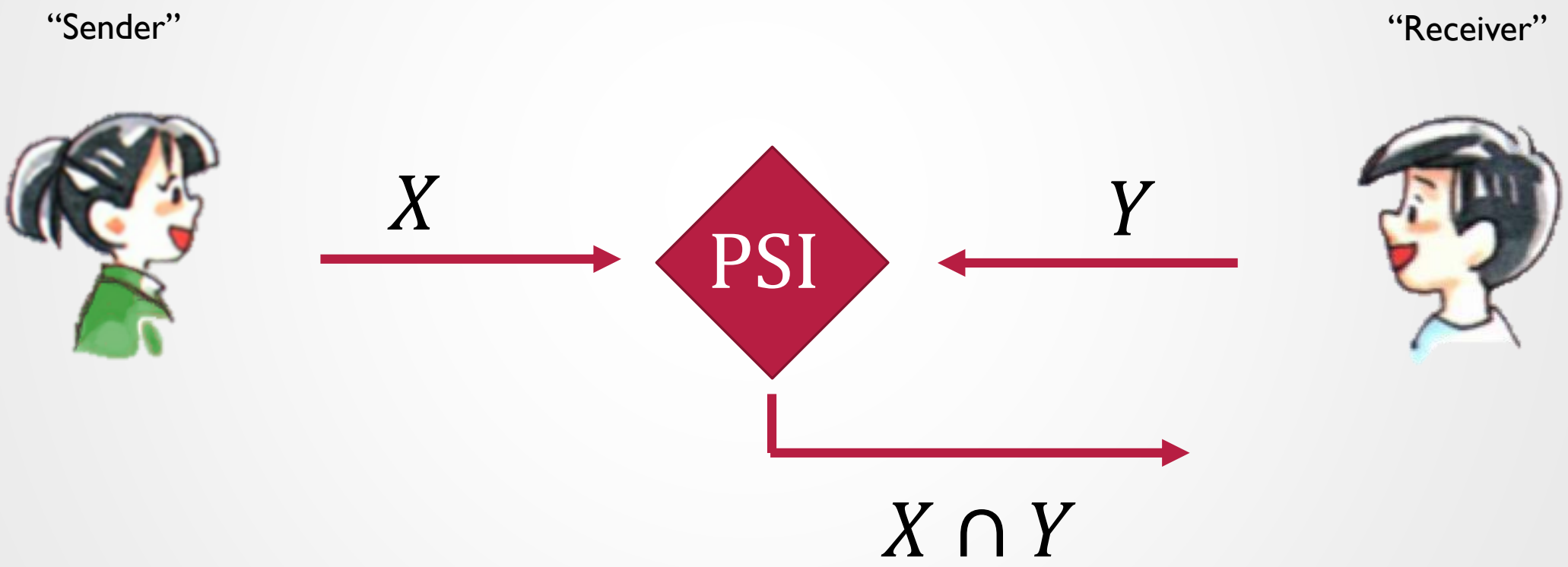
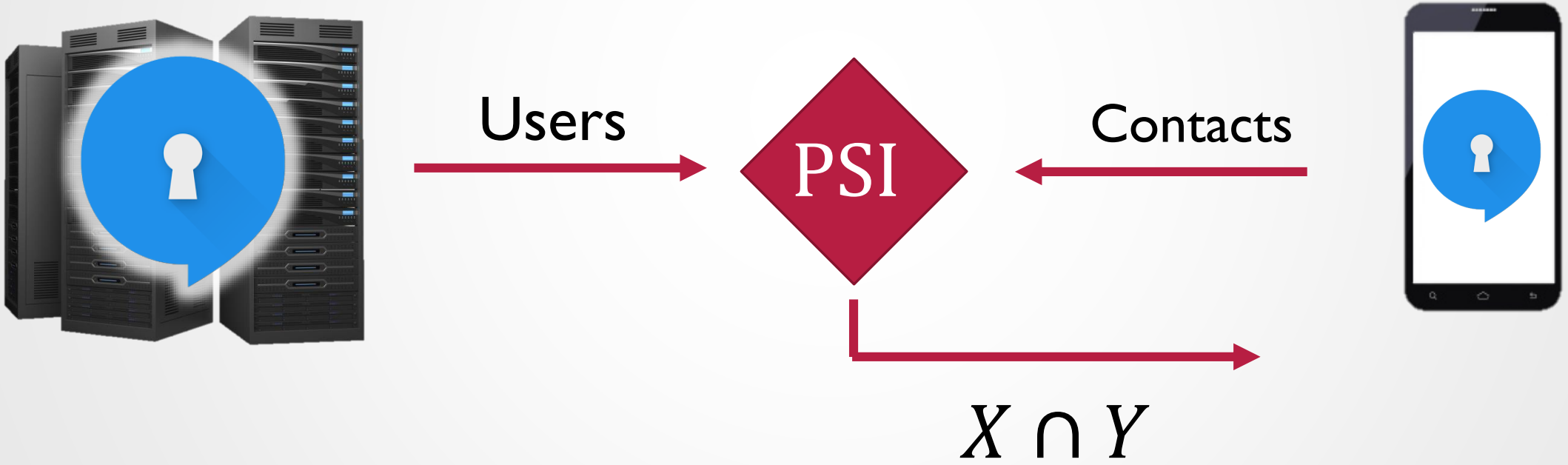
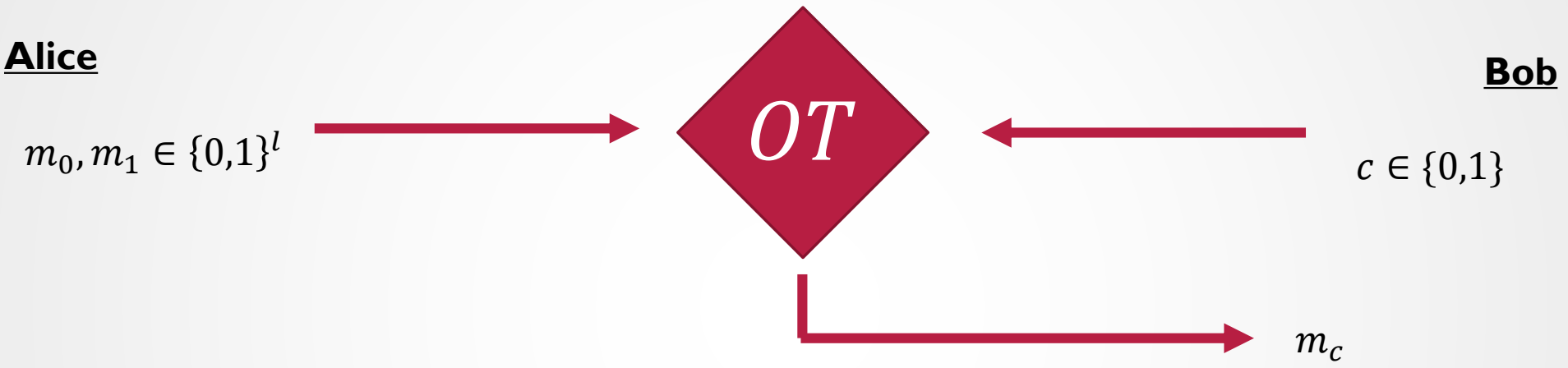


# App: Contact discovery



# Oblivious Transfer (OT)




- Highly efficient and secure protocols exists
- Motivates it use as the basis for PSI

# Bloom Filter

Plain text data structure similar to hash table

- Allows for testing set membership
- Paramerterized by hash functions $h_1, \ldots, h_k$

- , set
  - $b[h_i(x)] = 1 \quad , \forall i$

$b =$

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

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Plai$n\ i\ (x) = 1 \quad , \forall i$

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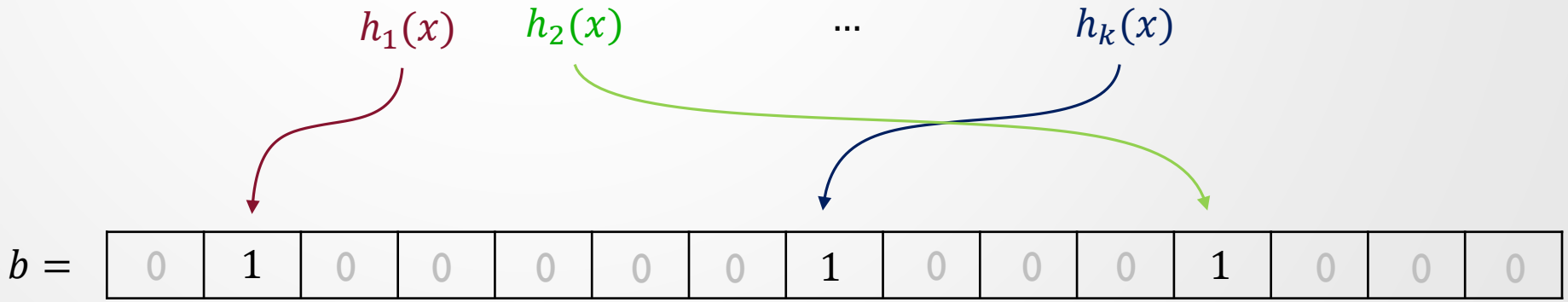

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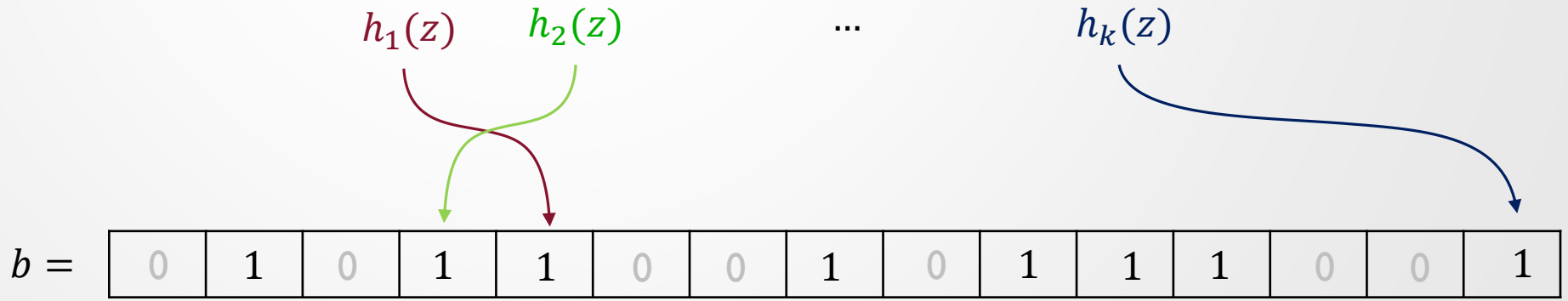

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- Plain text data structure similar to hash table
  - Allows for testing set membership
  - Paramerterized by hash functions $h_1, \ldots, h_n$

- To insert $x$, set
  - $b[h_i(x)] = 1 \quad , \forall i$
- To test membership
  - Return $\wedge_i \, b[h_i(x)]$

$b =$

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | 0 | 1 |

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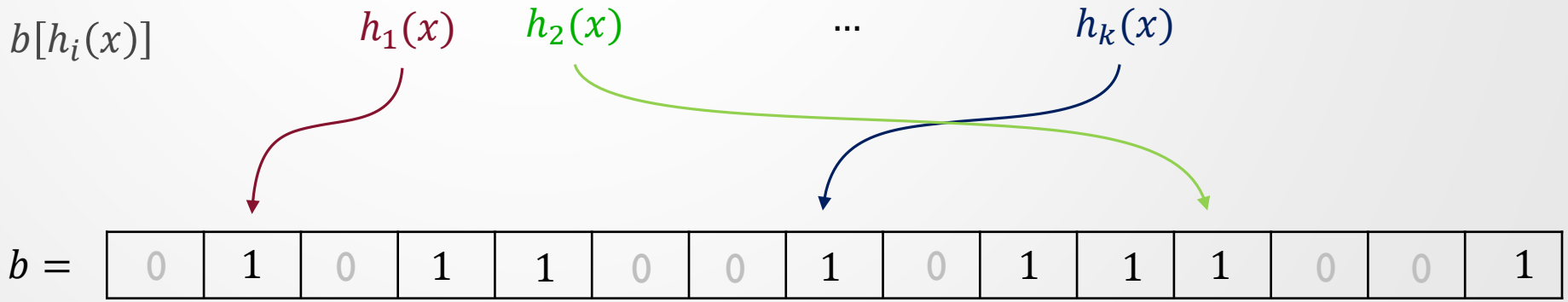
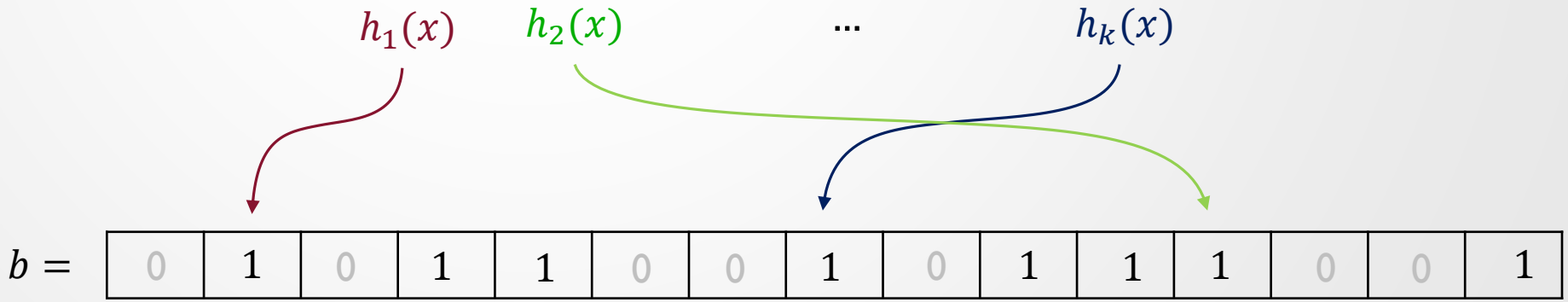


# Bloom Filter

$n$ items $\rightarrow$ Bloom filter with $m$ slots and $k$ hash functions

- Membership:
  - Pr[ false negatives ] = 0
  - $$\left(1 - e^{-kn/m}\right)^k$$
    $$\approx 2^{-k}$$



# Bloom Filter

$kn\,m\,kknn\,kn\,m\,mm\,kn\,m\;\;e\;-\;kn\,m\;\;1-\;e\;-\;kn\,m\;\;\;1-\;e\;-\;kn\,m\;\;\;k\,kk\;1-\;e\;-\;kn\,m\;\;\;k\;\;e\;-\;kn\,m\;\;ee\,e\;-\;kn\,m\;--\;1-\;e\;-\;kn\,m\;\;\;k\;1-\;e\;-\;kn\,m\;\;1n$ items → Bloom filter with $m$ slots and $k$ hash functions

- Membership:
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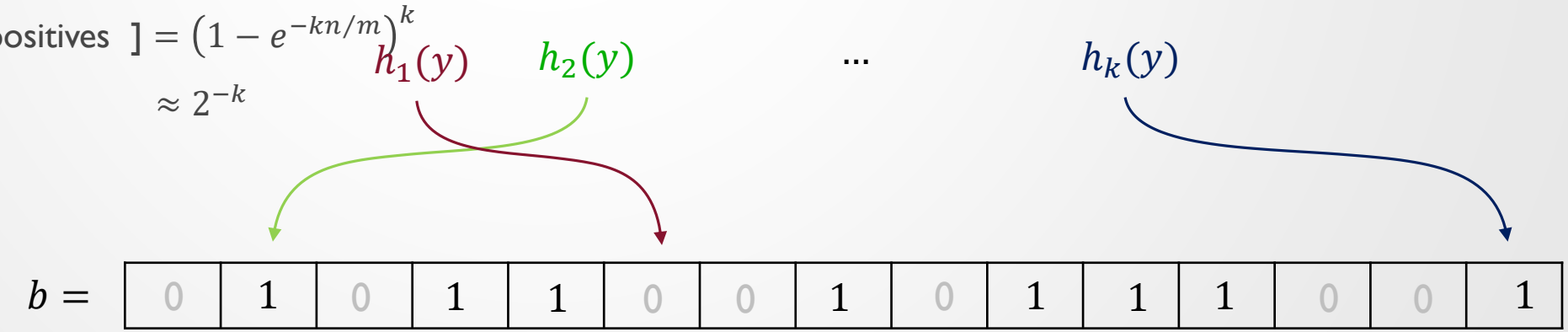
  - Pr[ false positives ] $= \left(1 - e^{-kn/m}\right)^k$
    $\approx 2^{-k}$



# Bloom Filter

$\approx 2\ -k\ 2\ 2\ -k\ -kk\ 2\ -k$

$\approx 2\ -k\ 2\ 2\ -k\ -kk\ 2\ -k$

$kn\ m\ kknn\ kn\ m\ mm\ kn\ m\ \ e\ -\ kn\ m\ \ 1-\ e\ -\ kn\ m\ \ \ 1-\ e\ -\ kn\ m\ \ \ k\ kk\ 1-\ e\ -\ kn\ m\ \ \ k\ \ e\ -$
$kn\ m\ \ ee\ e\ -\ kn\ m\ --\ 1-\ e\ -\ kn\ m\ \ \ k\ 1-\ e\ -\ kn\ m\ \ 1n$ items → Bloom filter with $m$ slots and $k$ hash functions

- Membership:

$\approx 2^{-k}\ \ \approx 2^{-k}$

$h_1(y)$ $\approx 2^{-k}$ $h_2(y)$ … $h_k(y)$

$b =$

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | 0 | 1 |

# Bloom Filter Intersection

- Bitwise AND $B_X \wedge B_Y$ is a Bloom filter for $X \cap Y$

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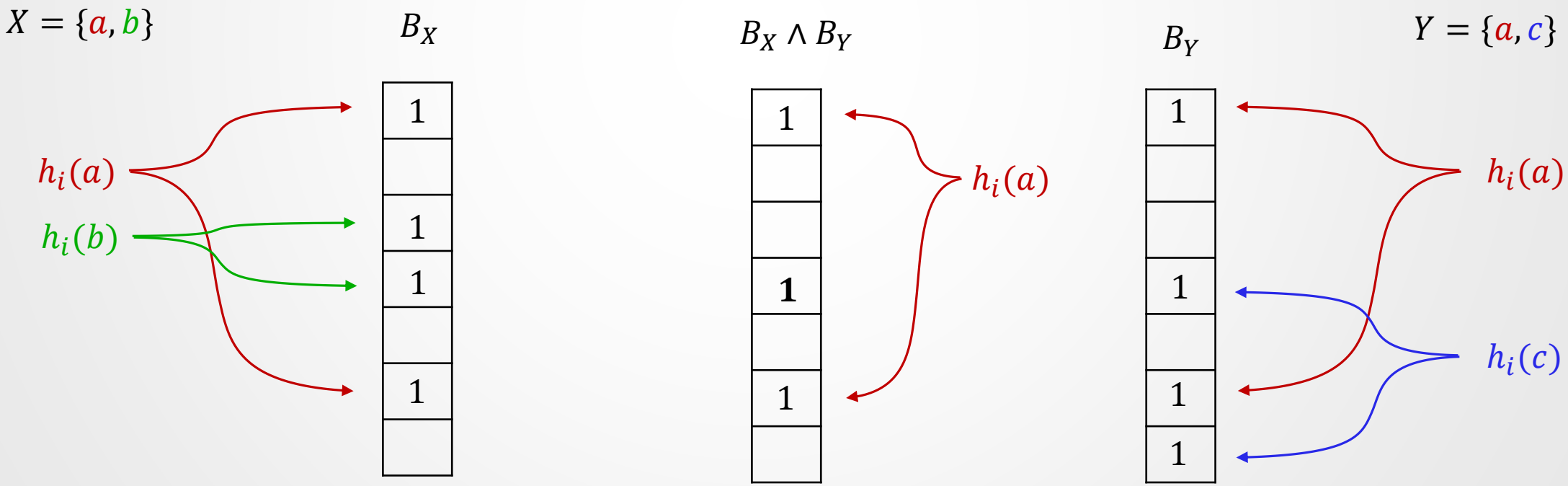

# Bloom Filter Protocol

[DongChenWen13, PinkasSchniederZohner14]

$m_i \leftarrow \{0,1\}^\kappa$

| |
|---|
| $m_0$ |
| $m_1$ |
| $m_2$ |
| $m_3$ |
| $m_4$ |
| $m_5$ |
| $m_6$ |

$B_Y$

$Y = \{a, c\}$

| |
|---|
| 1 |
| |
| |
| 1 |
| |
| 1 |
| 1 |

$h_i(a)$

$h_i(c)$

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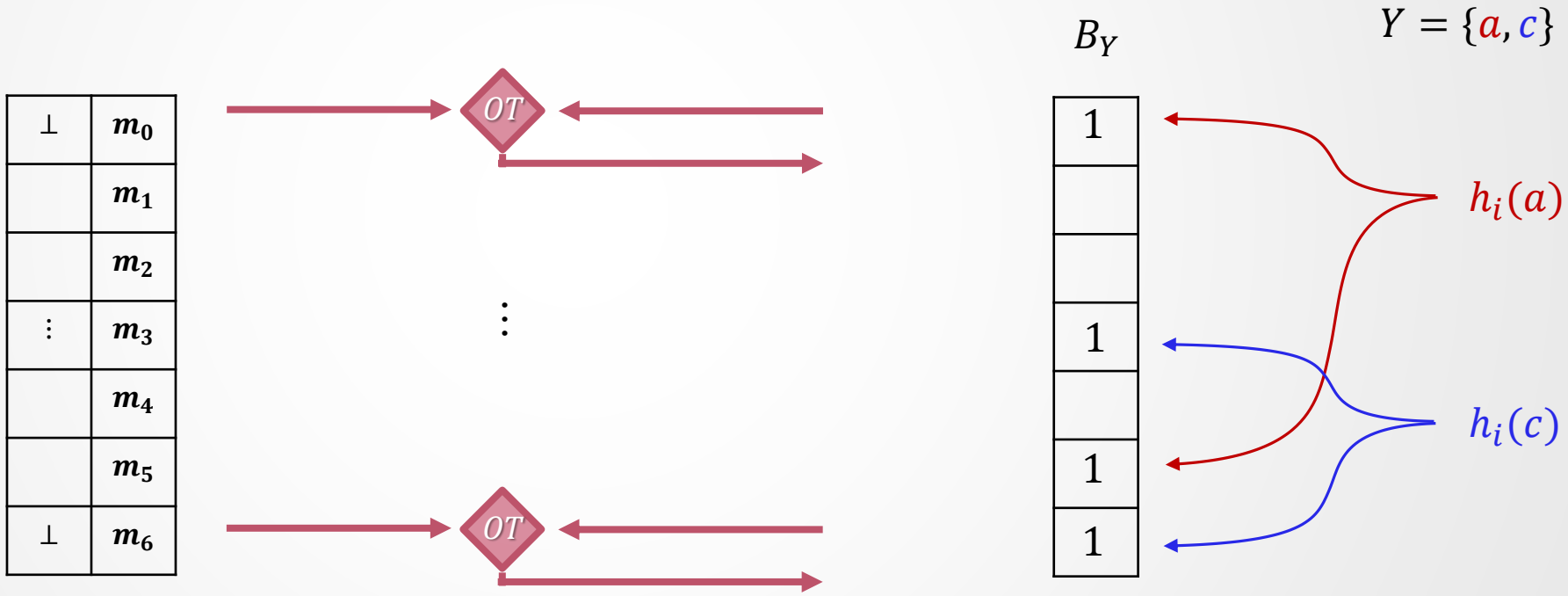

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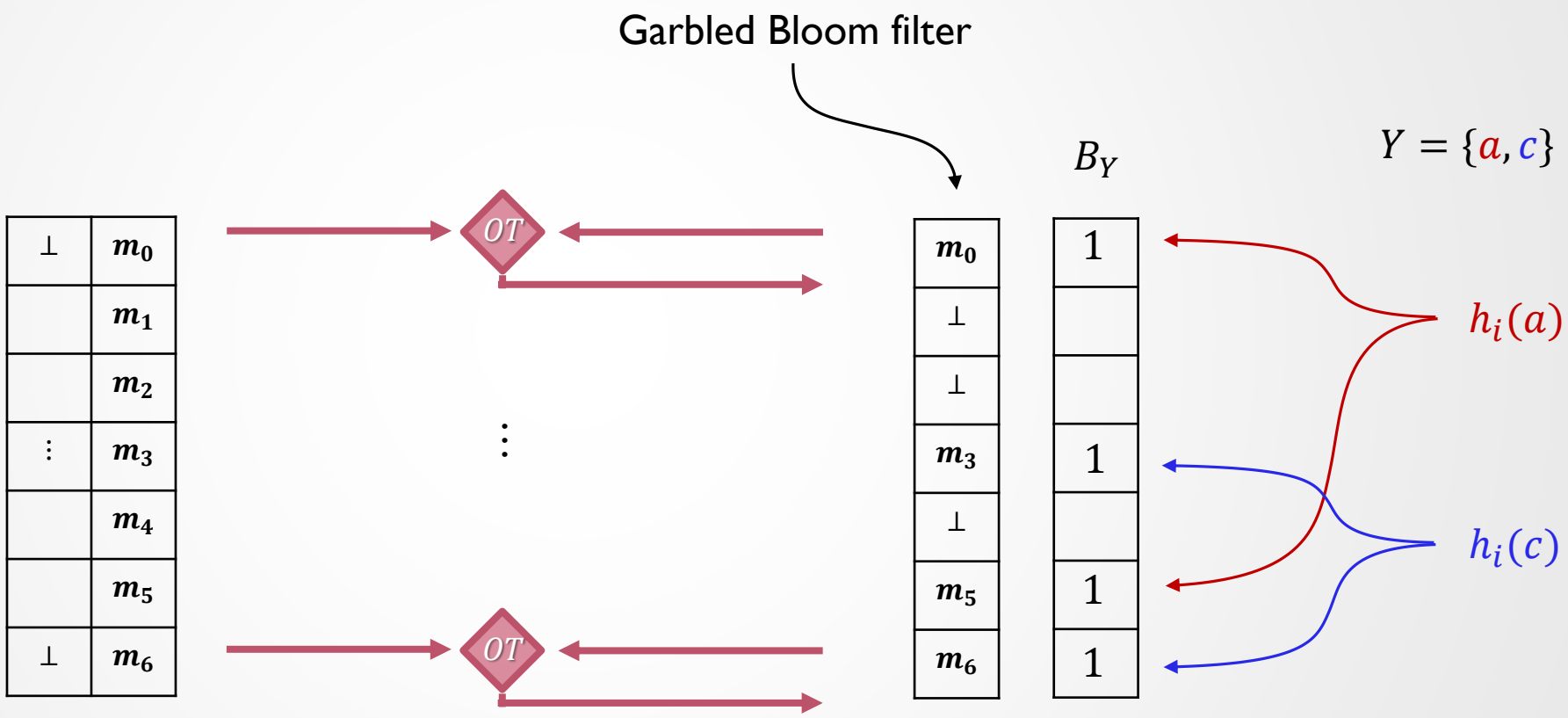

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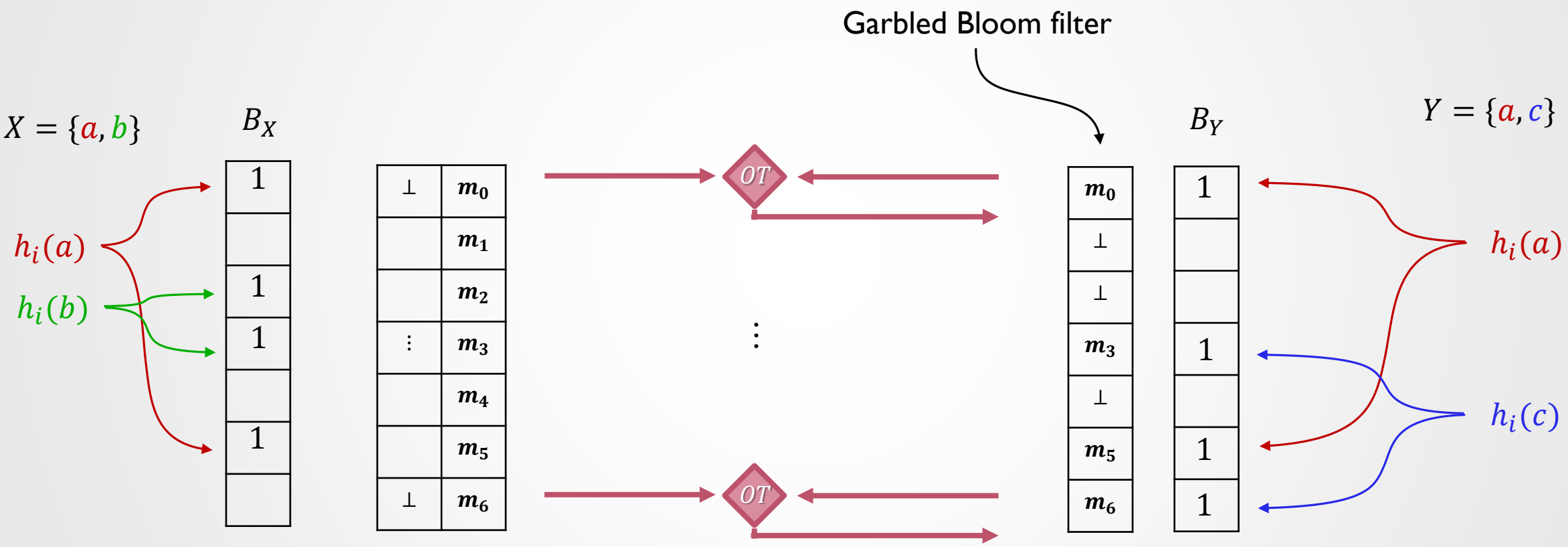

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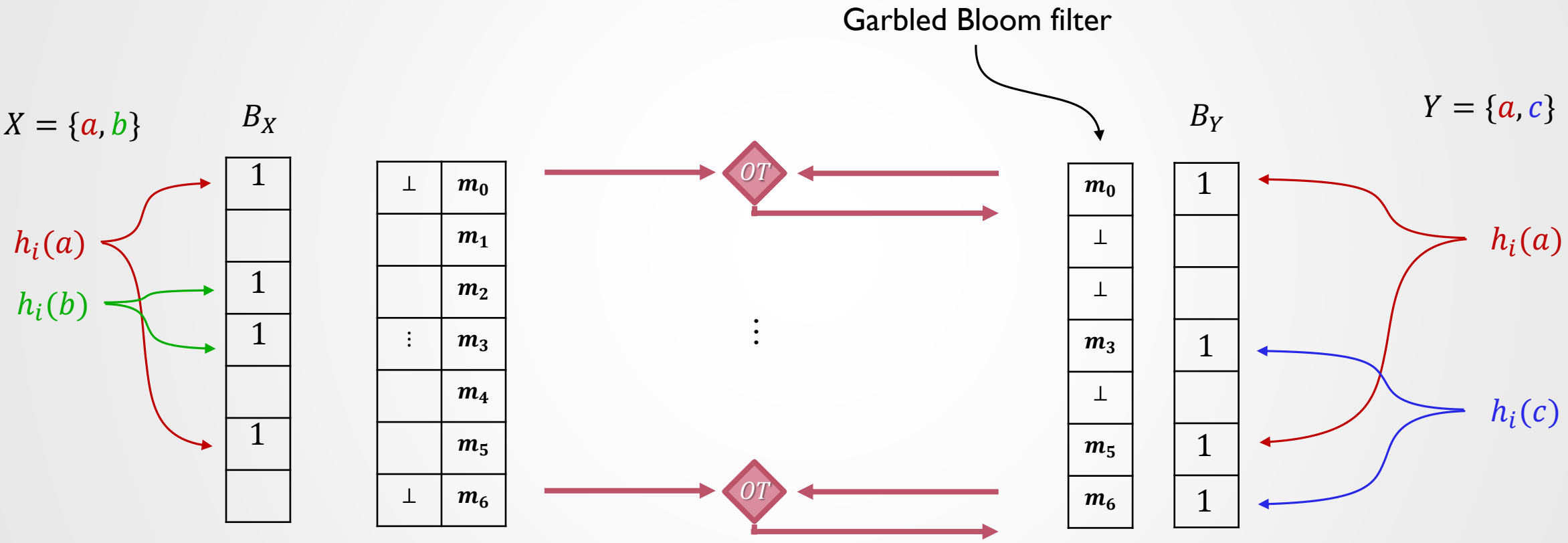


$$\widehat{X} = \{m_0 \oplus m_5, m_2 \oplus m_3\}$$

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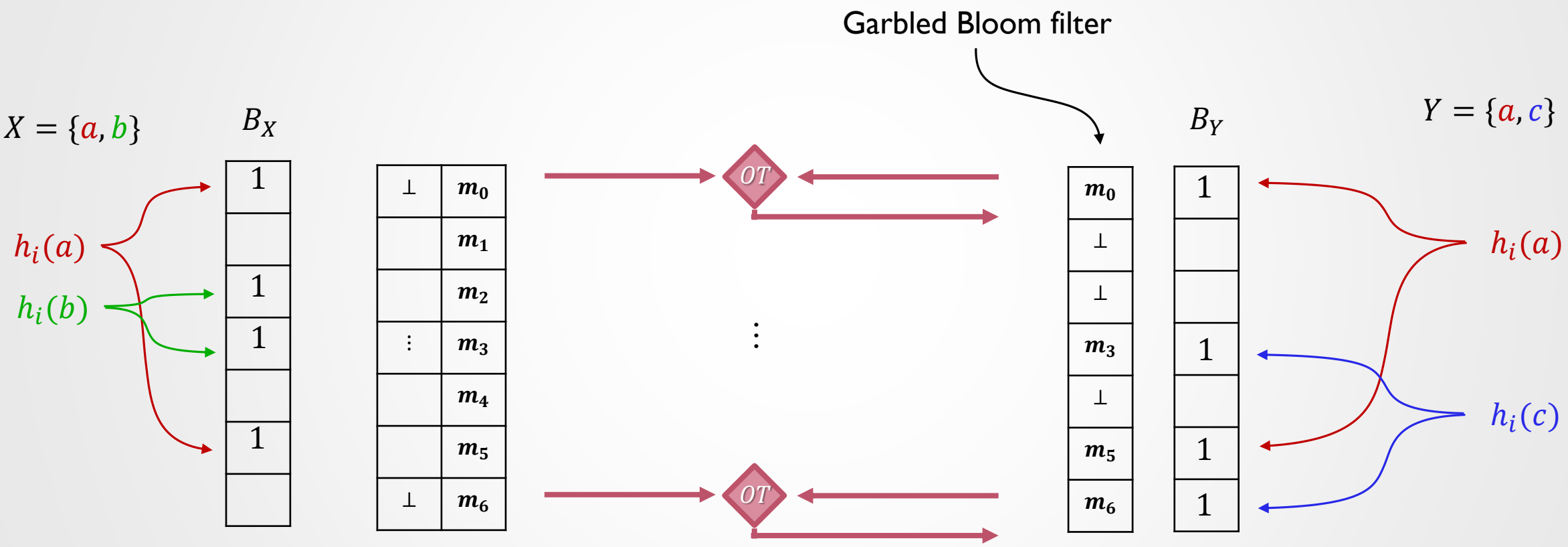



$$\widehat{X} = \{m_0 \oplus m_5, m_2 \oplus m_3\}$$

Output the intersection

$$\widehat{X} \cap \{m_0 \oplus m_5, m_3 \oplus m_6\}$$

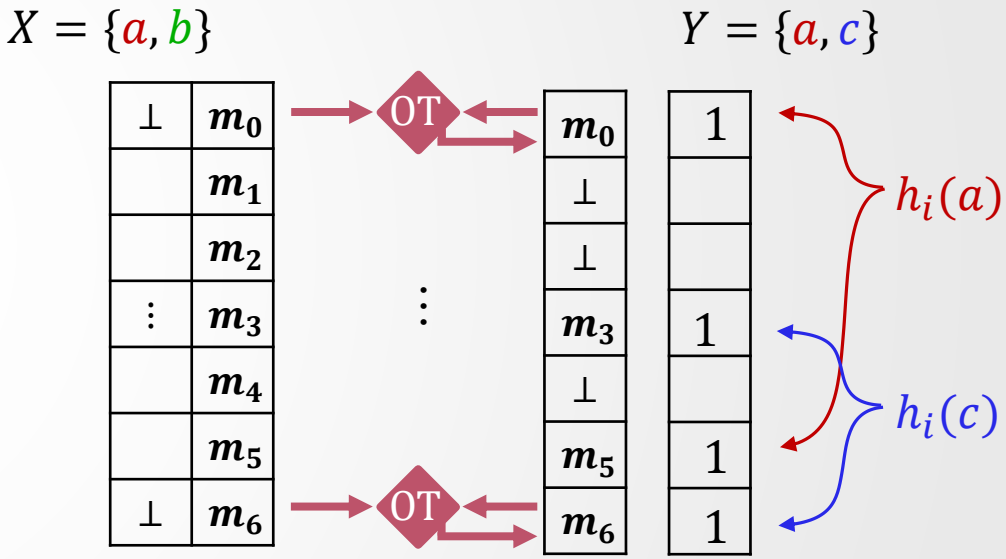
# Semi-Honest Security

[DongChenWen13, PinkasSchniederZohner14]

Naturally secure against Sender.

- OT hides select bits
- Final message sent to Receiver
- $\notin Y$, Receiver learns encoding

  e.g. $\text{Encode}(y') = m_3 \oplus m_4$
- DCW13 show equivalence to false positive in standard bloom filter
  - Pr[ false positives ] $\approx 2^{-k}$




Output:

$$\hat{X} = \begin{Bmatrix} m_0 \oplus m_5, \\ m_2 \oplus m_3 \end{Bmatrix} \longrightarrow \hat{X} \cap \begin{Bmatrix} m_0 \oplus m_5, \\ m_3 \oplus m_6 \end{Bmatrix}$$

# Semi-Honest Security

[DongChenWen13, PinkasSchniederZohner14]

$y' \notin YY$, Receiver learns encoding

Naturally secure against Sender.

- OT hides select bits
- Final message sent to Receiver

- Secure against Receiver
  - Attack: For $y''' \notin Y$, Receiver learns encoding

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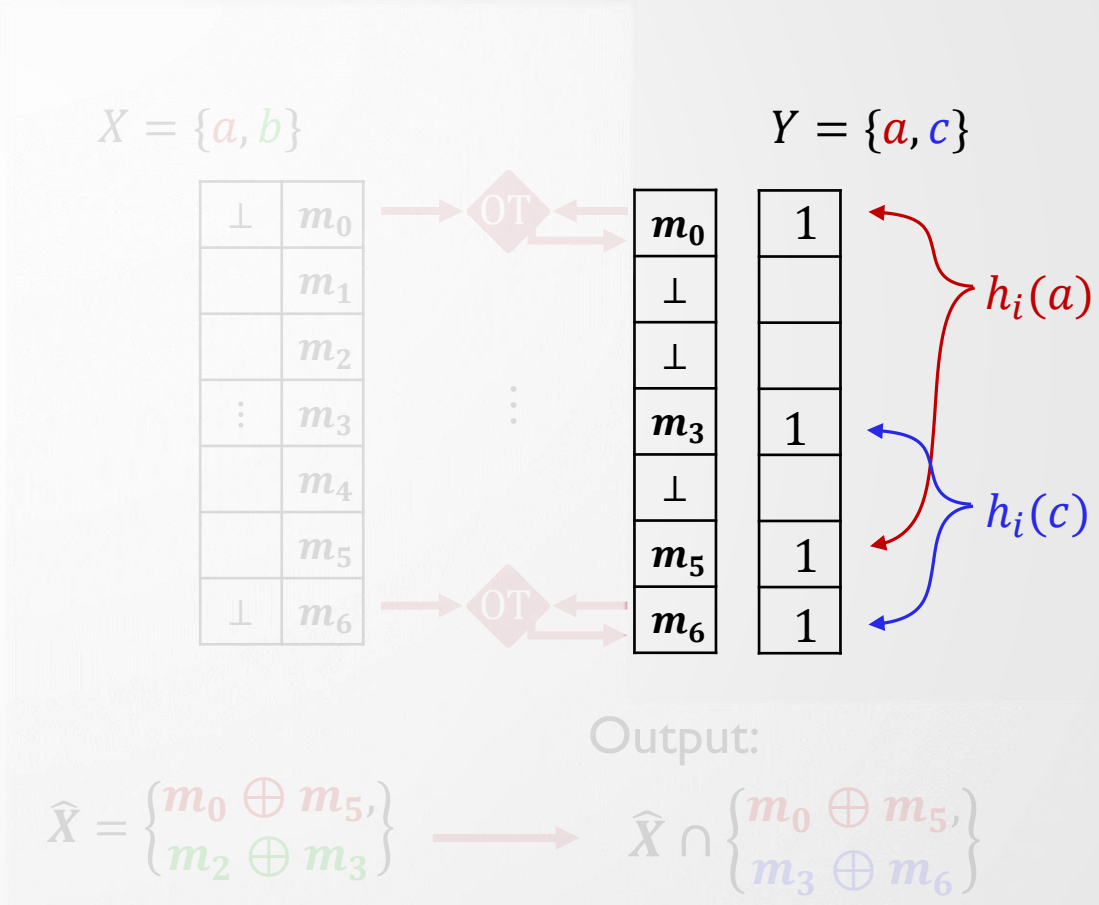

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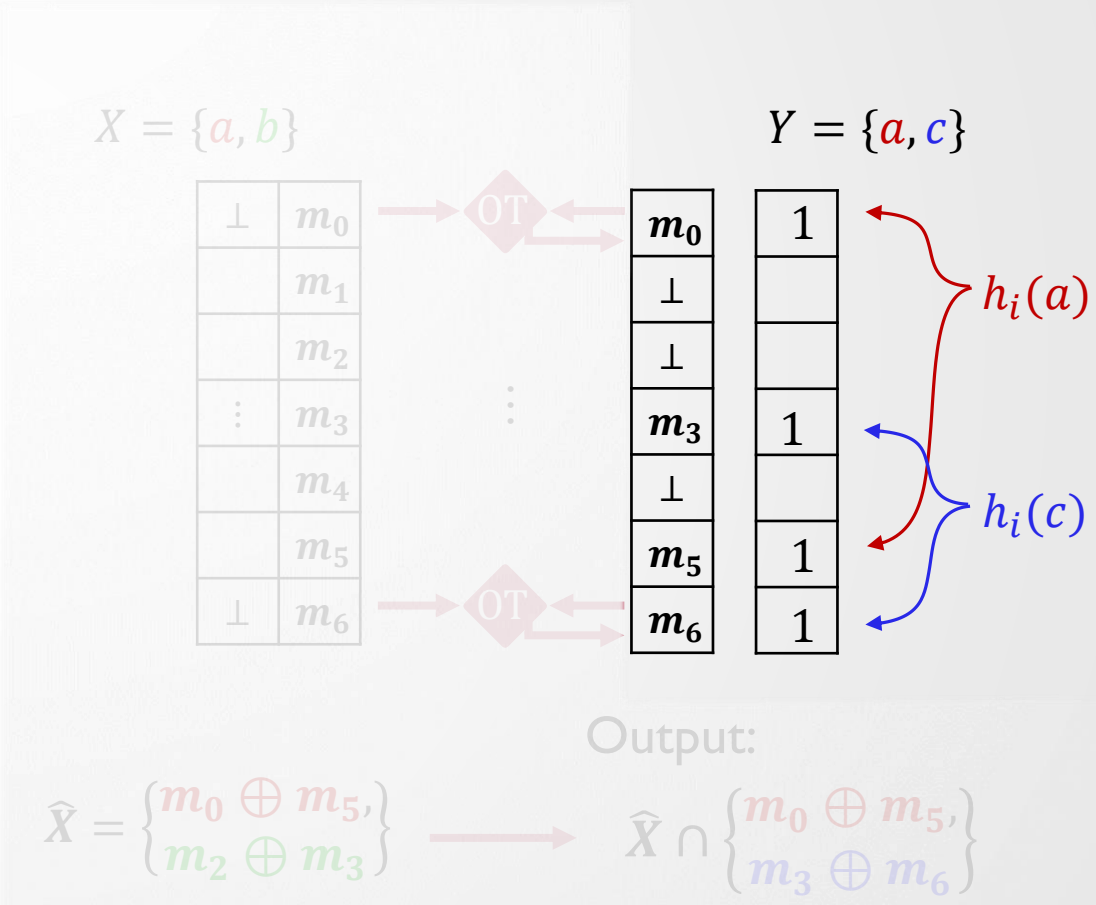

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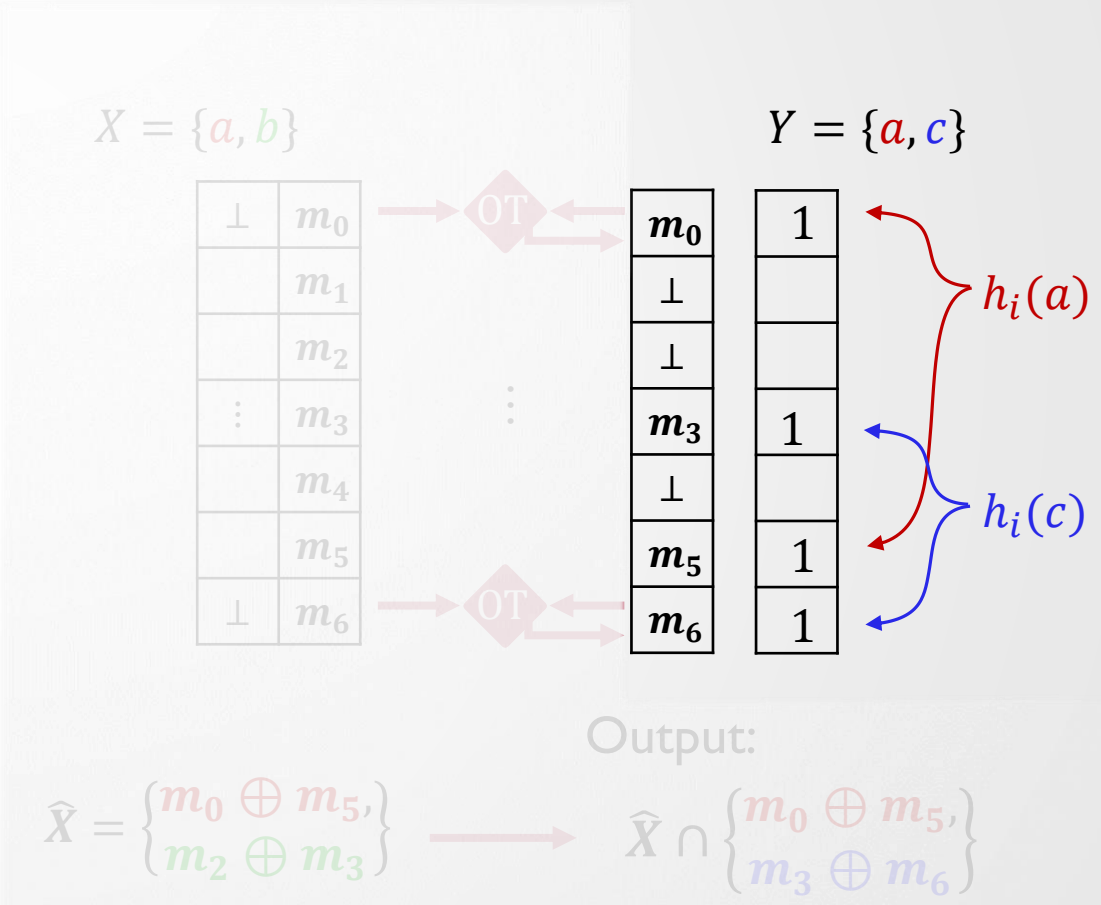

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$2\ -k\ 2\ 2\ -k\ -kk\ 2\ -k$

e.g. Encode $y'\ yy'\ y' = m\ 3\ mm\ m\ 3\ 3\ m\ 3 \oplus m\ 4\ mm\ m\ 4\ 4\ m\ 4$

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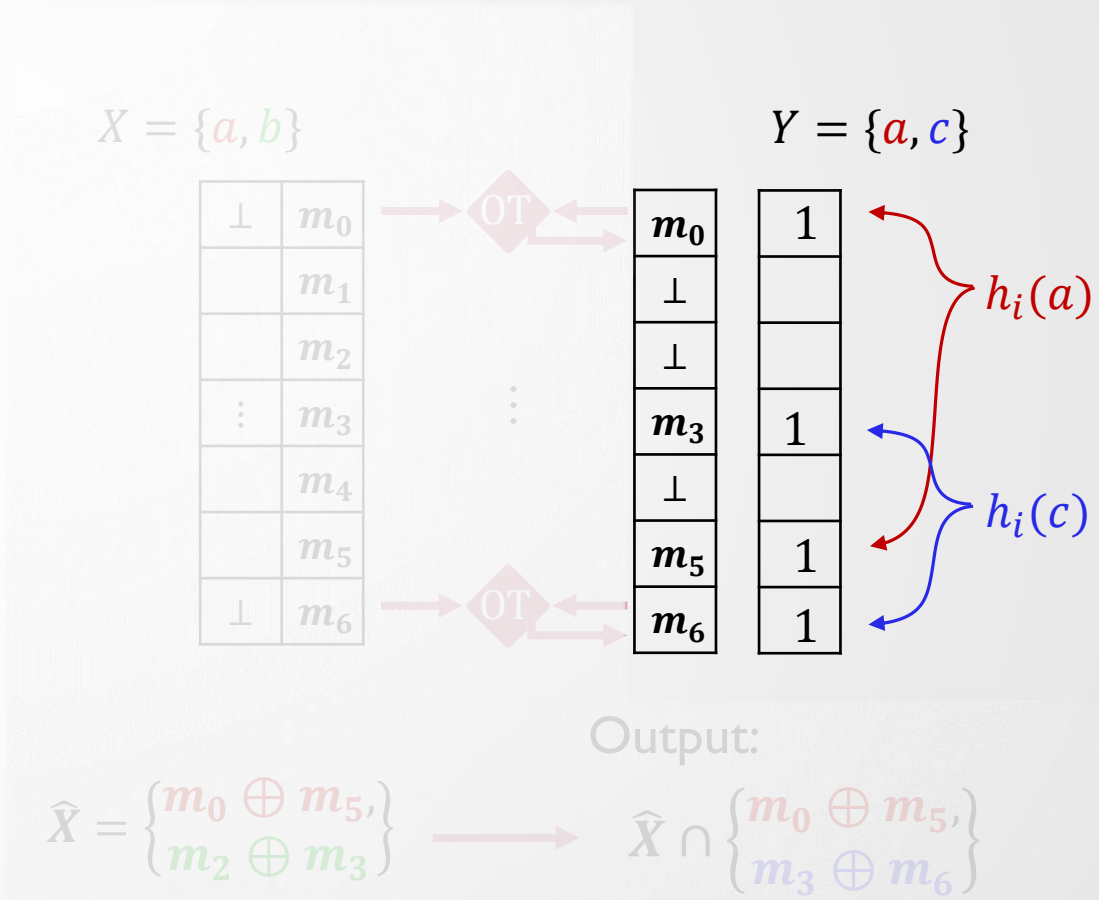
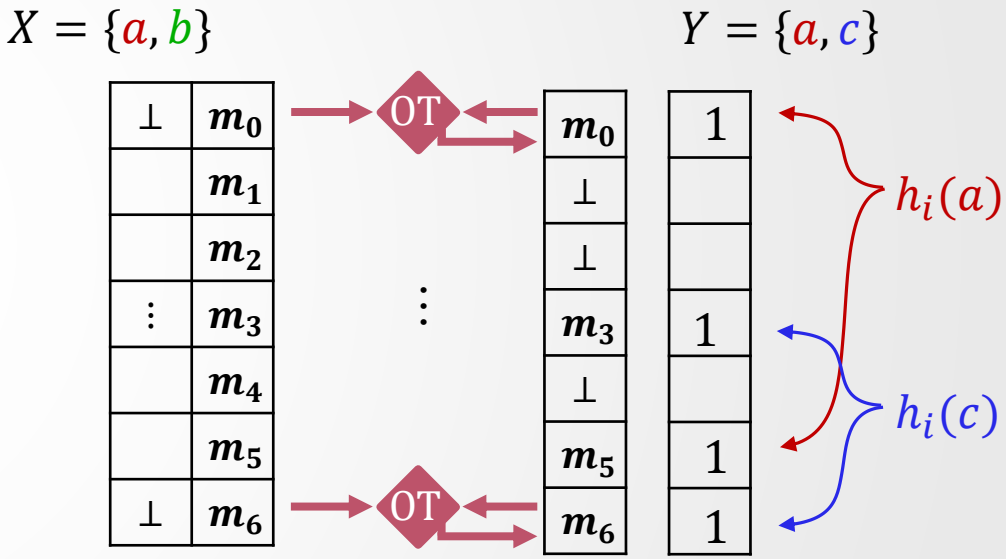


# Malicious Receiver

**Insecure** against Receiver

- Bloom filter
- Receiver will obtain all $m_i$
- Can probe for $\boldsymbol{m_2 \oplus m_3}$

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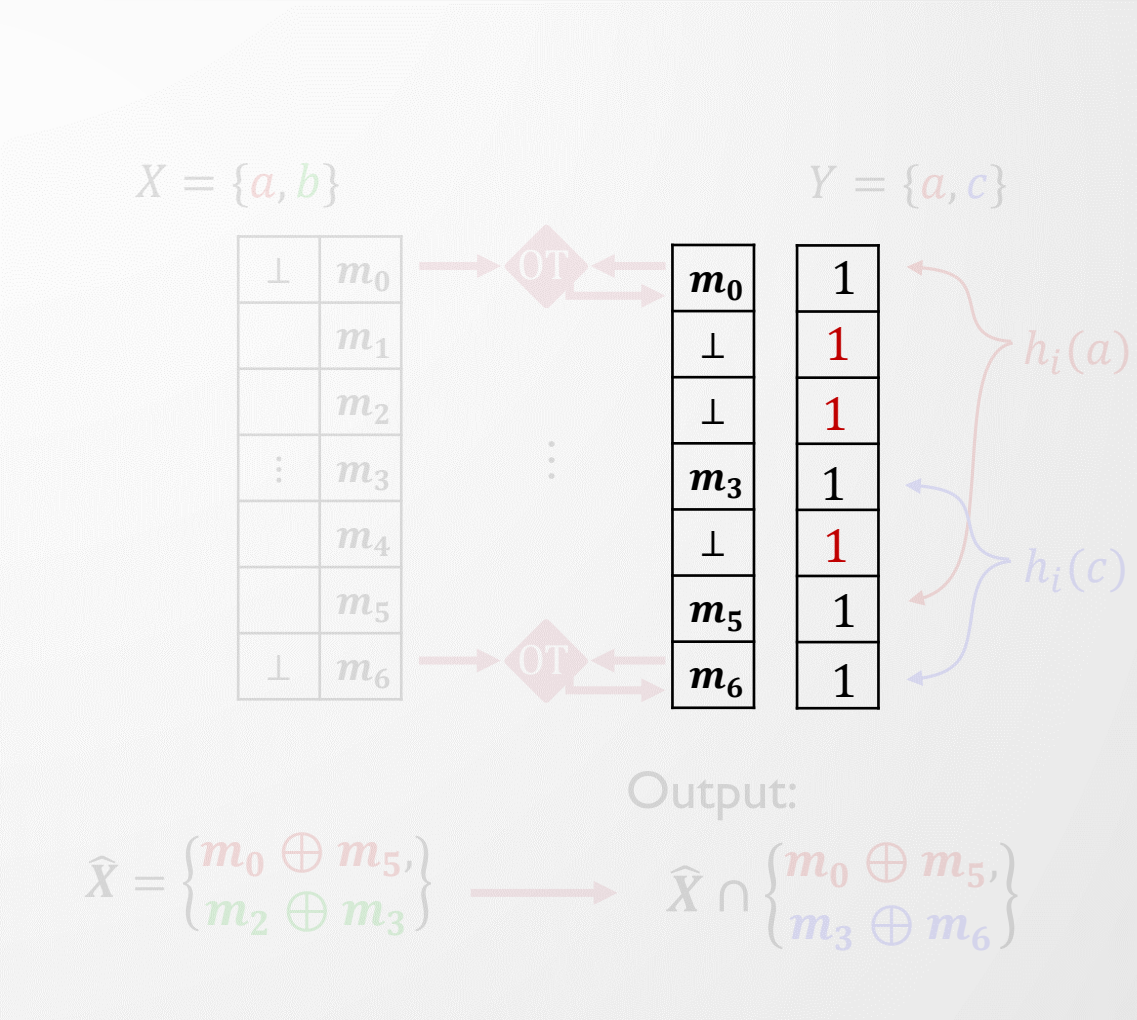
# Malicious Receiver

Bloom filter

Insecure against Receiver

- Consider all 1 Bloom filter
- Receiver will obtain all $m_i$
- Can probe for $\boldsymbol{m_2 \oplus m_3}$

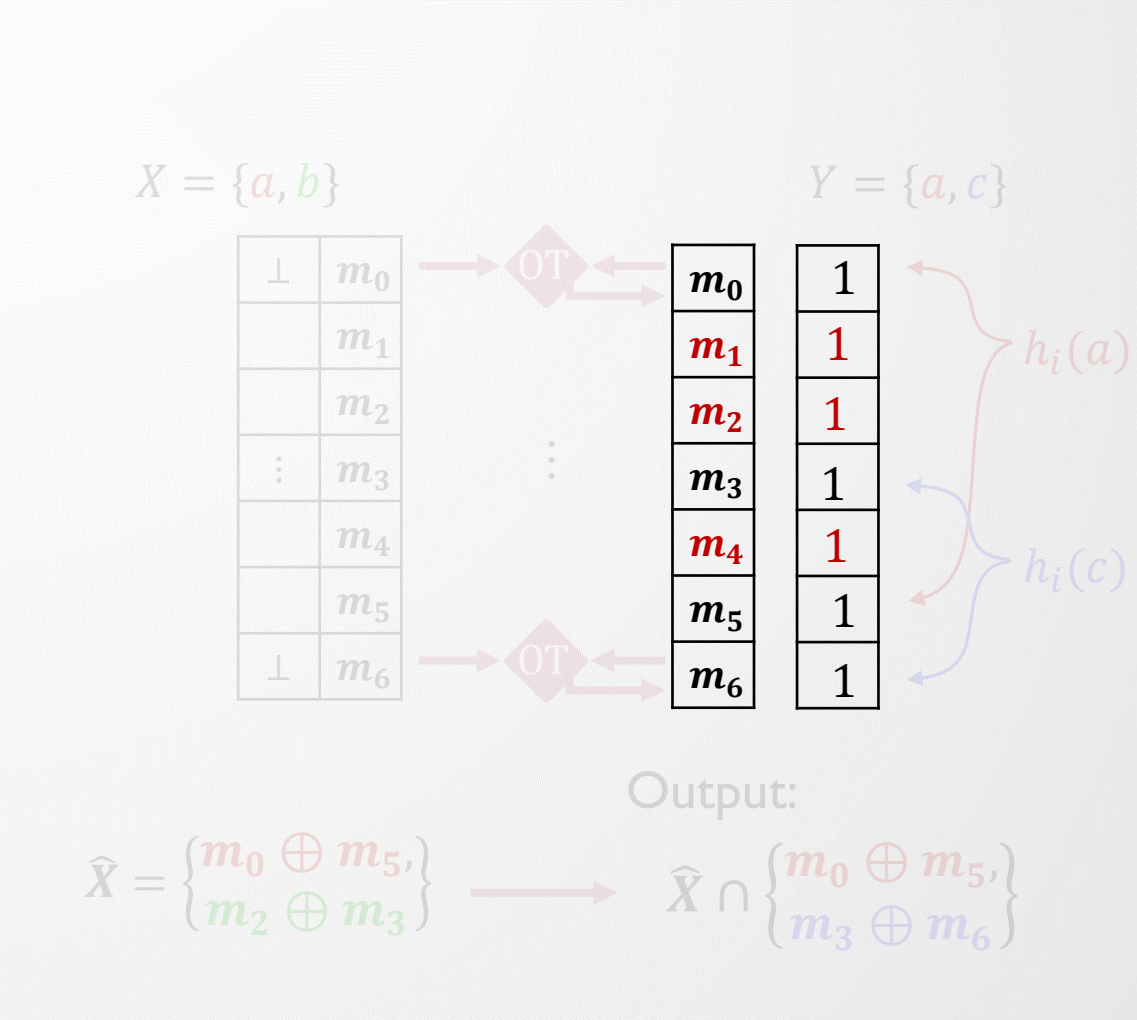


# Malicious Receiver

Bloom filter

Insecure against Receiver

- Receiver will obtain all $m\ i$
- $i\ i\ i$
- Receiver will obtain all $m_i$
- Can probe for $\boldsymbol{m_2 \oplus m_3}$



# Malicious Receiver

$\boldsymbol{m\ 3\ mm\ m\ 3\ 33\ m\ 3}$

Bloom filter

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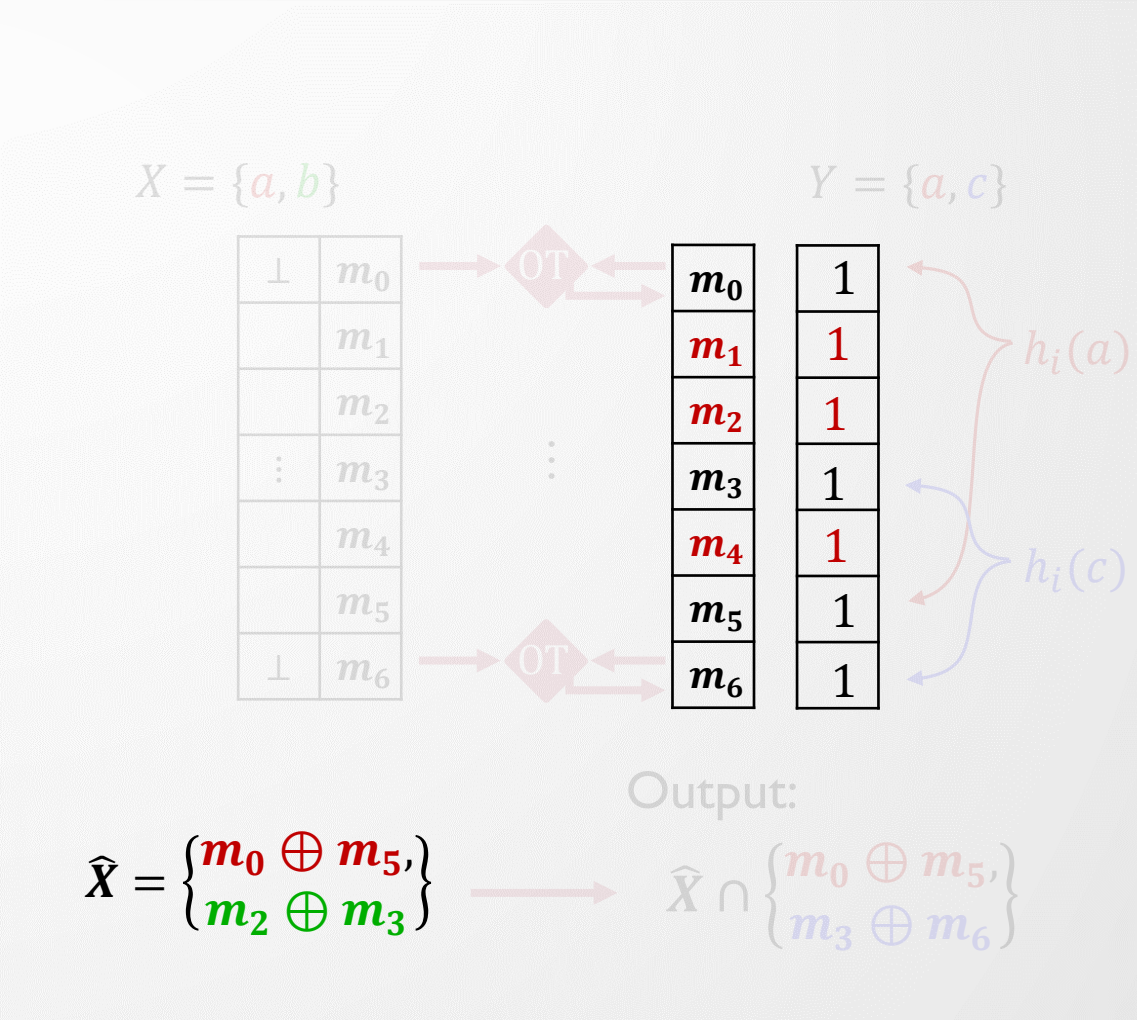
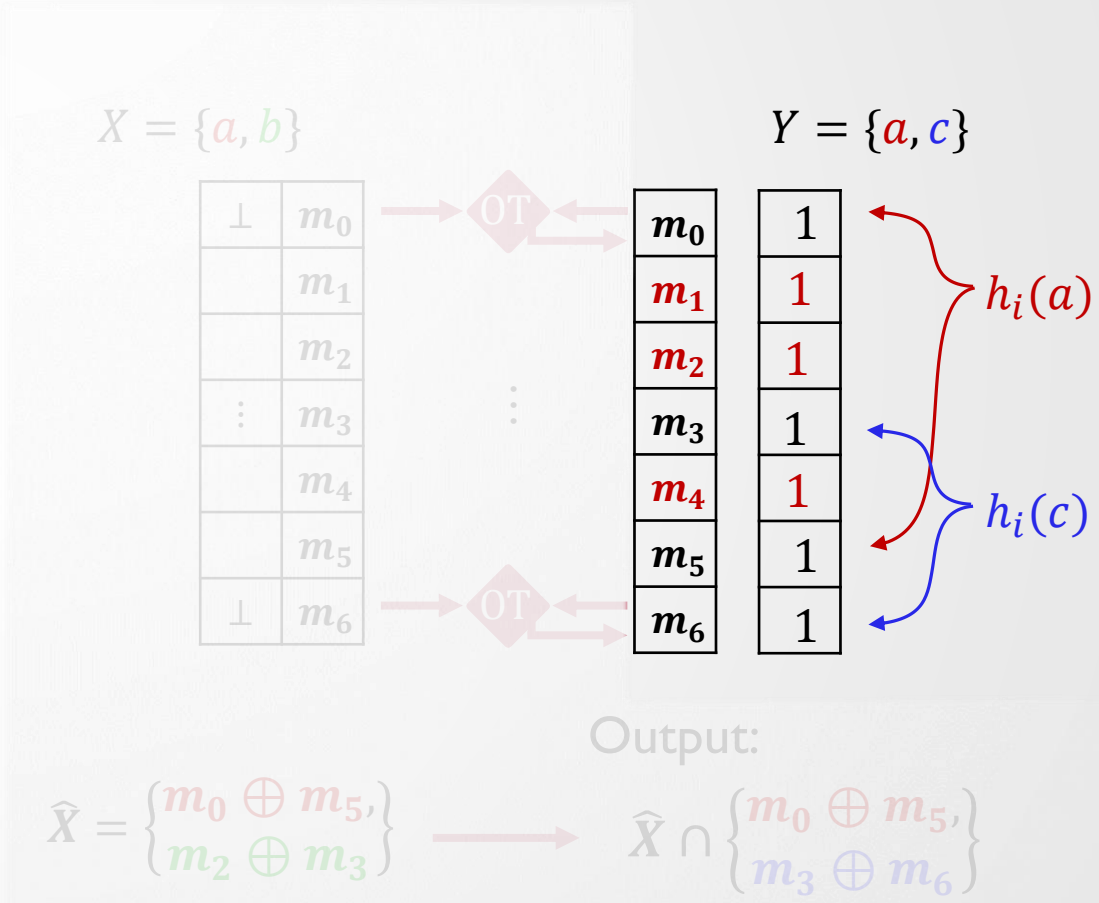


# Warm-Up: The DongChenWen13 Approach

Goal – restrict the Receiver to a valid Bloom filter

- bits contains $\frac{1}{2}m$ ones

- Make Receiver prove zero choice bits
  - Sample random key $s \leftarrow \{0,1\}^{\kappa}$
  - Generate a $\frac{m}{2}$ out of $m$ secret sharing of $s$
    - $s_1, \ldots, s_m$
  - Transmit $s_i$ as the $i$th zero OT message
  - Encrypt summary values with $s$



# Warm-Up: The DongChenWen13 Approach

1 2 *2 1 2 mm* ones

Goal – restrict the Receiver to a valid Bloom filter

- Bloom filter of $m$ bits contains 1 2 $m$ ones

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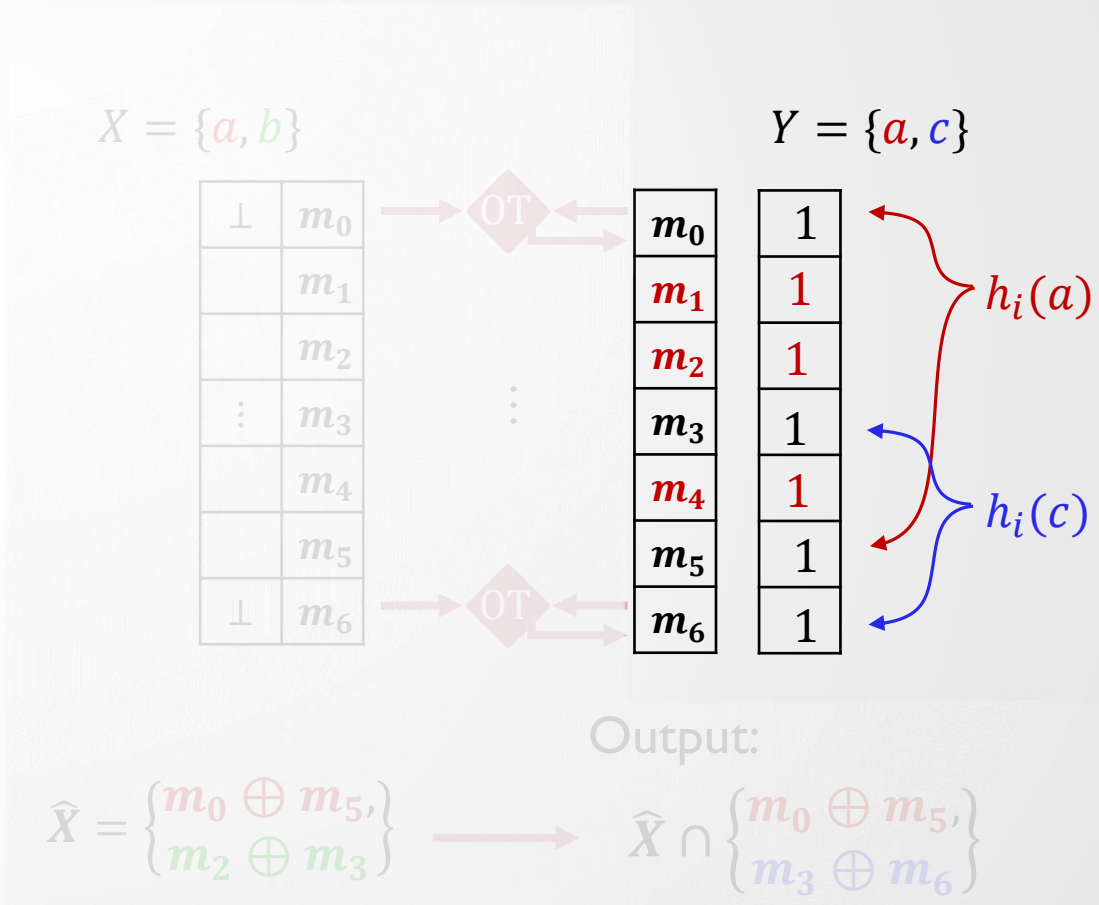

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0,1 $\kappa$ 0,1 0,1 0,1 0,1 $\kappa$ $\kappa\kappa$ 0,1 $\kappa$

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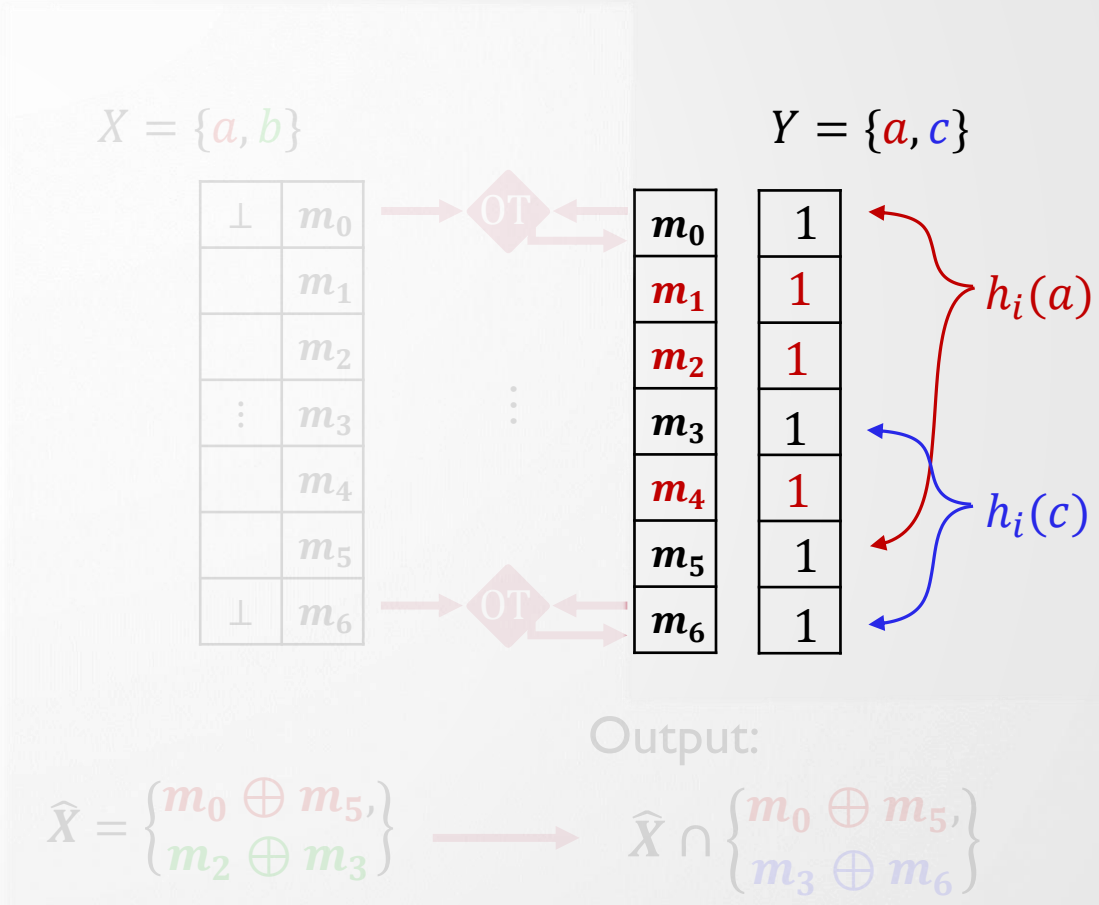

# Warm-Up: The DongChenWen13 Approach

$s$ 1 ,…, $s$ $m$ $ss$ $s$ $m$ $mm$ $s$ $m$

$m$ 2 out of $mm$ secret sharing of $ss$

0,1 $\kappa$ 0,1 0,1 0,1 0,1 $\kappa$ $\kappa\kappa$ 0,1 $\kappa$

1 2 $2$ $1$ $2$ $mm$ ones

Goal – restrict the Receiver to a valid Bloom filter

- Bloom filter of $m$ bits contains 1 2 $m$ ones

- Make Receiver prove zero choice bits
  - $s$ 1 1 1 , … , $s_m$
  - Sample random key $s \leftarrow \{0,1\}^\kappa$
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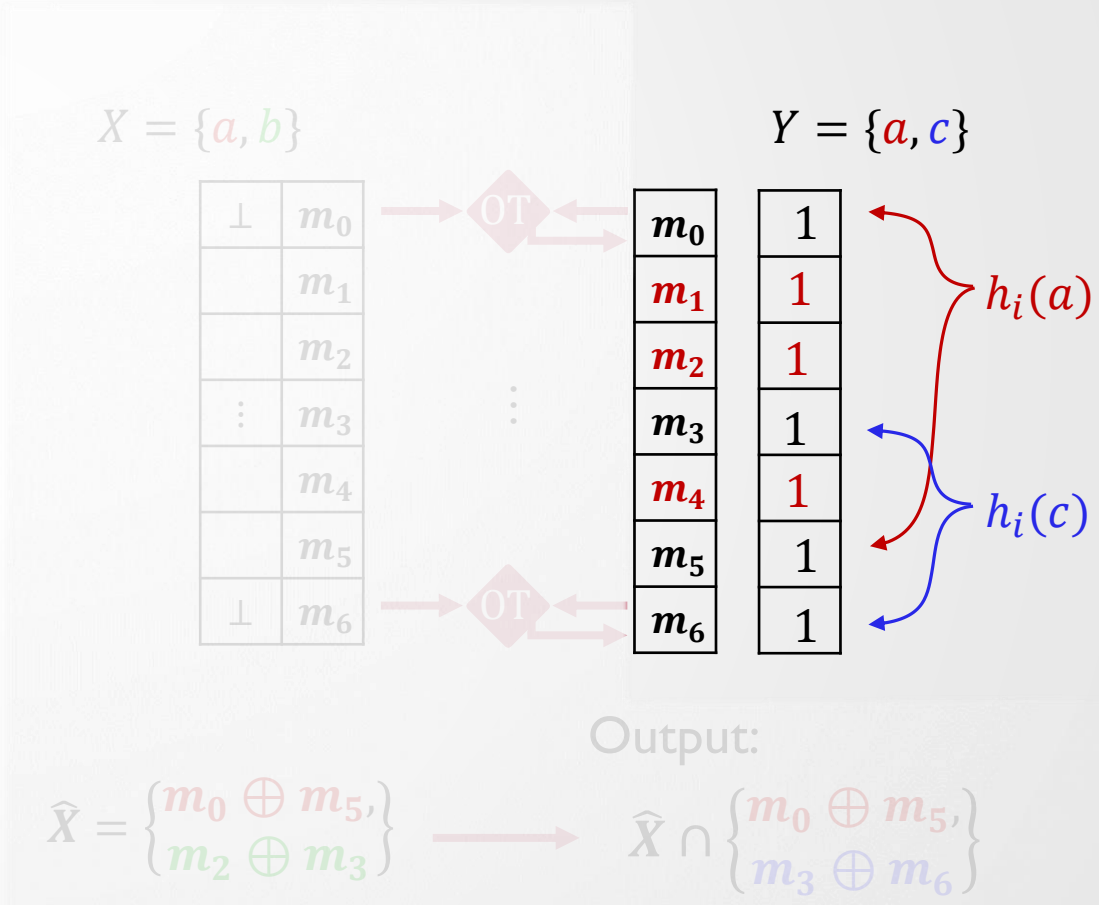

# Warm-Up: The DongChenWen13 Approach

as the $ii$th zero OT message

$s$ 1 ,…, $s$ $m$ $ss$ $s$ $m$ $mm$ $s$ $m$

$m$ 2 out of $mm$ secret sharing of $ss$

0,1 $\kappa$ 0,1 0,1 0,1 0,1 $\kappa$ $\kappa\kappa$ 0,1 $\kappa$

1 2 $2$ $1$ $2$ $mm$ ones

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- Make Receiver prove zero choice bits
  - $s$ 1 1 1 , … , $s_m$
  - Transmit $s$ $i$ $i$ $i$ $i$ as the $i$th zero OT message
  - Generate a $\frac{m}{2}$ out of $m$ secret sharing of $s$
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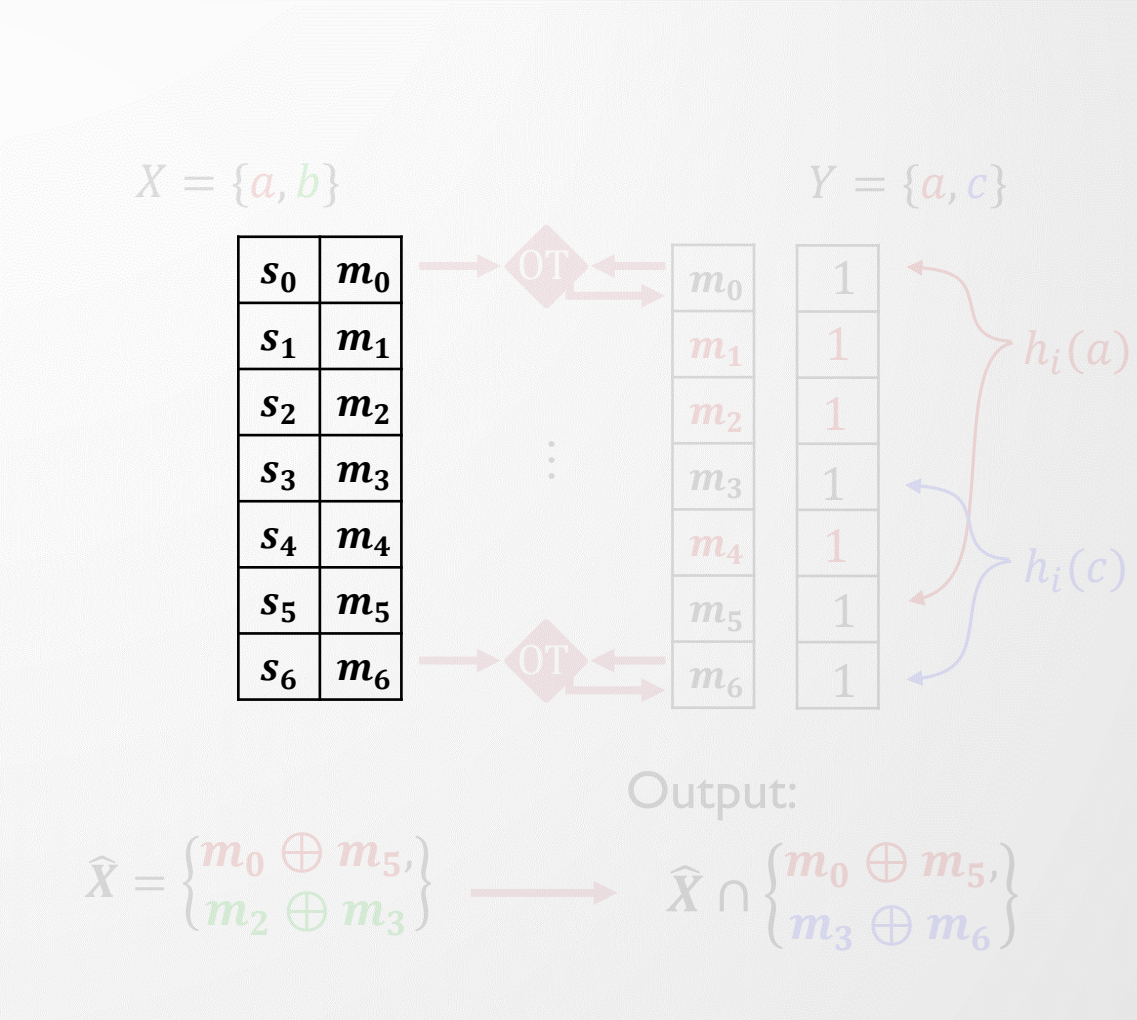

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as the $ii$th zero OT message

$s$ 1 ,…, $s$ $m$ $ss$ $s$ $m$ $mm$ $s$ $m$

$m$ 2 out of $mm$ secret sharing of $ss$

0,1 $\kappa$ 0,1 0,1 0,1 0,1 $\kappa$ $\kappa\kappa$ 0,1 $\kappa$

1 2 $2$ $1$ $2$ $mm$ ones

Goal – restrict the Receiver to a valid Bloom filter

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  - $s$ 1 1 1 , … , $s_m$
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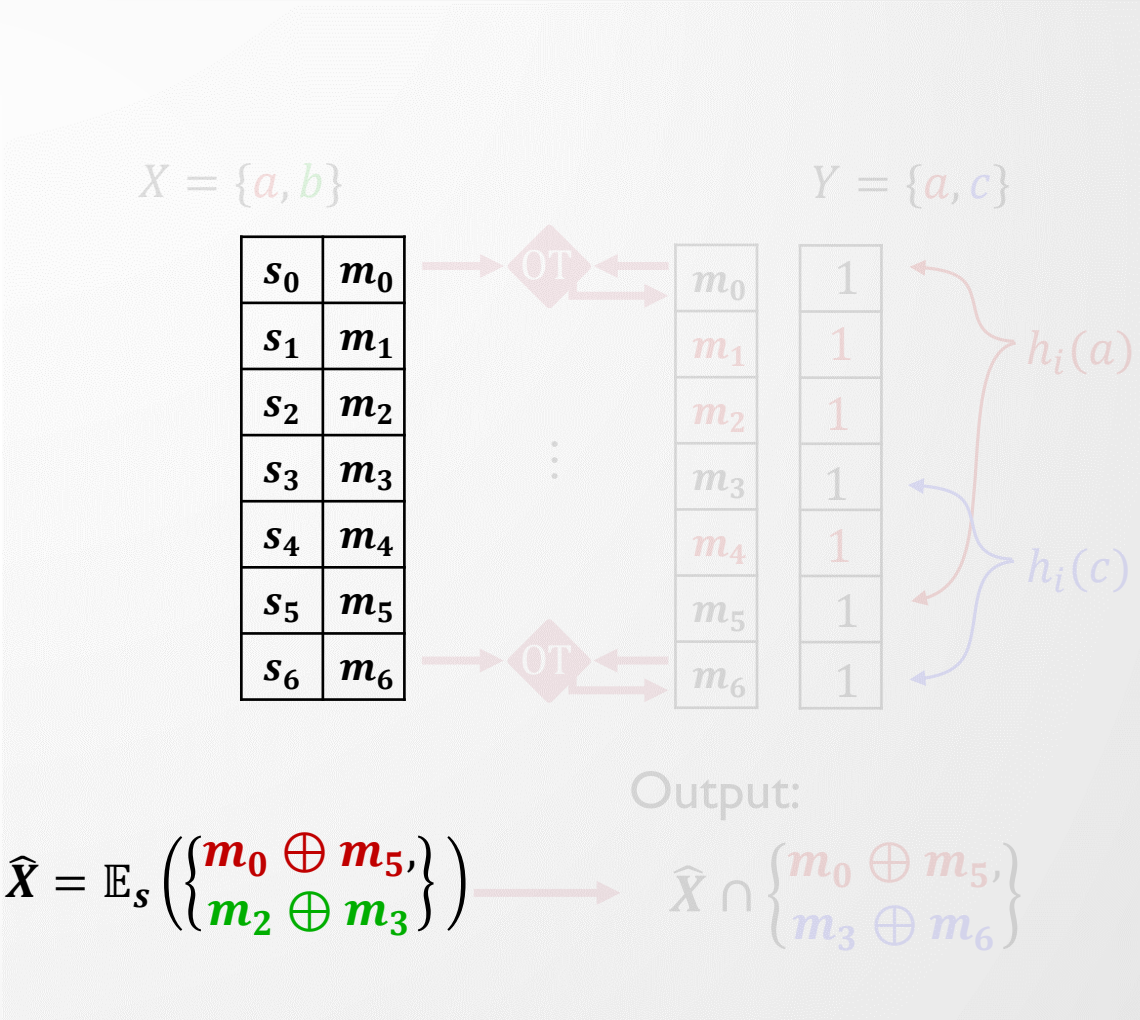

# Warm-Up: The DongChenWen13 Approach

as the $ii$th zero OT message

$s$ 1 ,…, $s$ $m$ $ss$ $s$ $m$ $mm$ $s$ $m$

$m$ 2 out of $mm$ secret sharing of $ss$

0,1 $\kappa$ 0,1 0,1 0,1 0,1 $\kappa$ $\kappa\kappa$ 0,1 $\kappa$

1 2 $2$ $1$ $2$ $mm$ ones

Goal – restrict the Receiver to a valid Bloom filter

- Bloom filter of $m$ bits contains 1 2 $m$ ones

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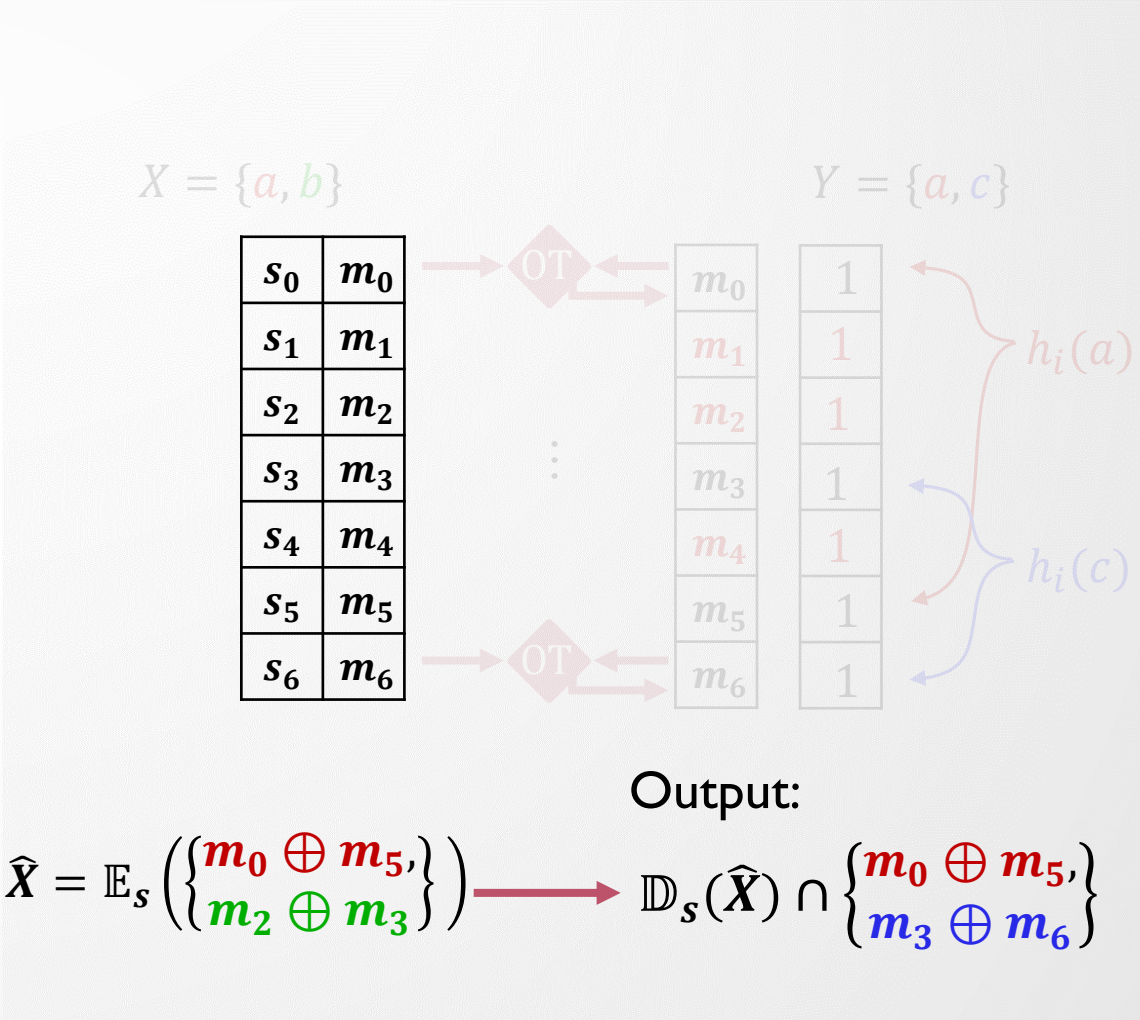

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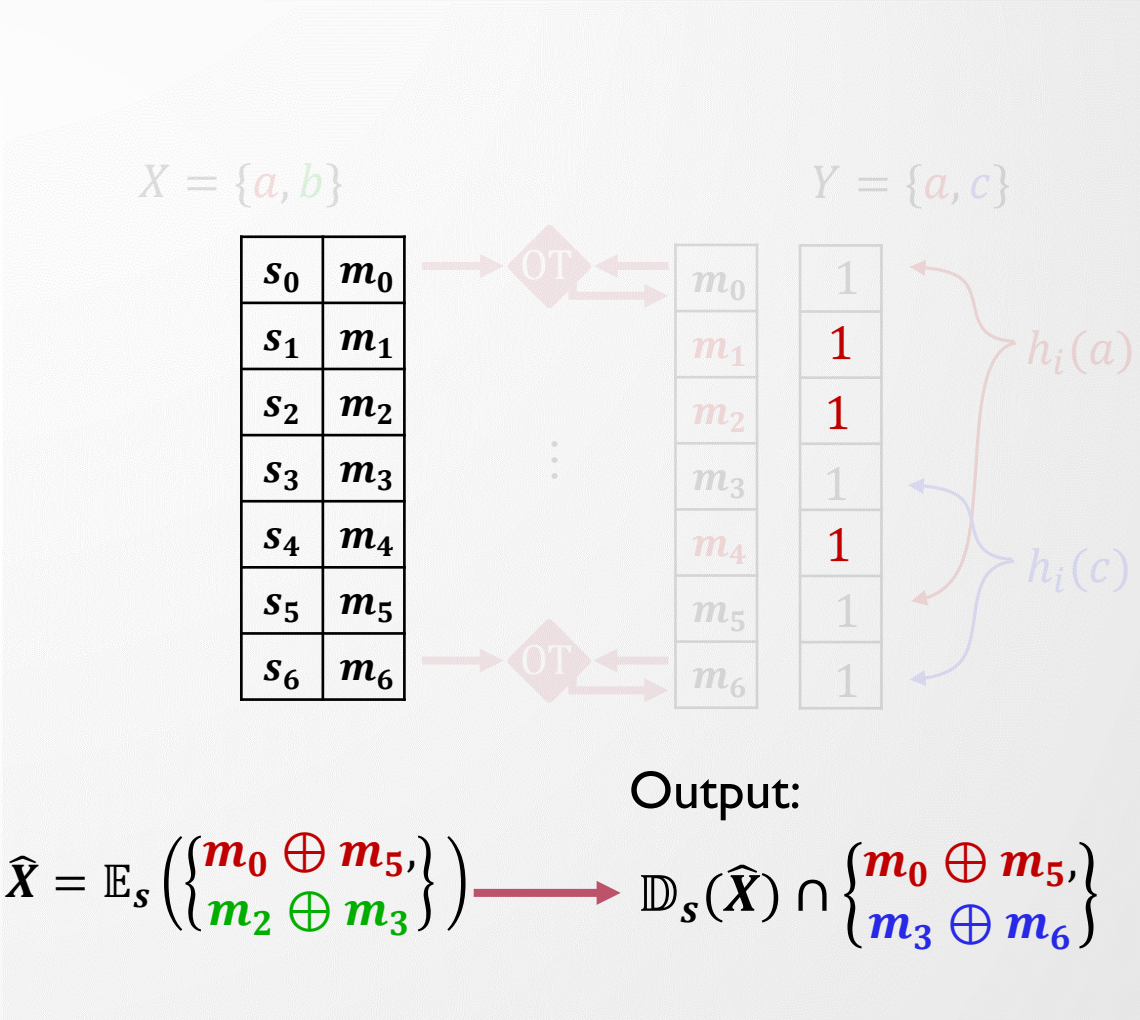

# Warm-Up: The DongChenWen13 Approach

- Goal – restrict the Receiver to a valid Bloom filter
  - Bloom filter of $m$ bits contains $\frac{1}{2}m$ ones
- Make Receiver prove zero choice bits
  - Sample random key $s \leftarrow \{0,1\}^{\kappa}$
  - Generate a $\frac{m}{2}$ out of $m$ secret sharing of $s$
    - $s_1, \ldots, s_m$
  - Transmit $s_i$ as the $i$th zero OT message
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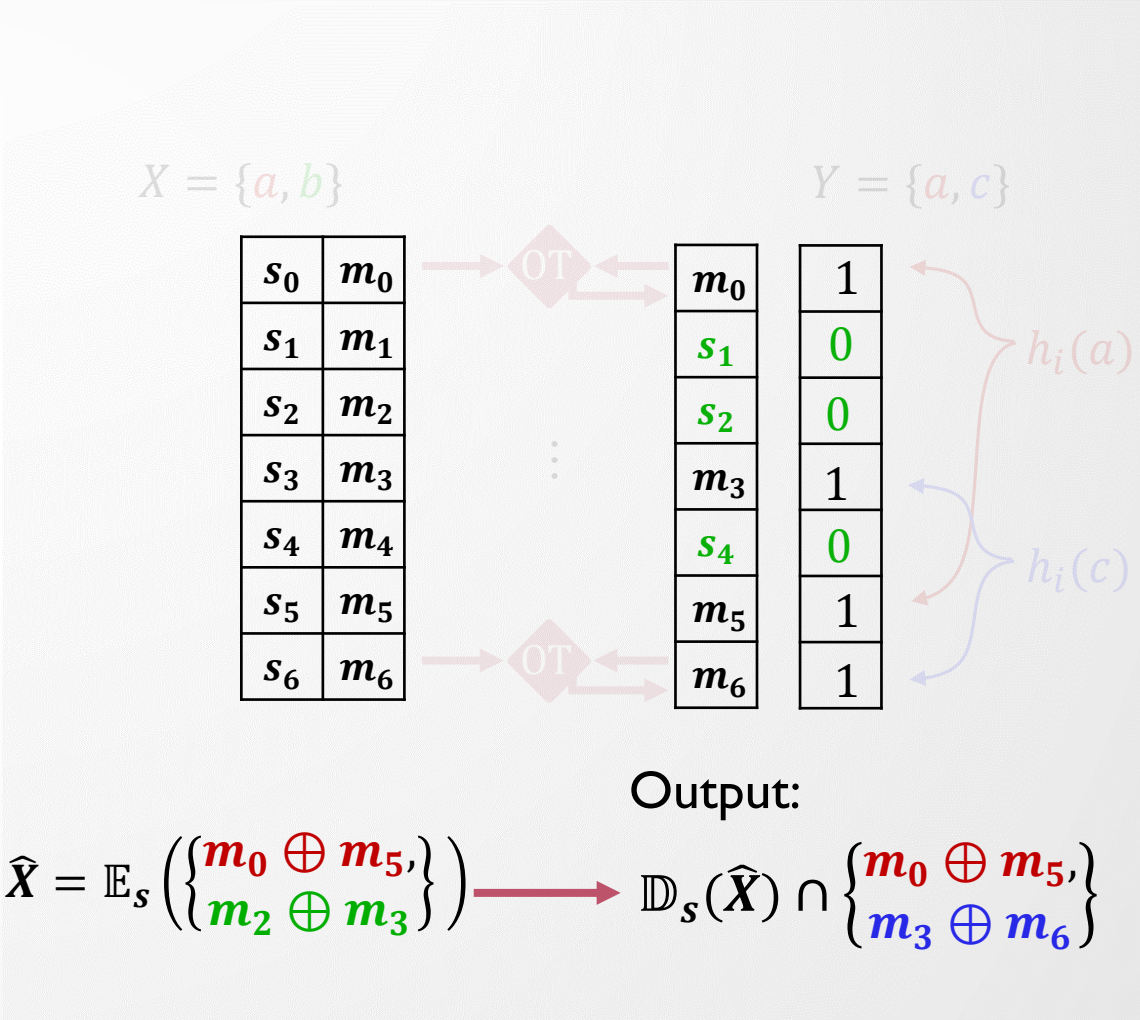


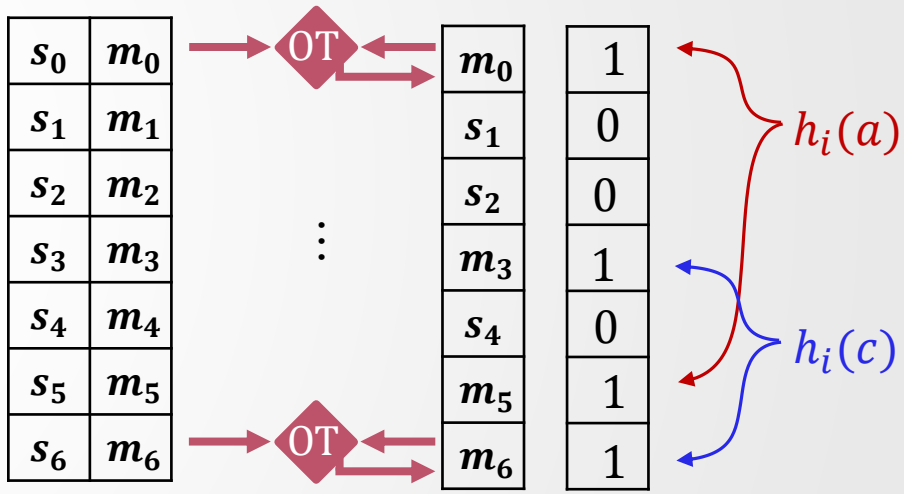
# Warm-Up: The DongChenWen13 Approach

[RindalRosulek17, Lambaek17]

Is this secure?

- $\frac{m}{2}$ ones
- Selective failure attack by the Sender…

$X = \{a, b\}$ $\qquad$ $Y = \{a, c\}$




Output:

$$\widehat{X} = \mathbb{E}_s\left(\begin{Bmatrix} m_0 \oplus m_5, \\ m_2 \oplus m_3 \end{Bmatrix}\right) \longrightarrow \mathbb{D}_s(\widehat{X}) \cap \begin{Bmatrix} m_0 \oplus m_5, \\ m_3 \oplus m_6 \end{Bmatrix}$$

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[RindalRosulek17, Lambaek17]

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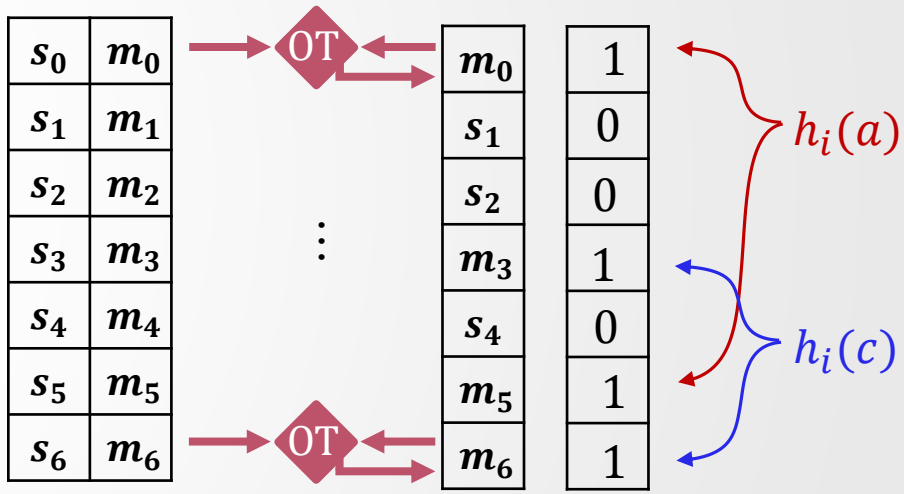


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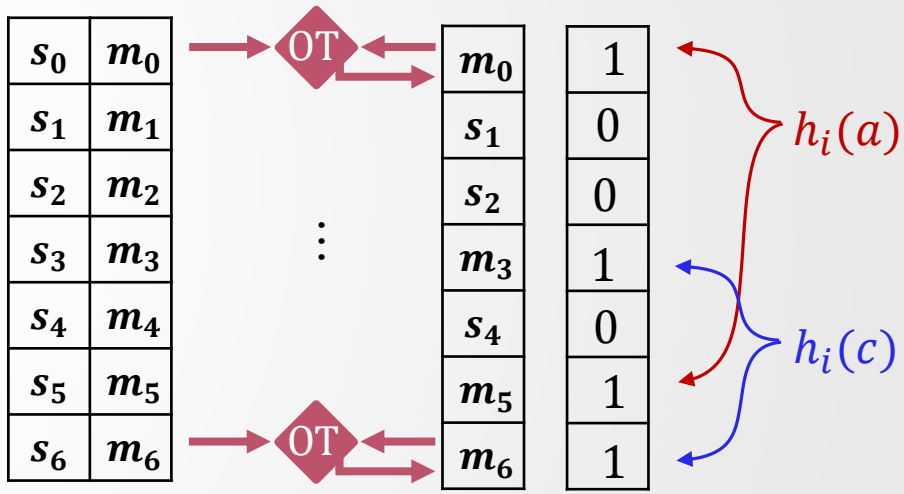


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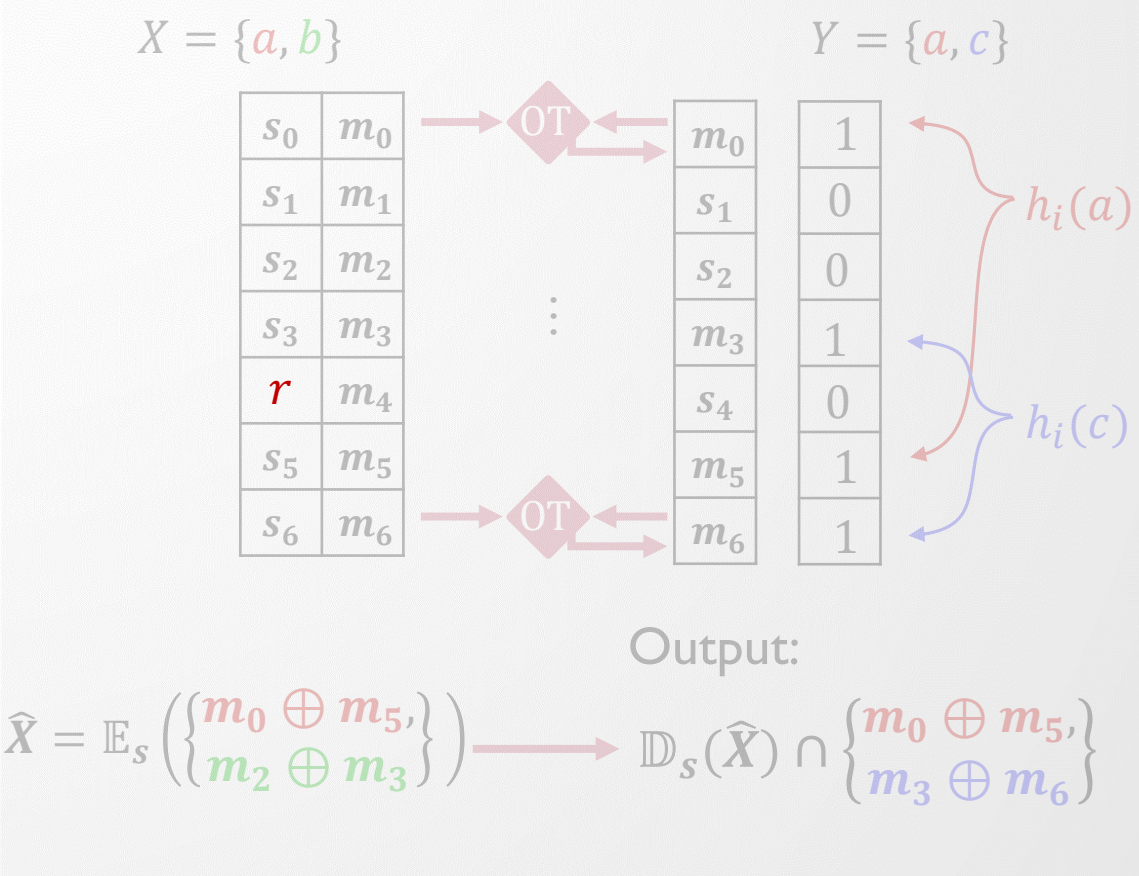

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  - replace $s_4$ with random value $r$
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- $$\forall y \in Y \quad : \quad h_i(y) \neq 4$$
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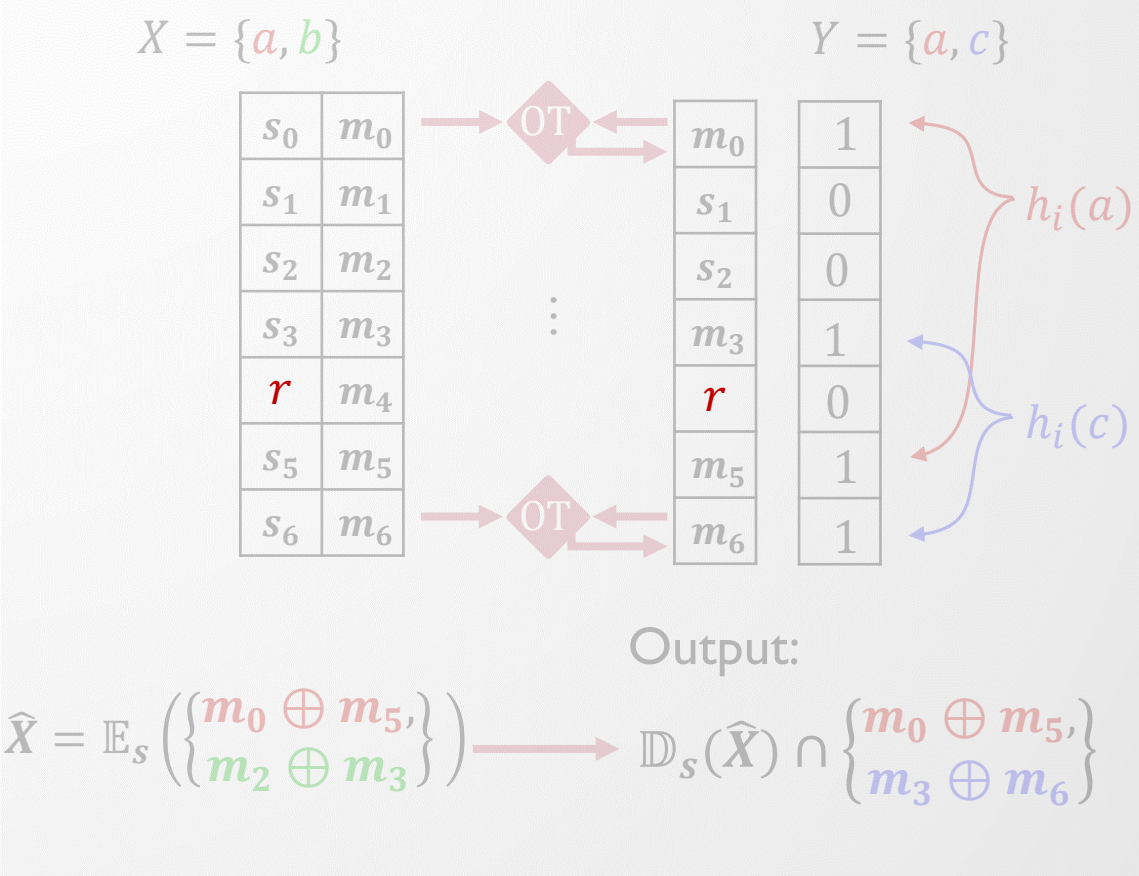

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[RindalRosulek17, Lambaek17]

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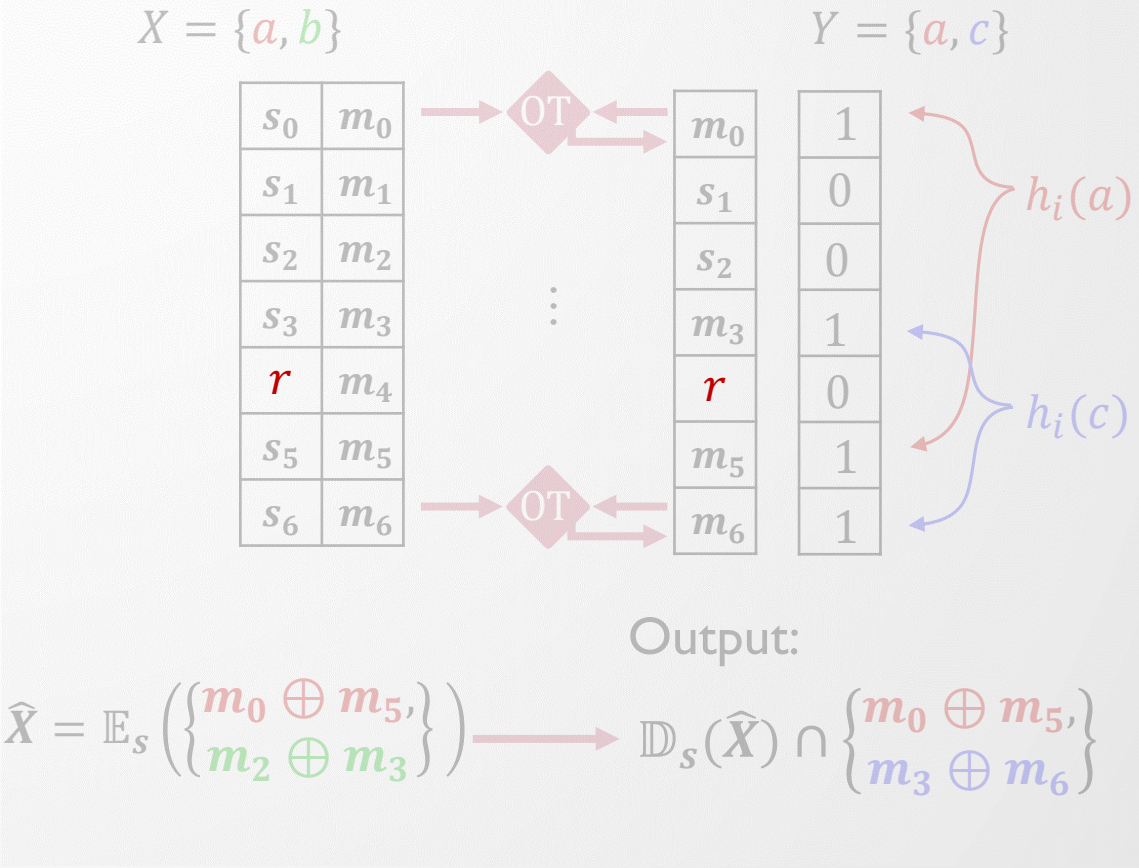
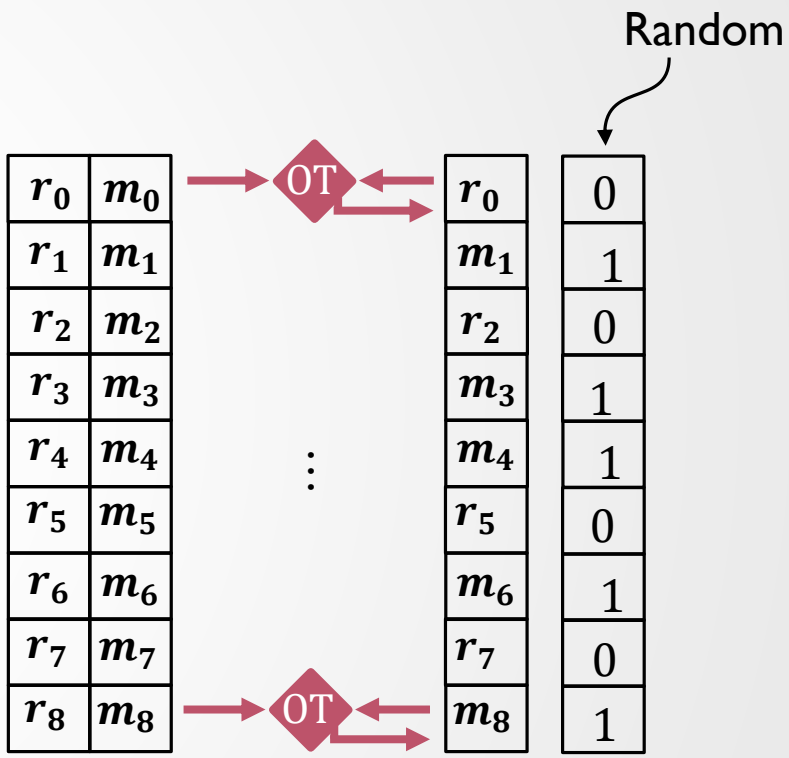


# Cut and Choose Approach

[RindalRosulek17]

Make Receiver p

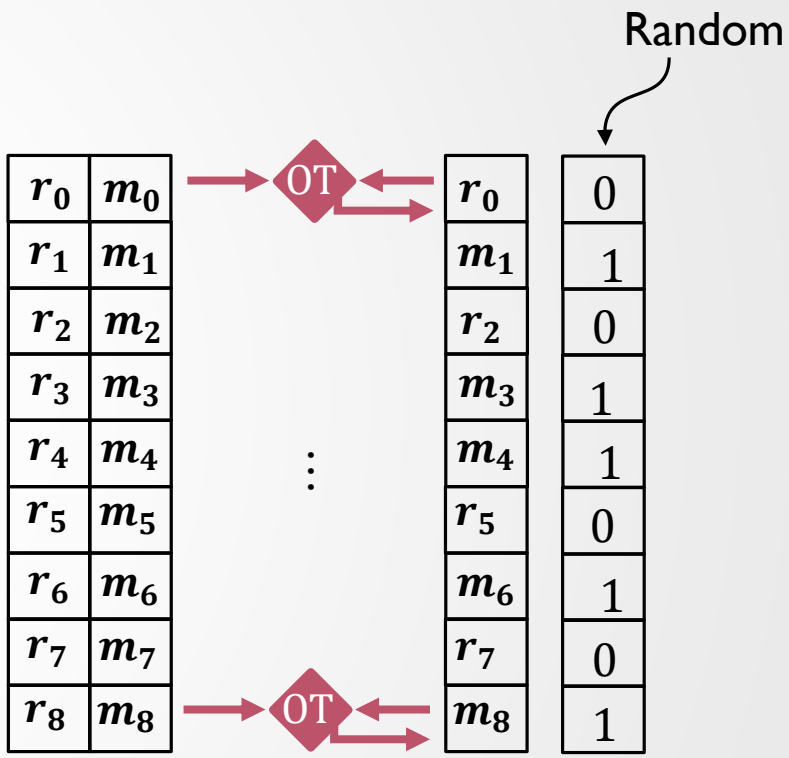


# Cut and Choose Approach

[RindalRosulek17]

Make Receiver p

- Sender challenges on a subset of OT
  - Receiver must reveal select bits



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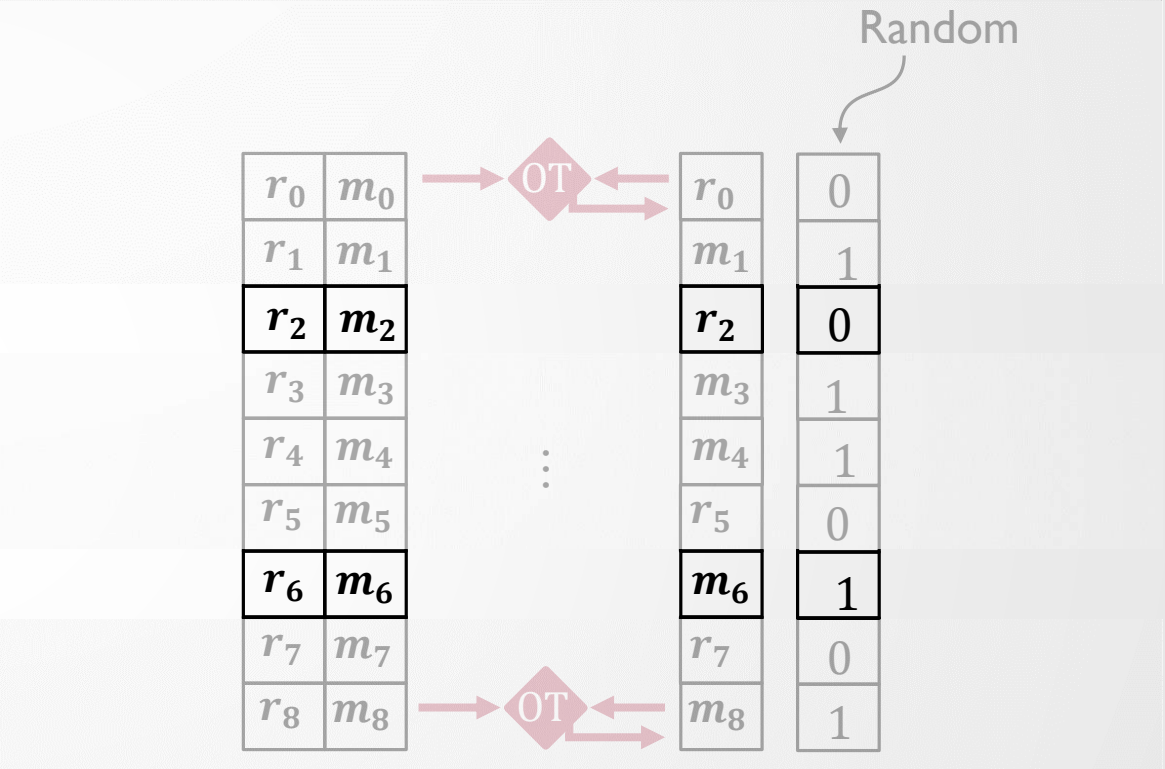

# Cut and Choose Approach

[RindalRosulek17]

Make Receiver p 2 zero bits, aborts otherwise

- Sender challenges on a subset of OT
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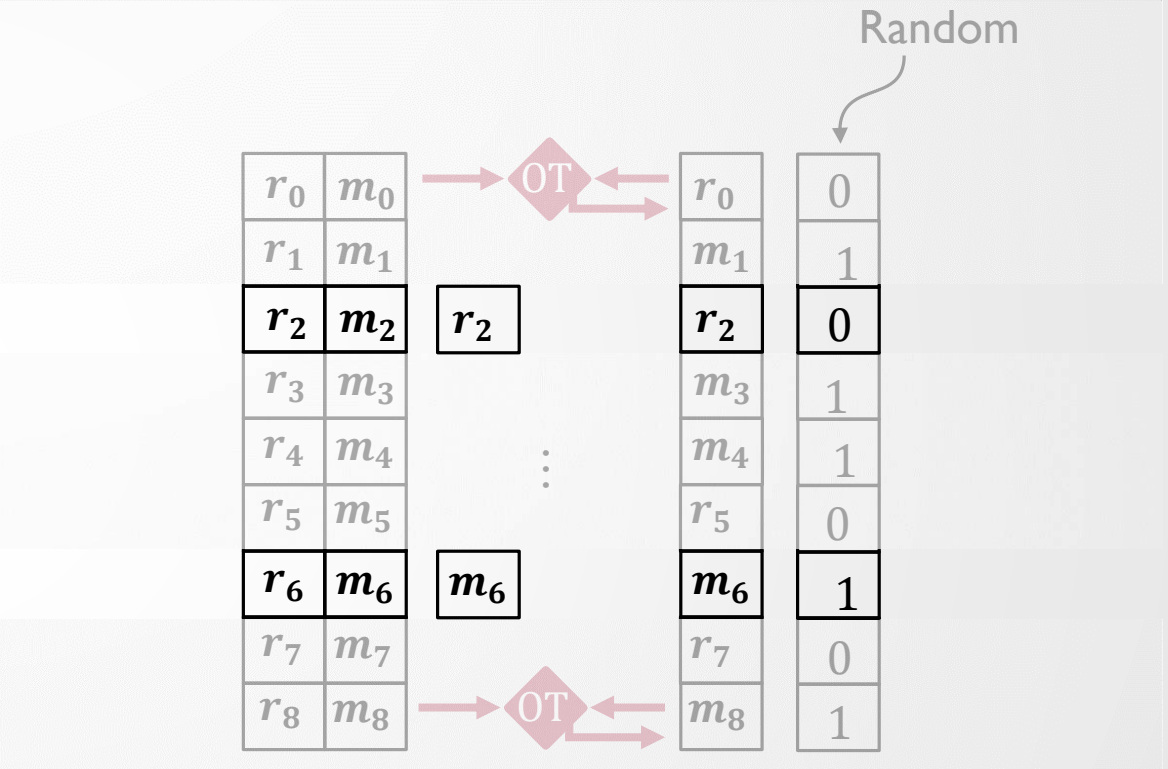
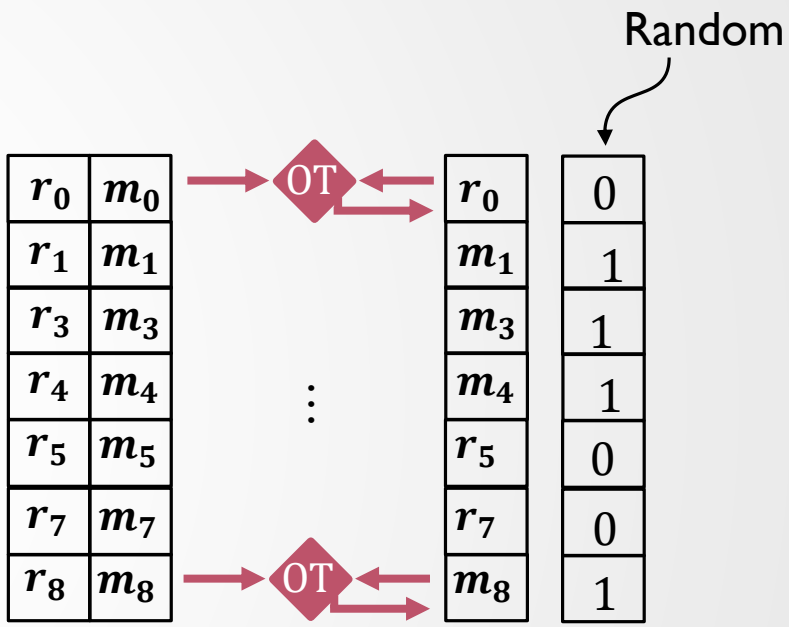


# Cut and Choose Approach

[RindalRosulek17]

- Make Receiver prove zero bits in an input-independent way

- Receiver uses *random* OT select bits
- Sender challenges on a subset of OT
  - Receiver must reveal select bits
  - Expect to see $\frac{1}{2}$ zero bits, aborts otherwise



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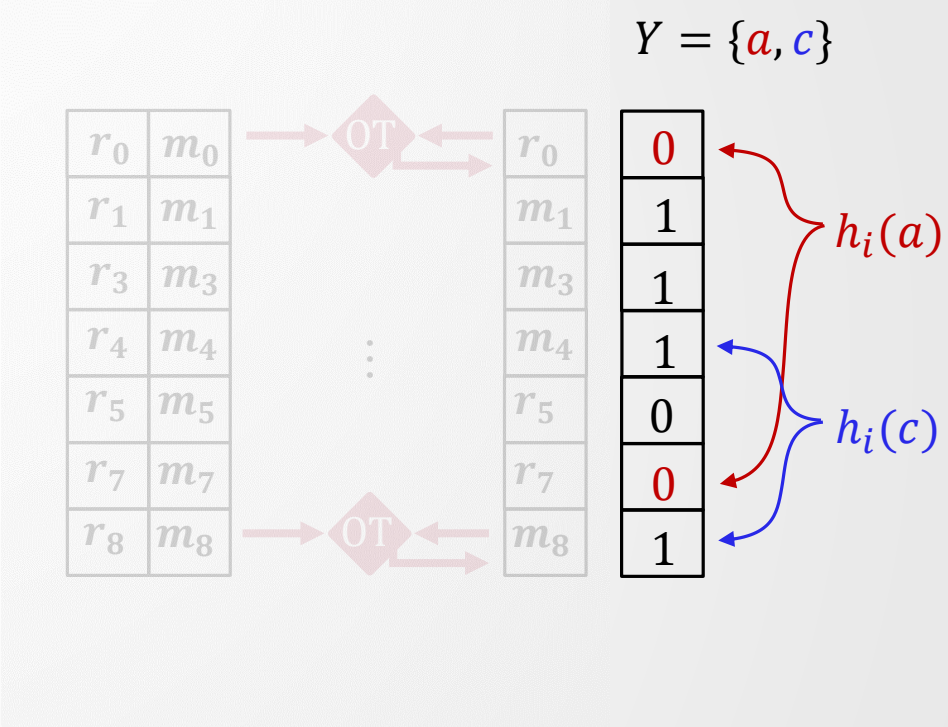

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[RindalRosulek17]

1

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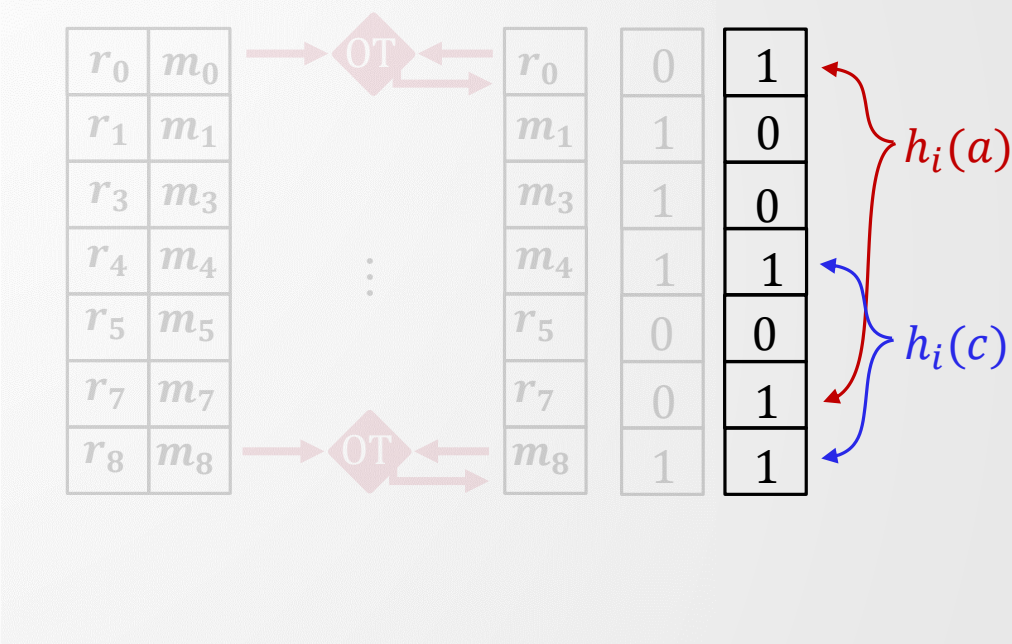
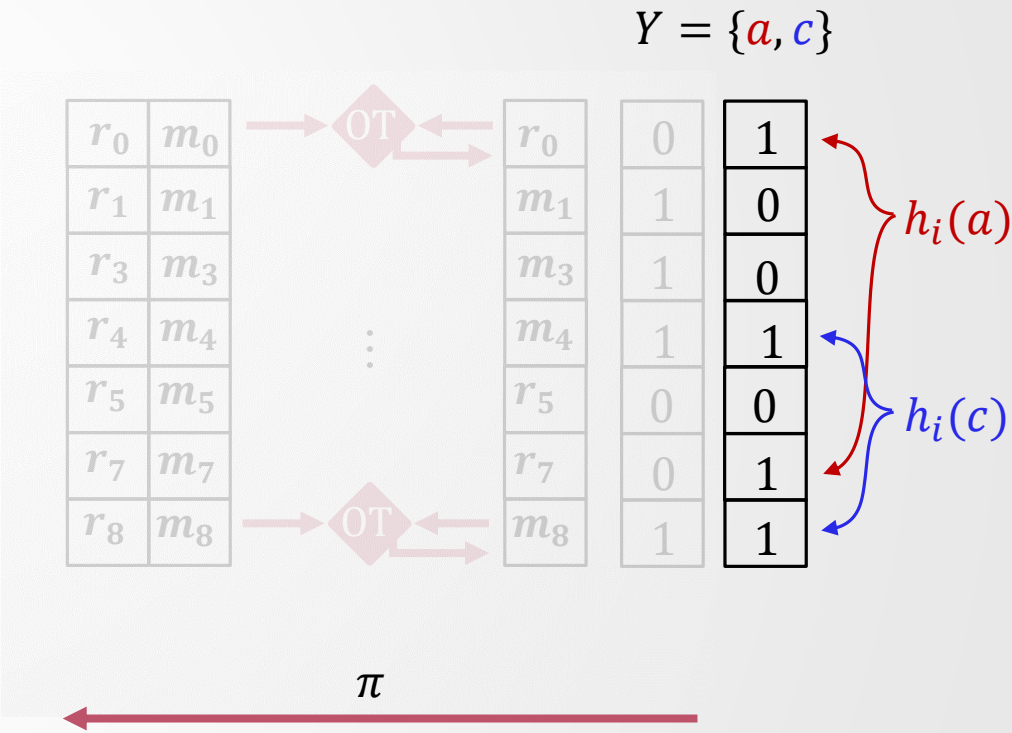


# Cut and Choose Approach

[RindalRosulek17]

l andom OTs →desired $BF$

- Randomly permute OTs to form Bloom filter
  - $\pi$ random OTs → desired $BF$



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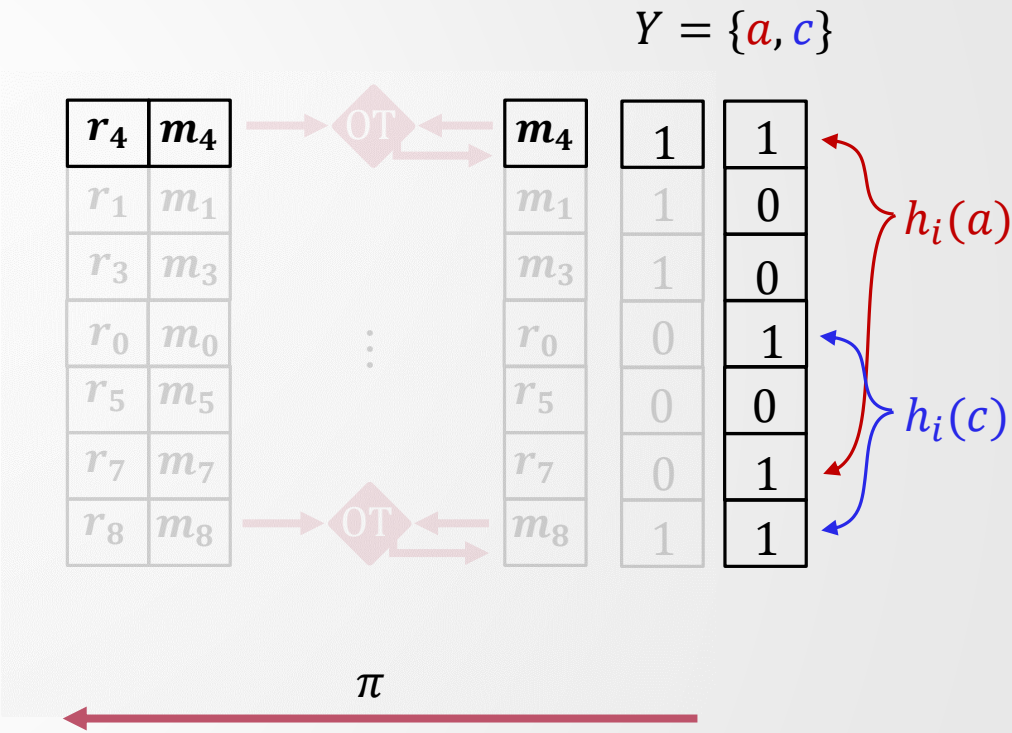

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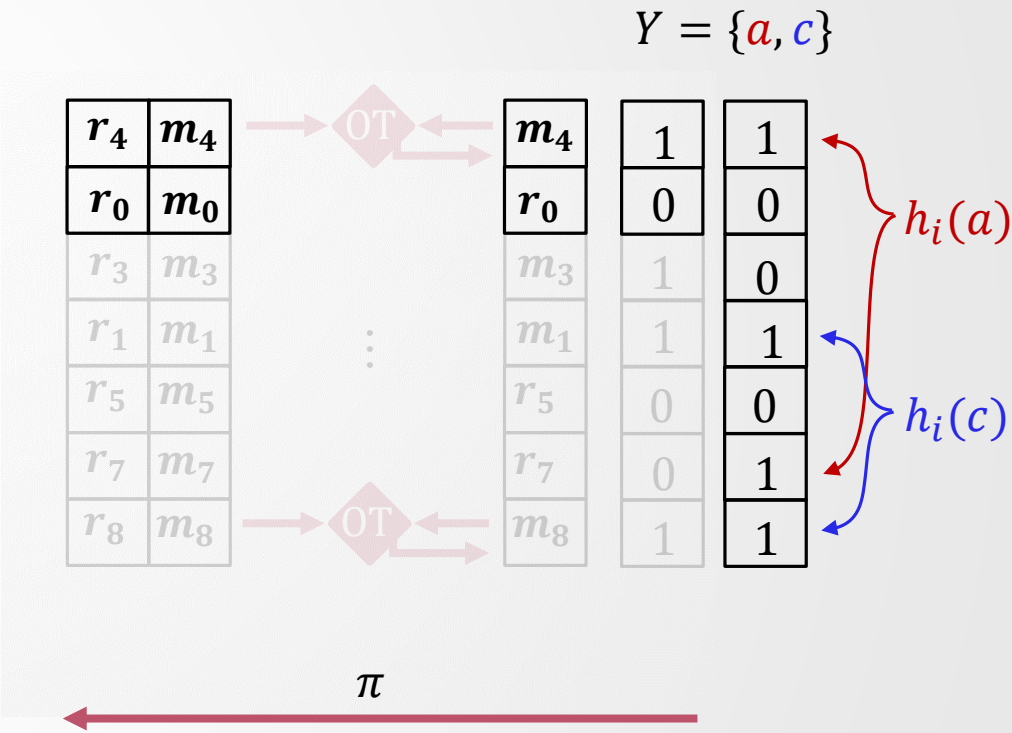

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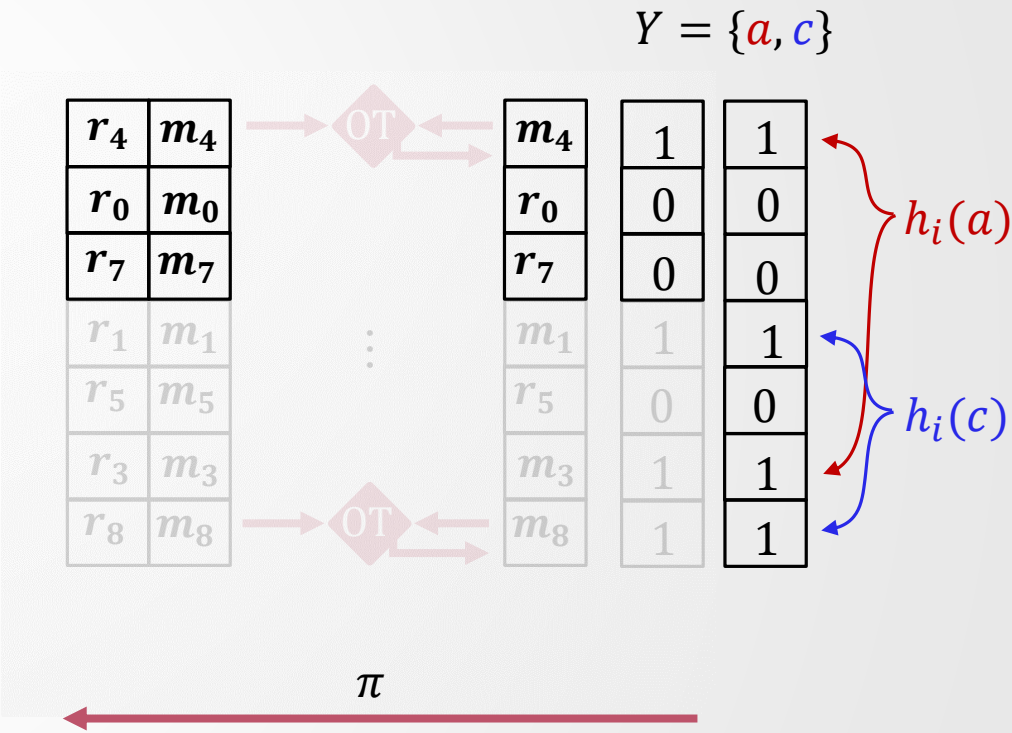

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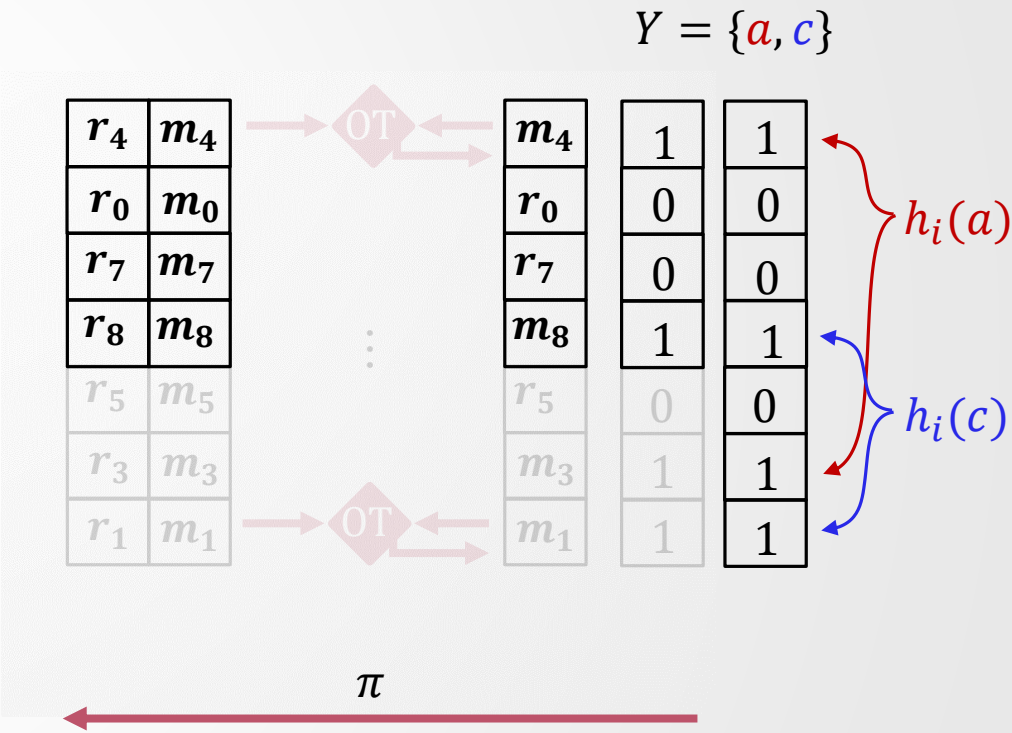

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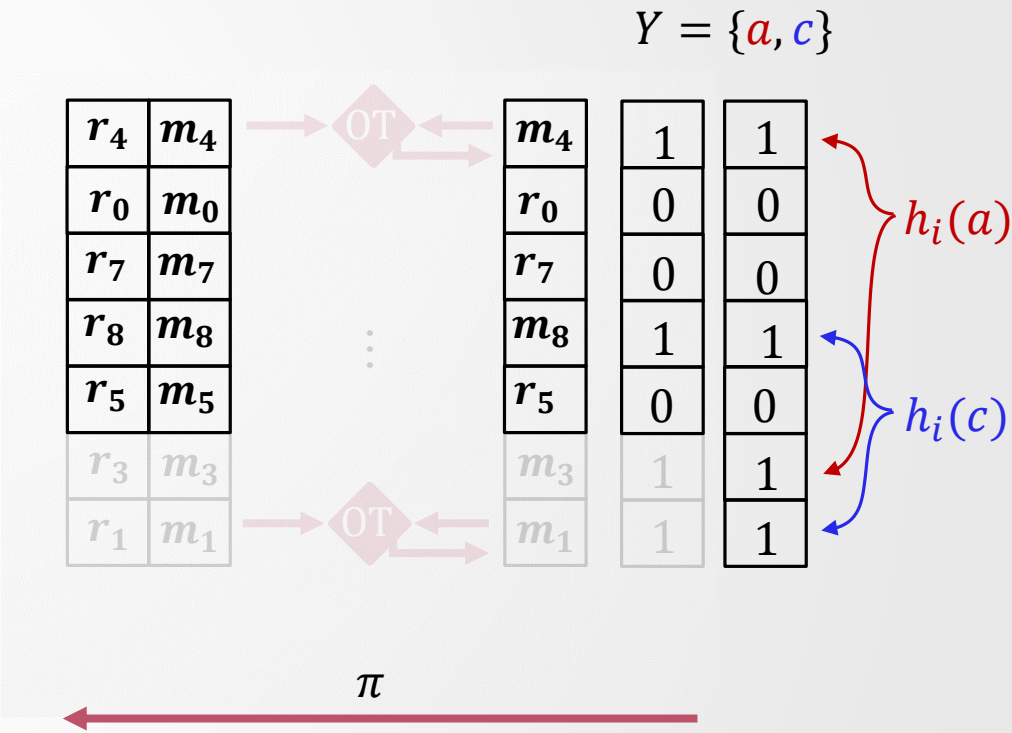

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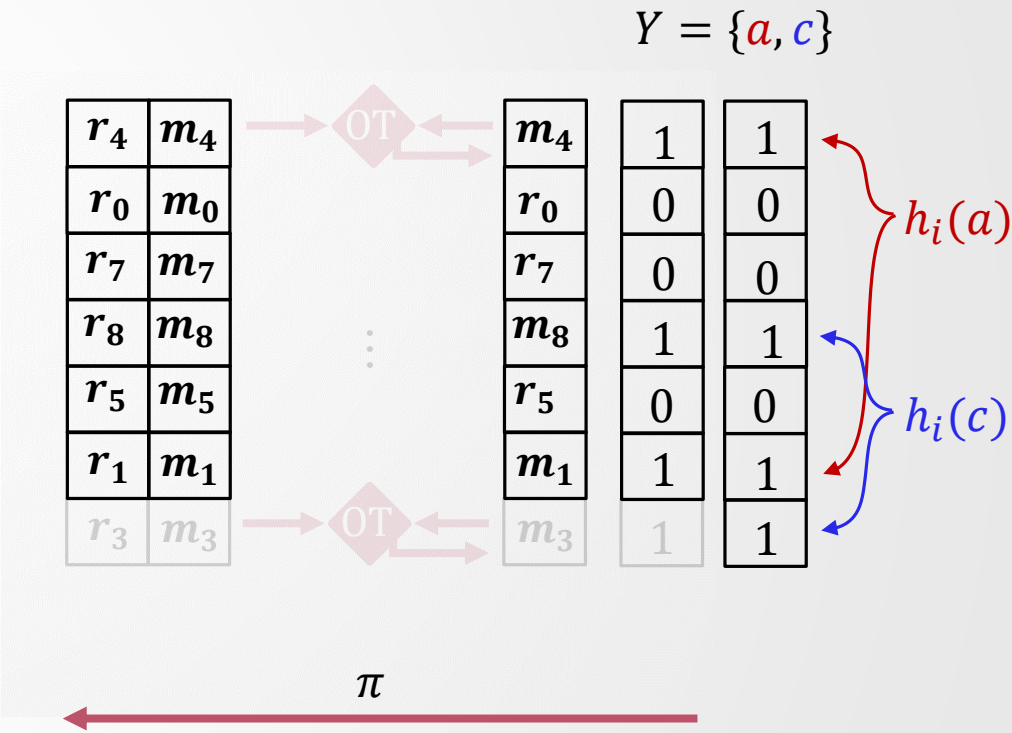

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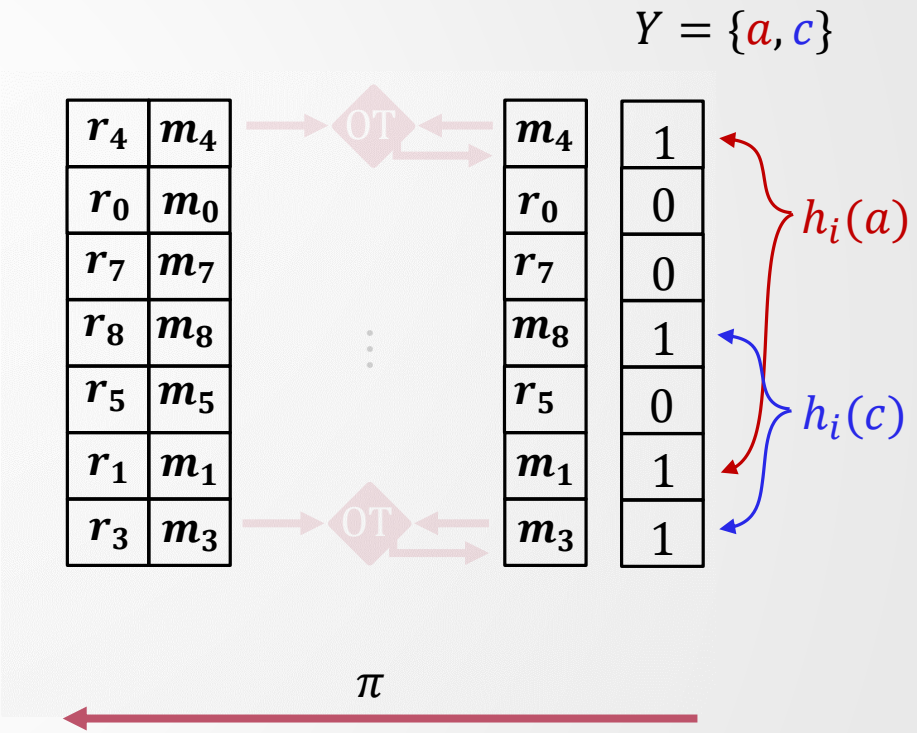

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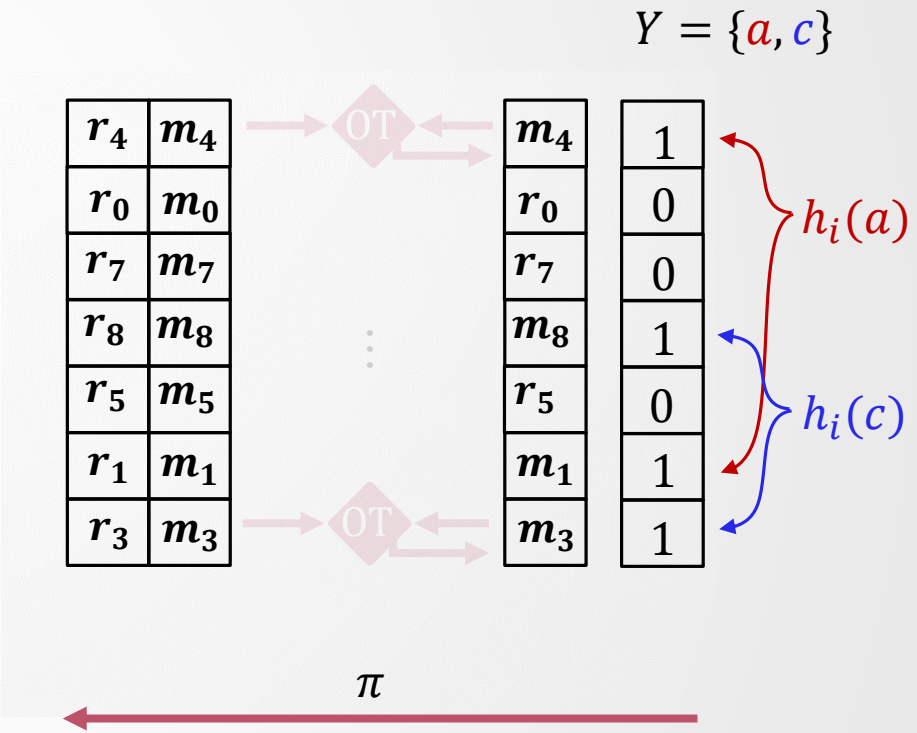


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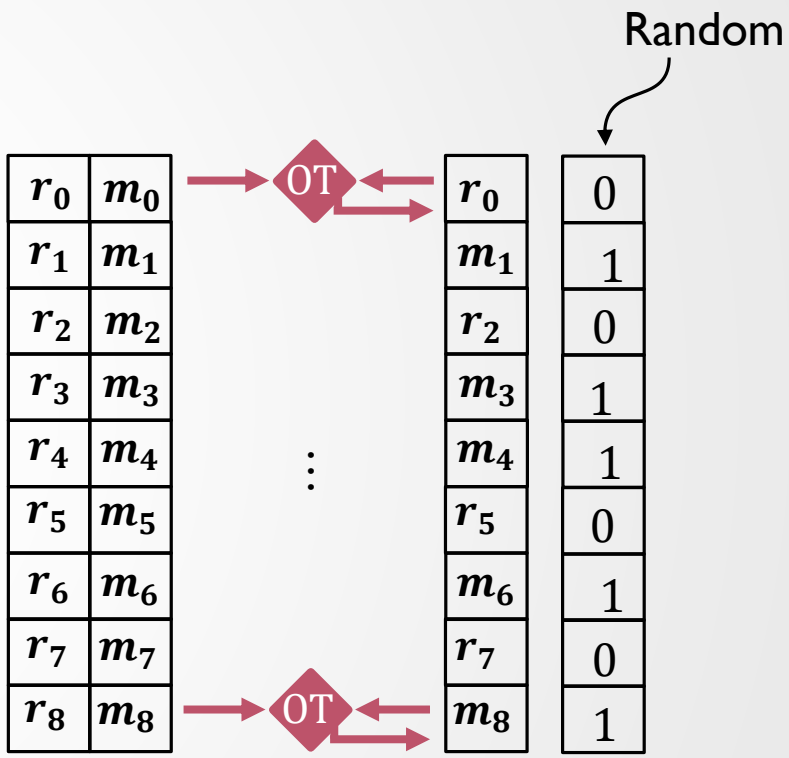

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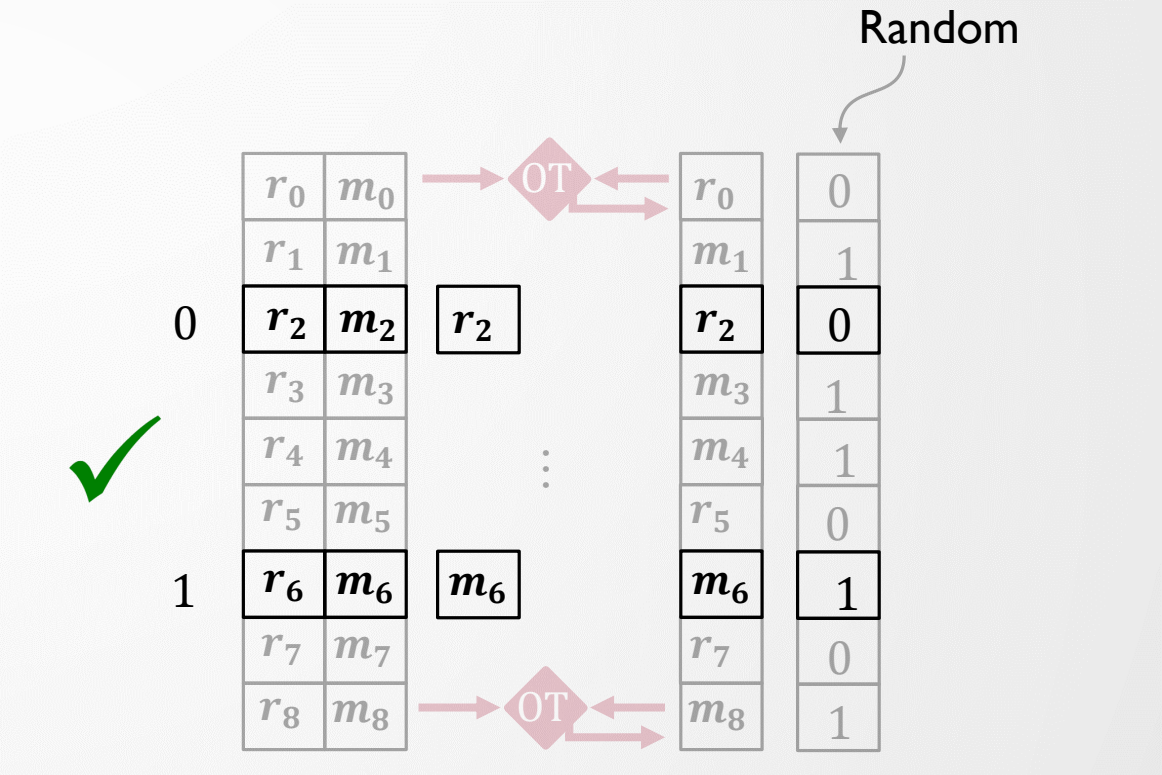

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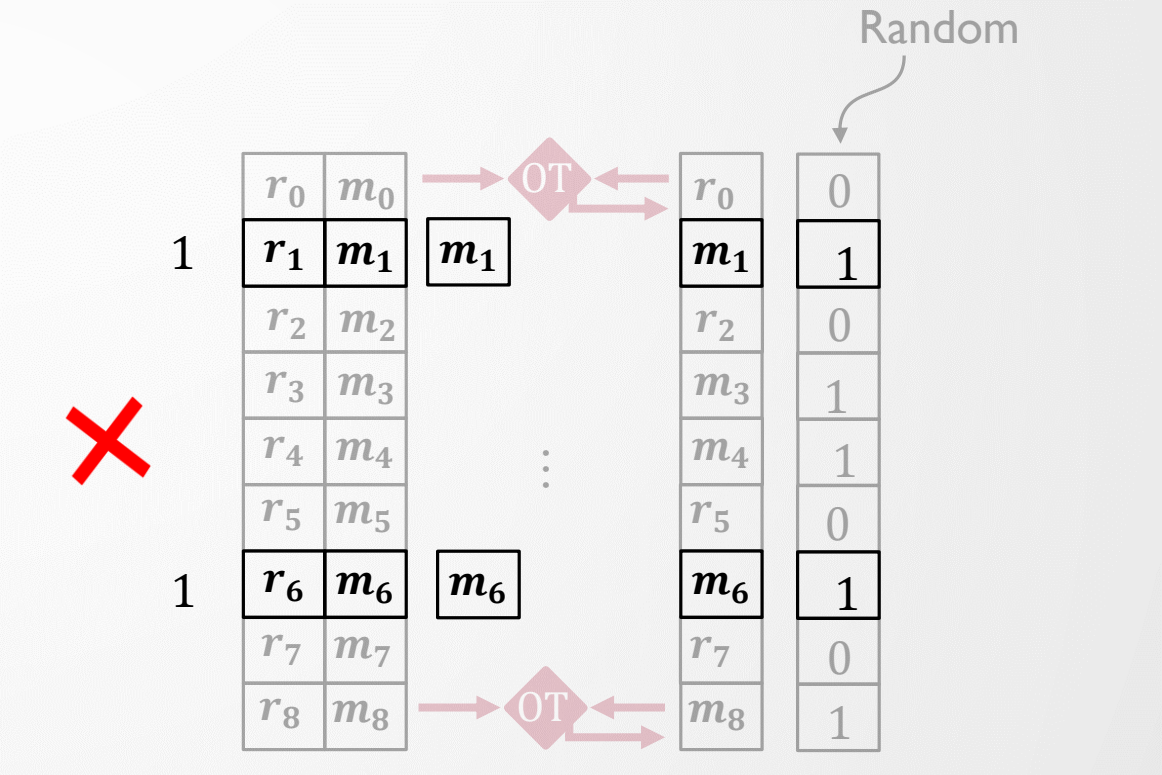

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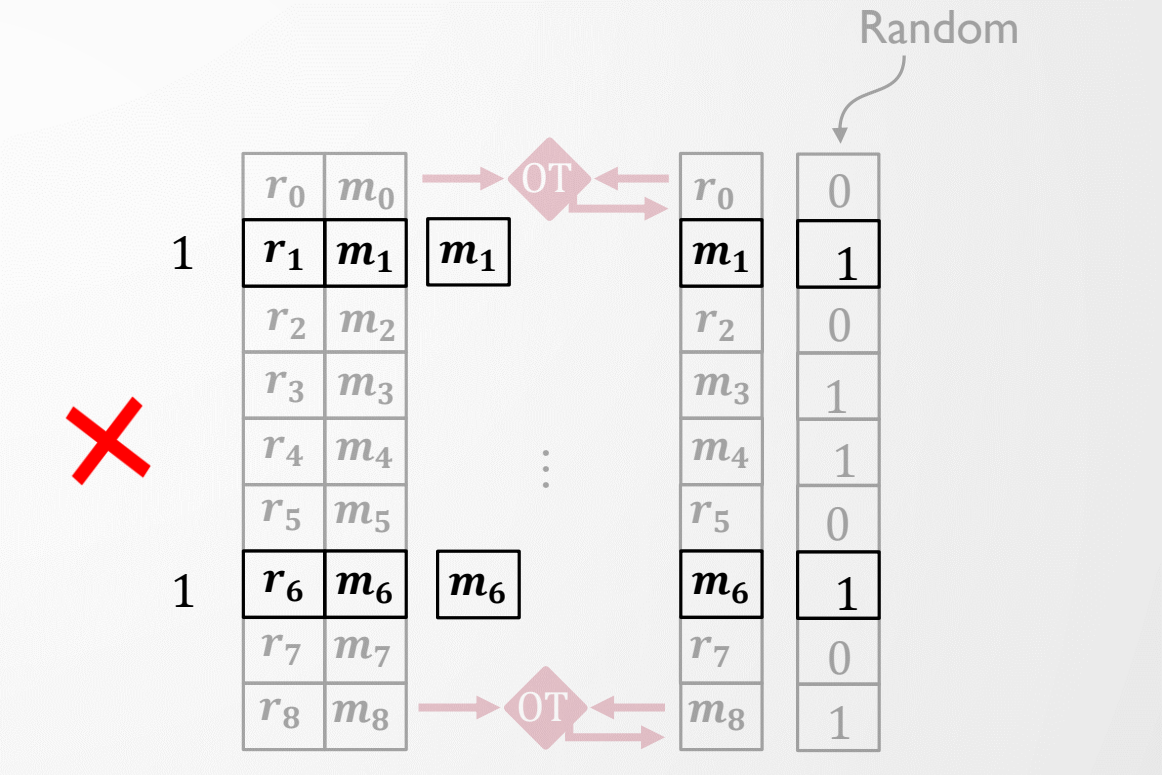

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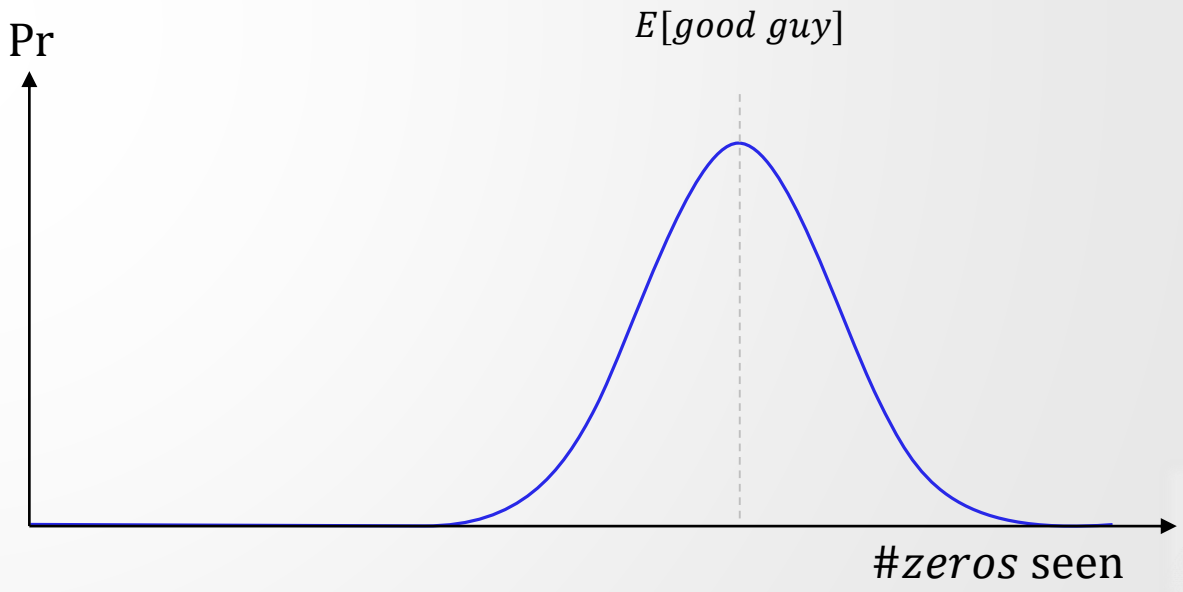

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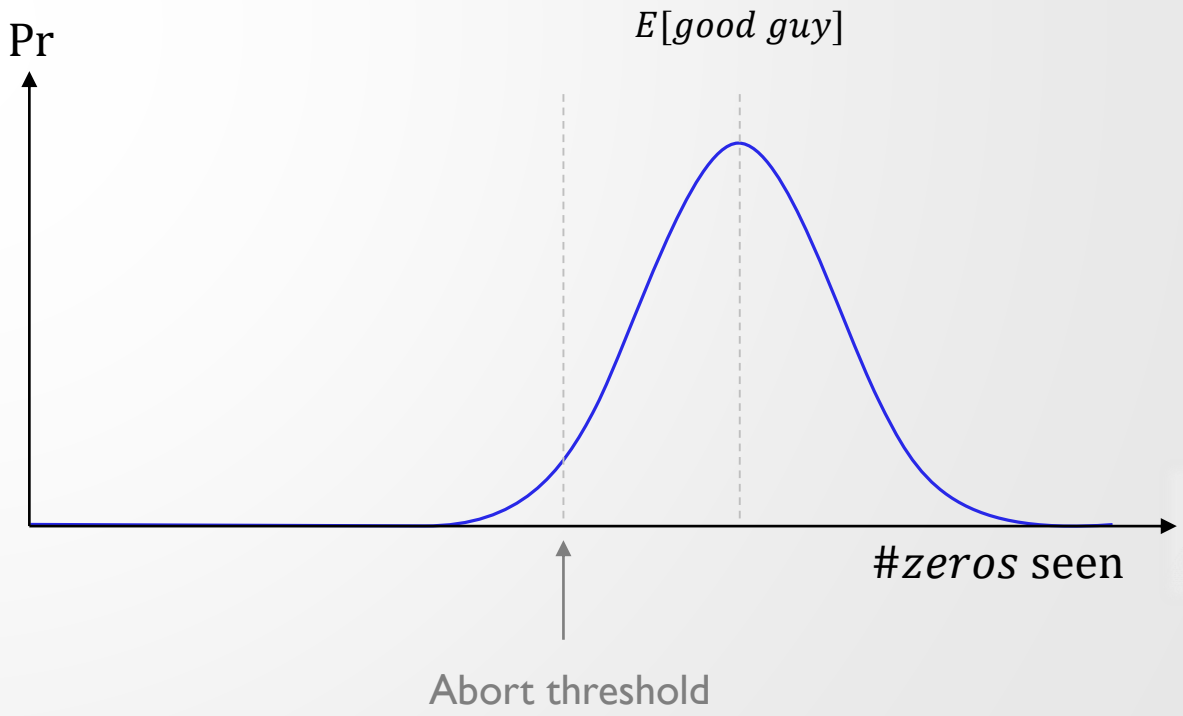

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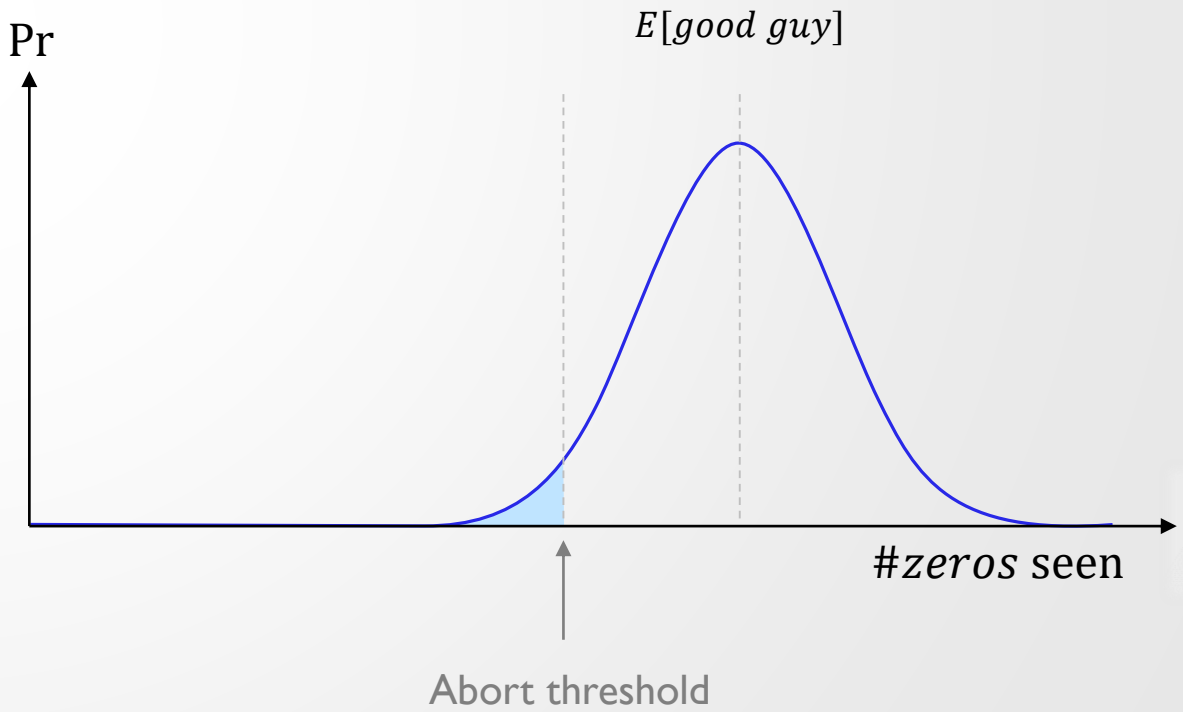

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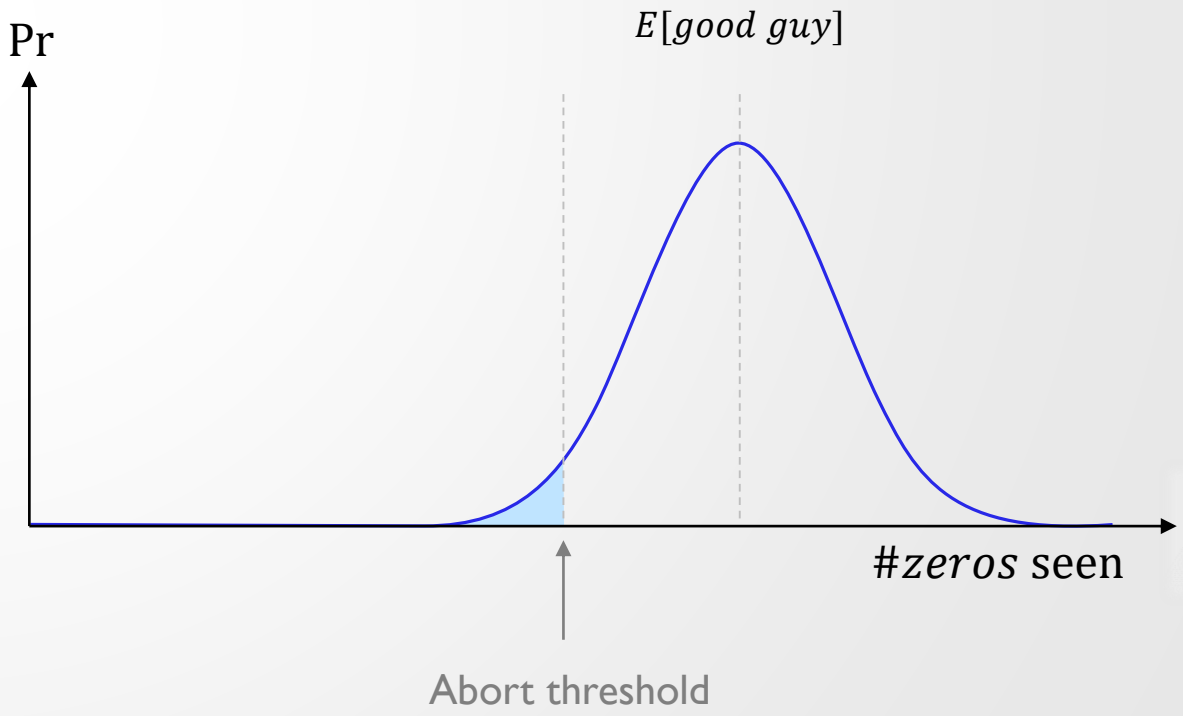

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[RindalRosulek17]

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$add\ gguuyy\ nnoott\ ccaauugghtt\ Bad\ guy\ not\ caught \leq nne$
$egg(\kappa\kappa)$ $Pr\ \ Bad\ guy\ not\ caught \leq neg(\kappa)$

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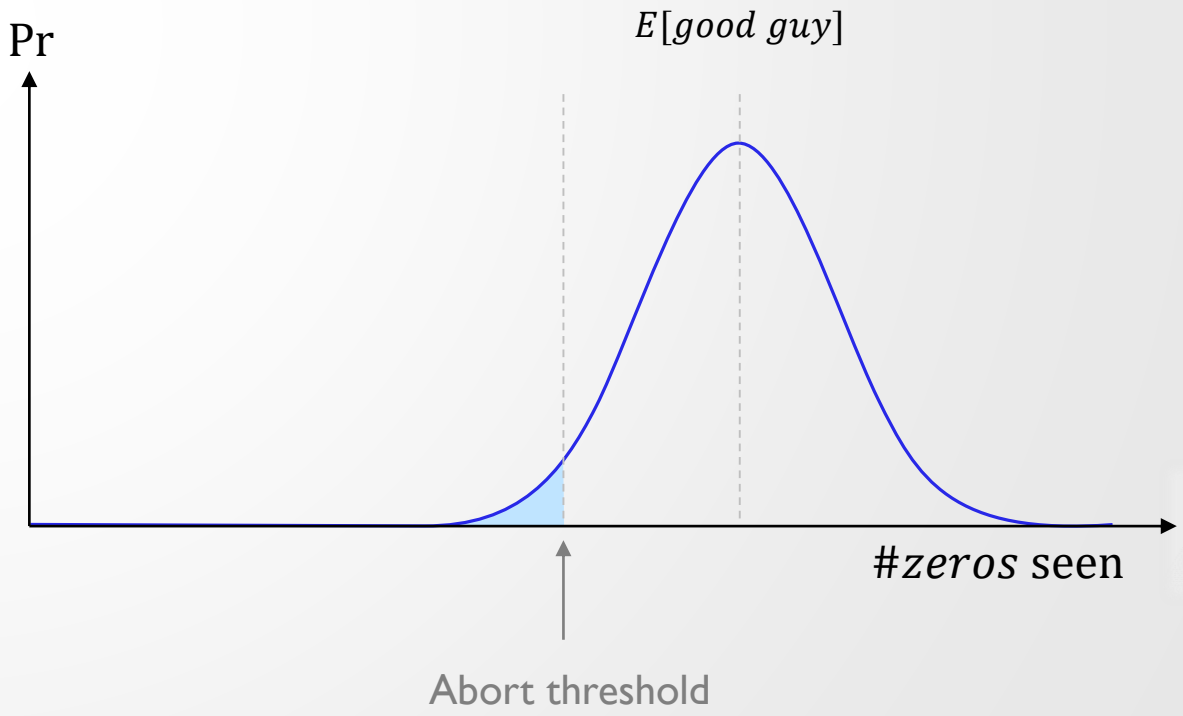

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r Pr $Bad\ guy\ not\ caught \leq neg(\kappa)$ $Bad\ guy\ not\ caught\ BBa$
$add\ gguuyy\ nnoott\ ccaauugghtt\ Bad\ guy\ not\ caught \leq nne$
$egg(\kappa\kappa)\ Pr\ \ Bad\ guy\ not\ caught \leq neg(\kappa)$

*Issue*: Random OTs/Cut-and-Choose may not result in exactly $\frac{1}{2}$ zero select bits!

- Need robust checking of zero bits
- Desired properties:
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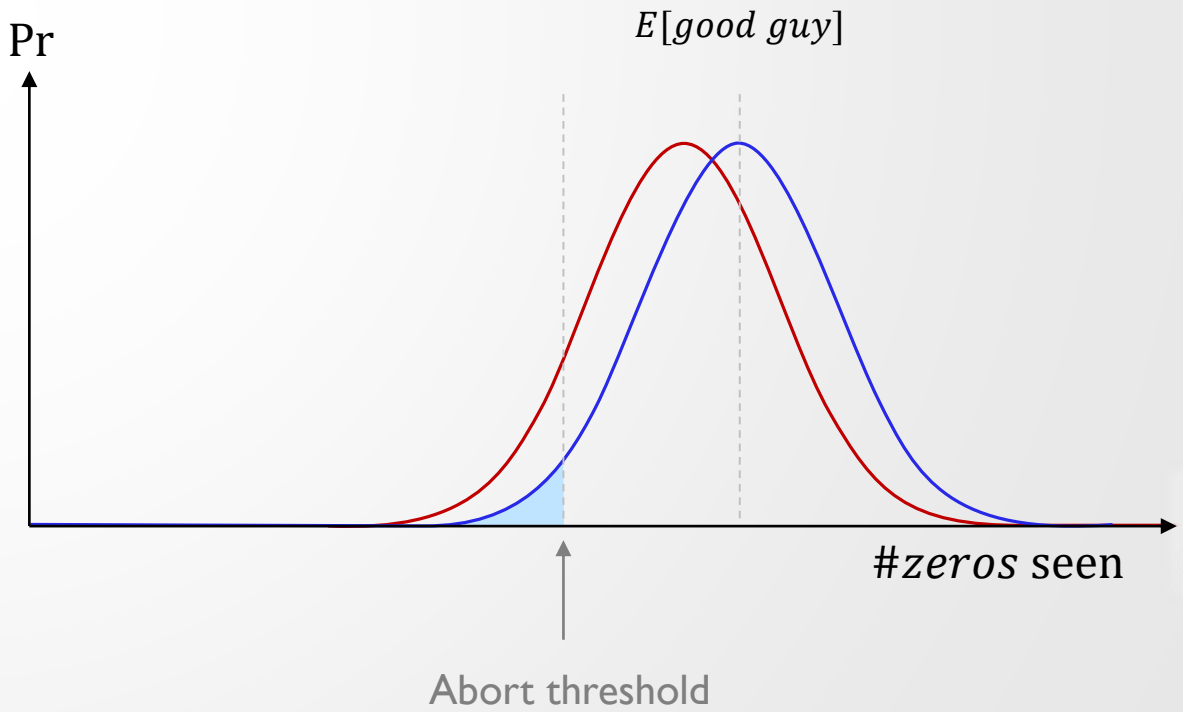
- Use Chernoff Bounds
- Sufficient to check 1% of the OTs!



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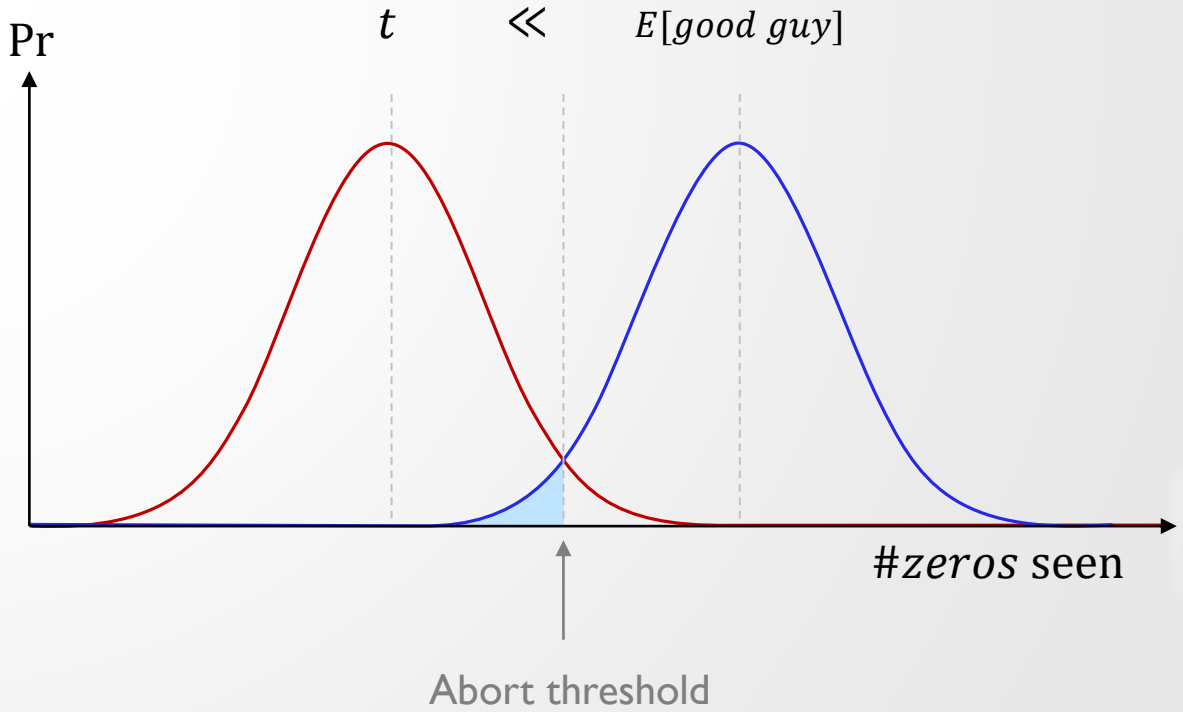

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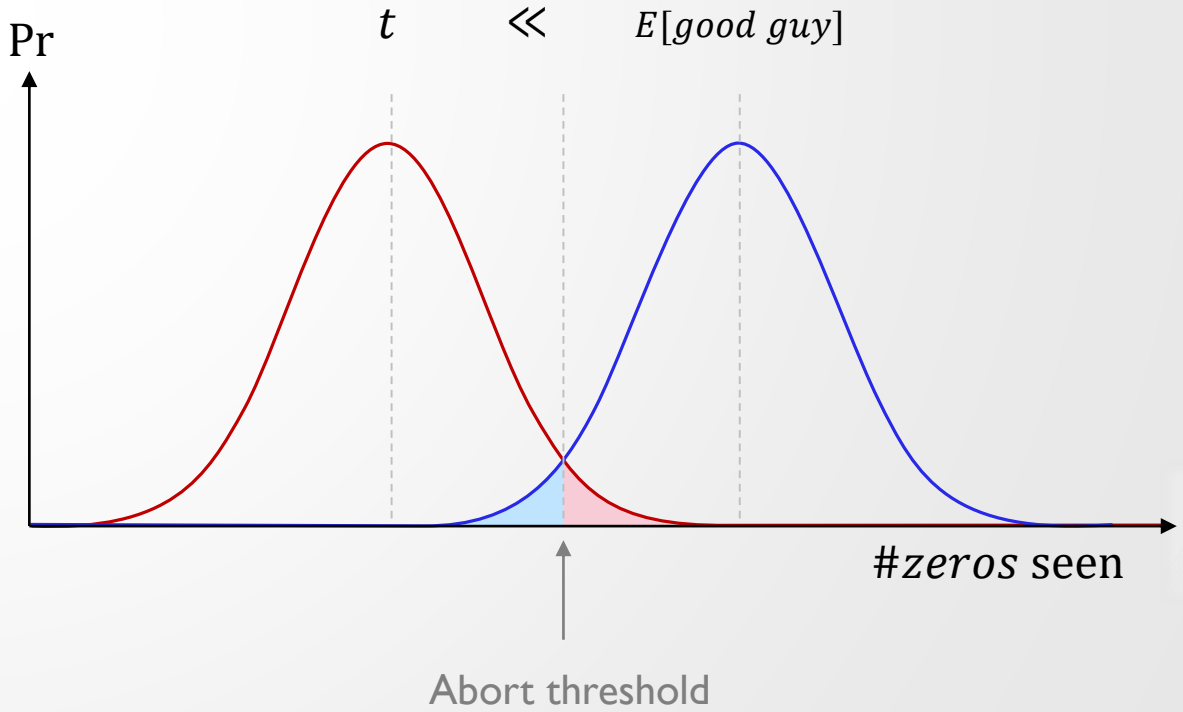

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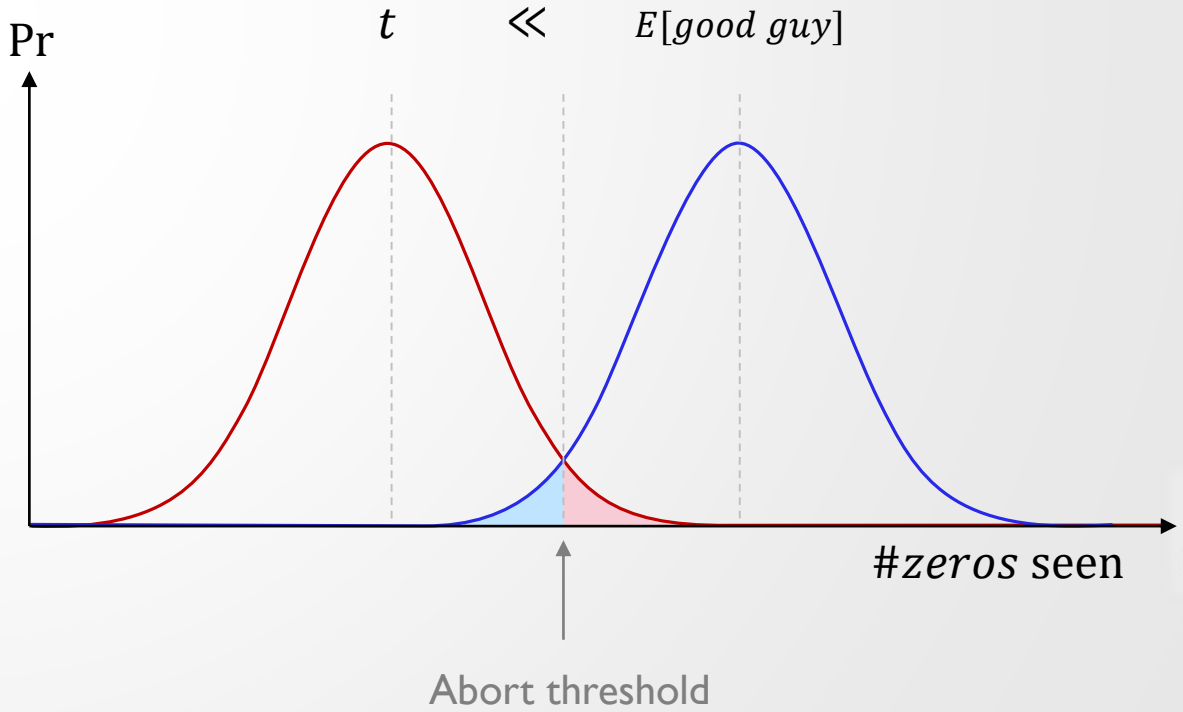

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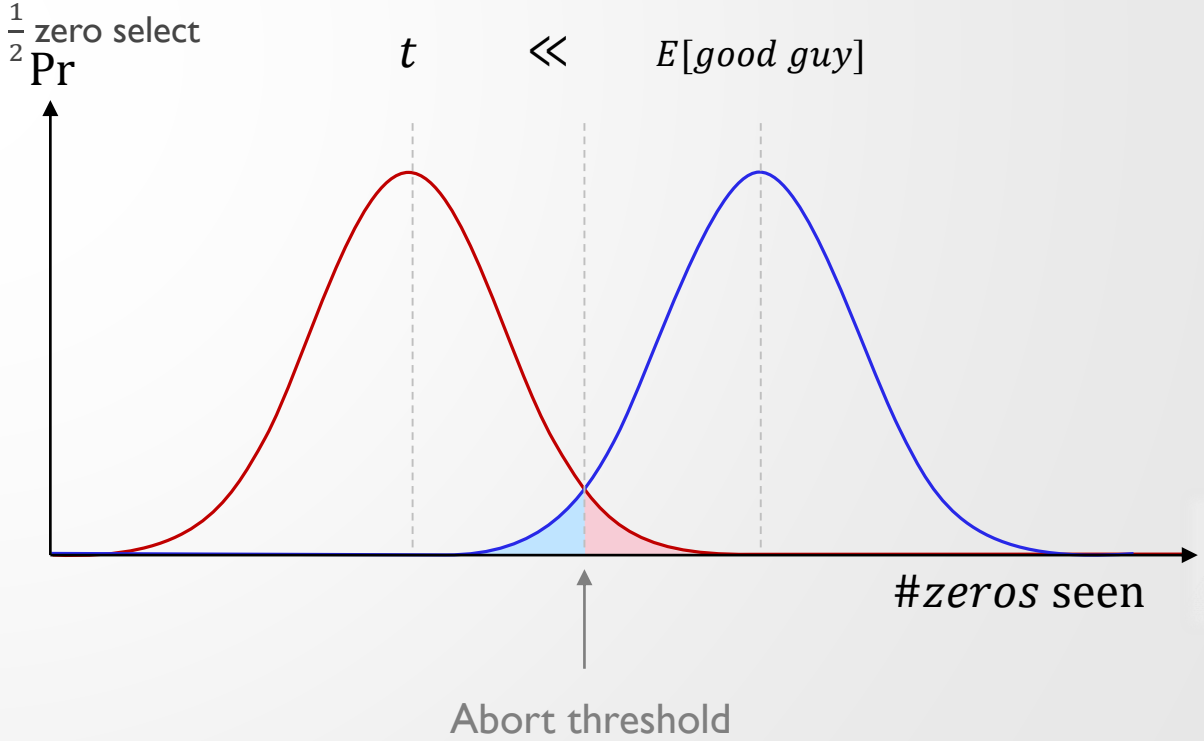
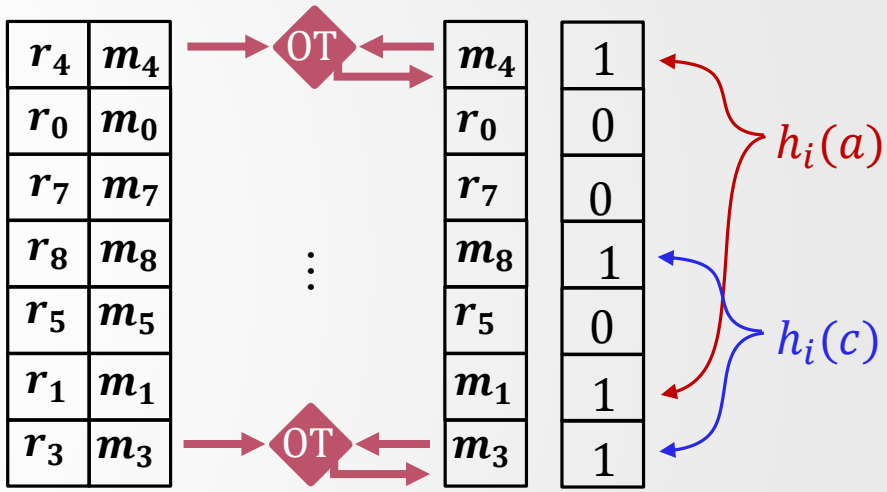


# Extracting $Y$ with Random Oracle

[RindalRosulek17]

Simulator must extract the effective input $Y$

$Y = \{a, c\}$




- $F$ is not naturally invertible
- $BF$ may be malformed…

- Solution:
  - Model hash function $h_i(\cdot)$ as Random Oracle
  - Non-programmable RO

Output:

$$\hat{X} = \begin{Bmatrix} m_4 \oplus m_1, \\ m_2 \oplus m_3 \end{Bmatrix} \longrightarrow \hat{X} \cap \begin{Bmatrix} m_4 \oplus m_1, \\ m_8 \oplus m_3 \end{Bmatrix}$$

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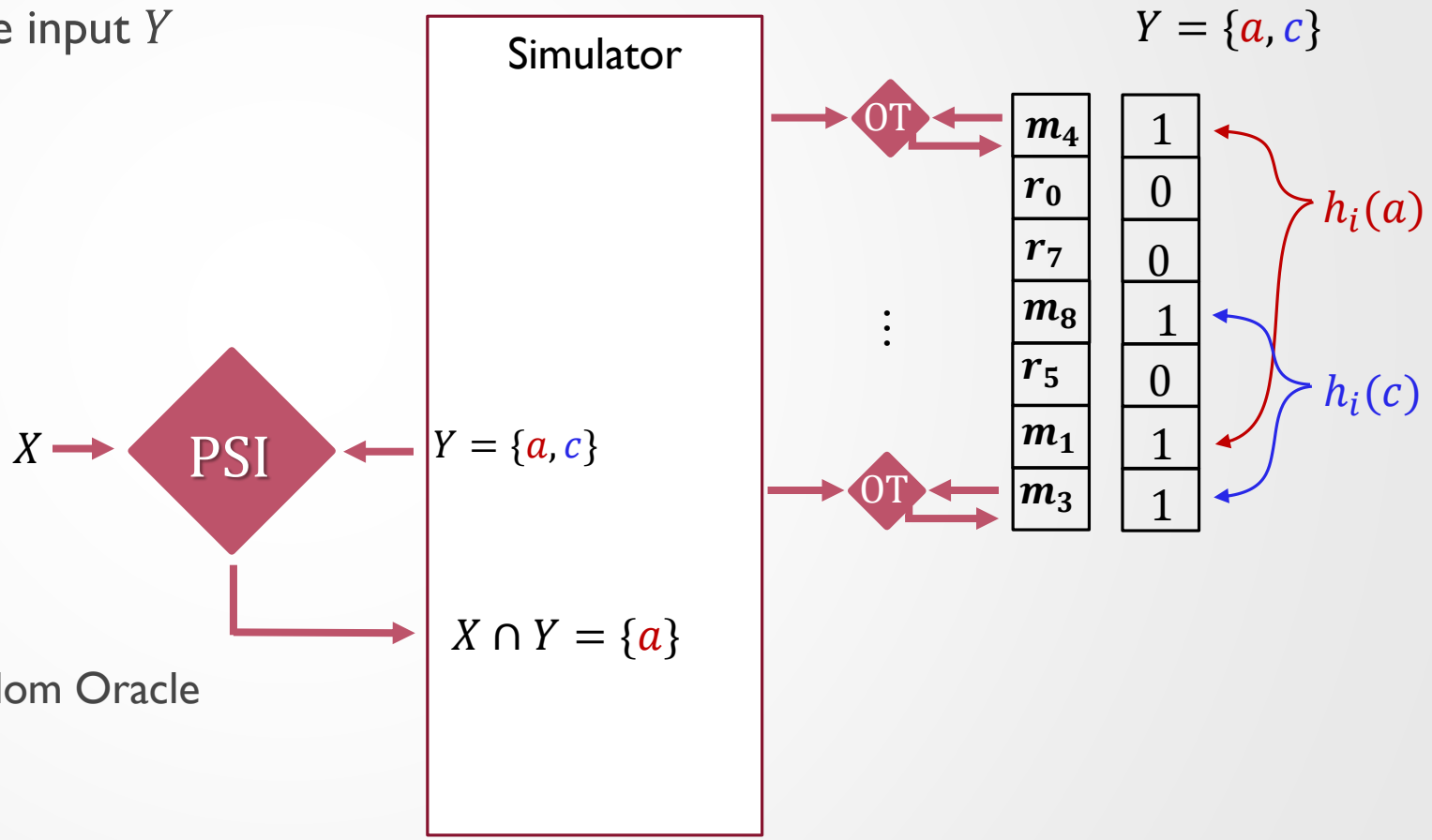

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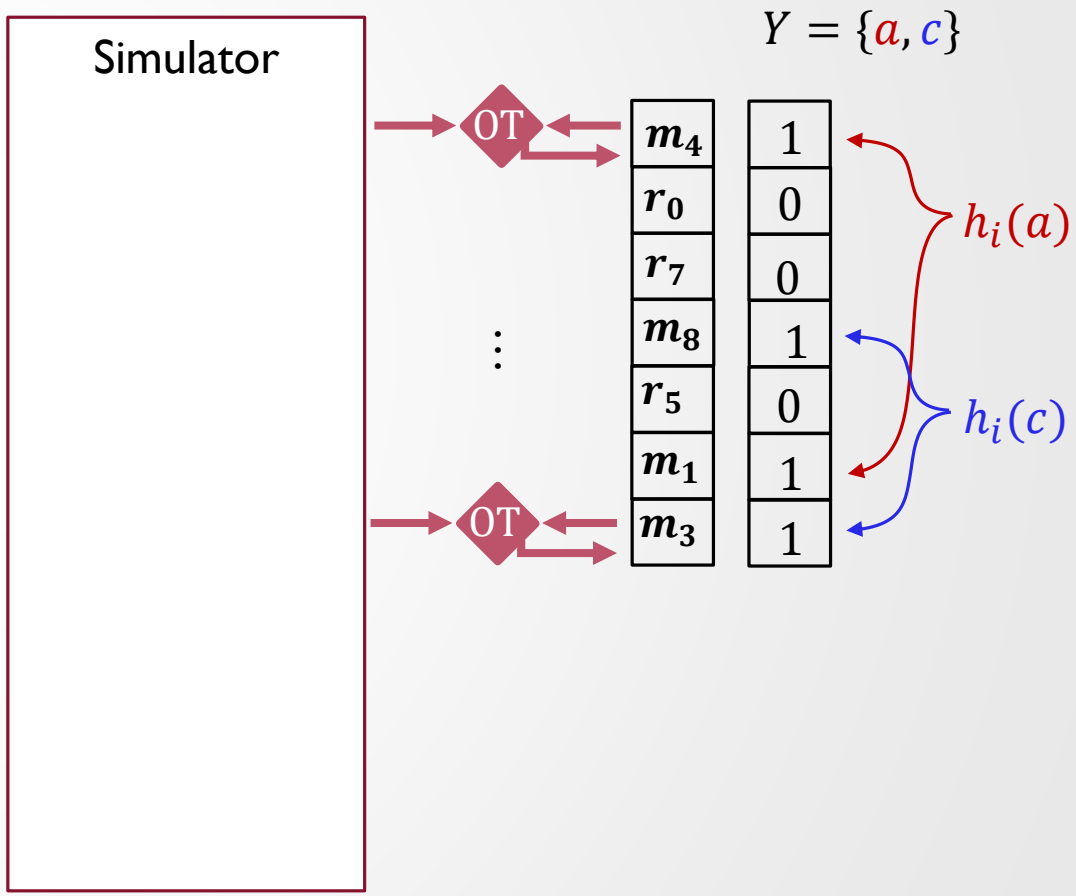

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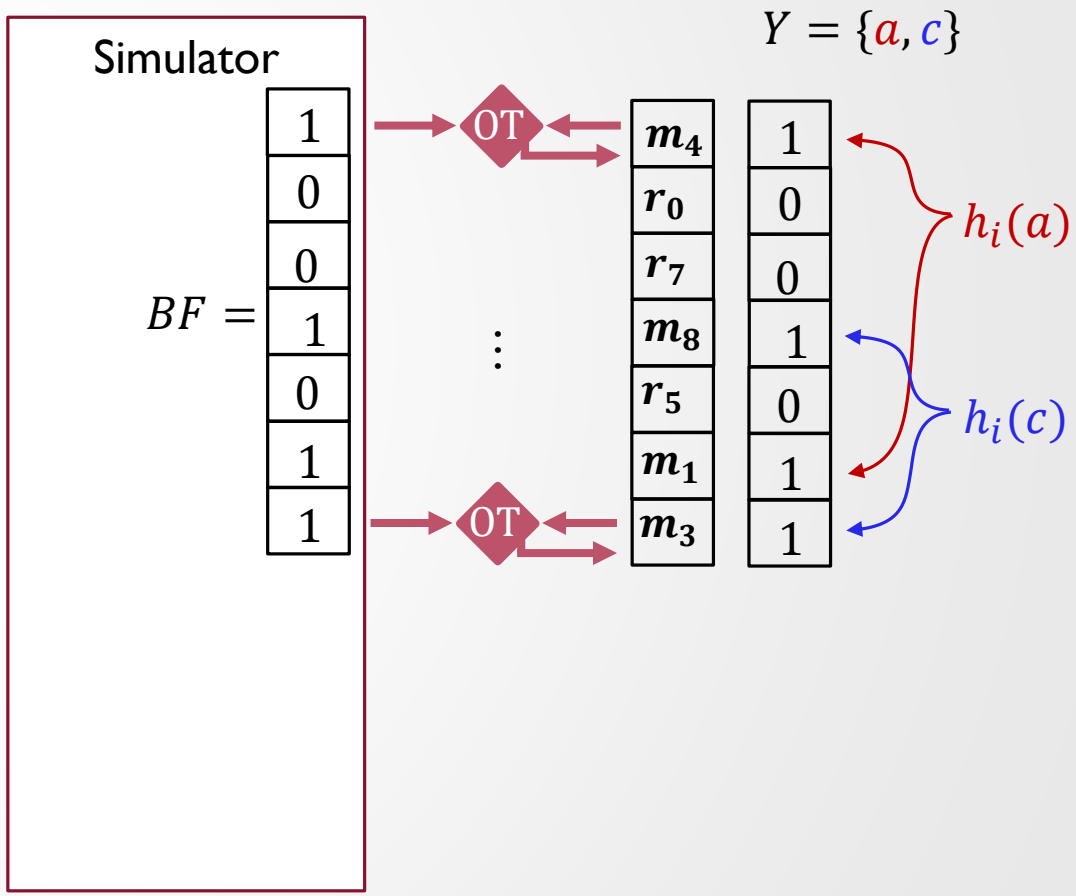

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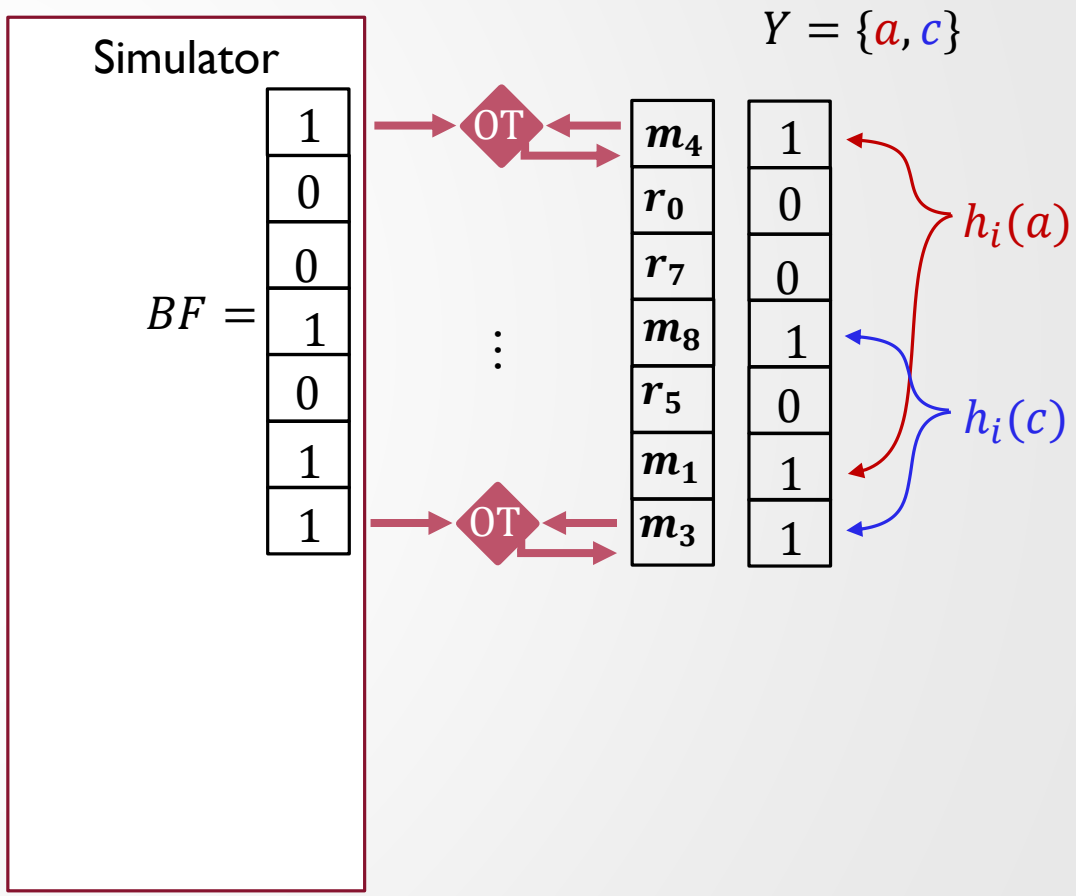

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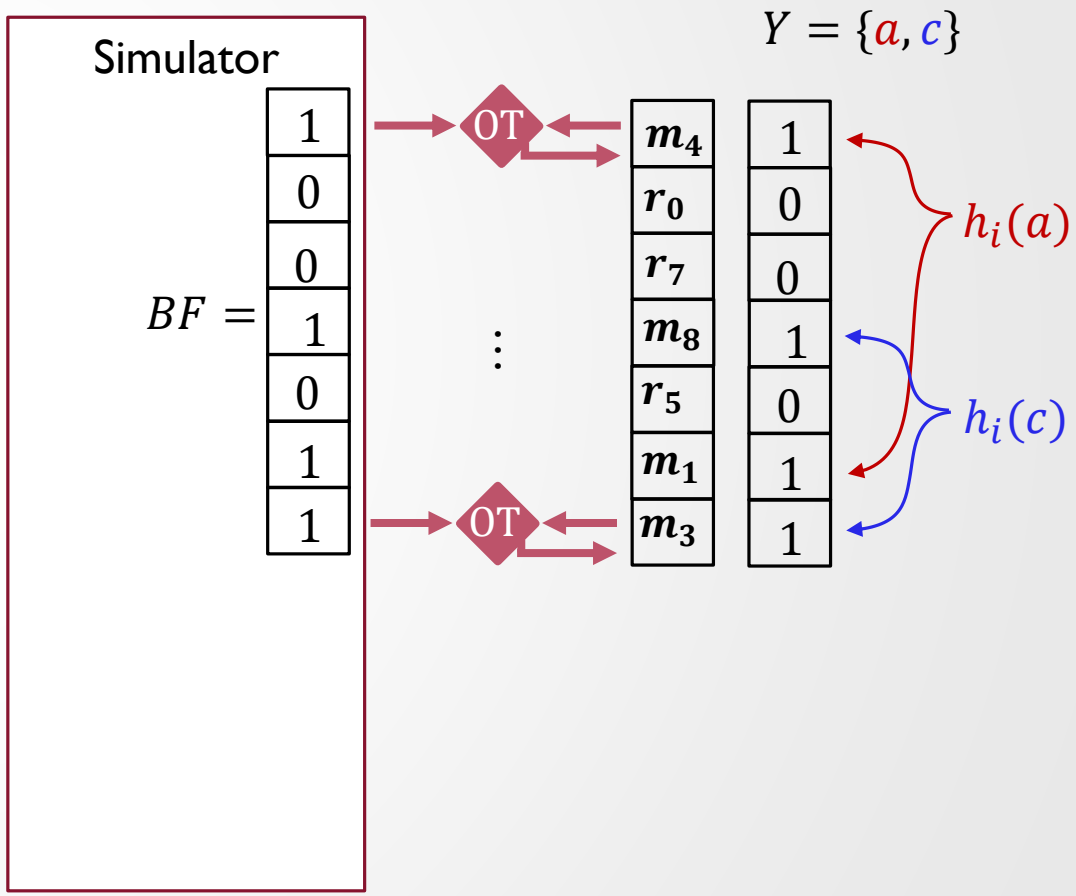

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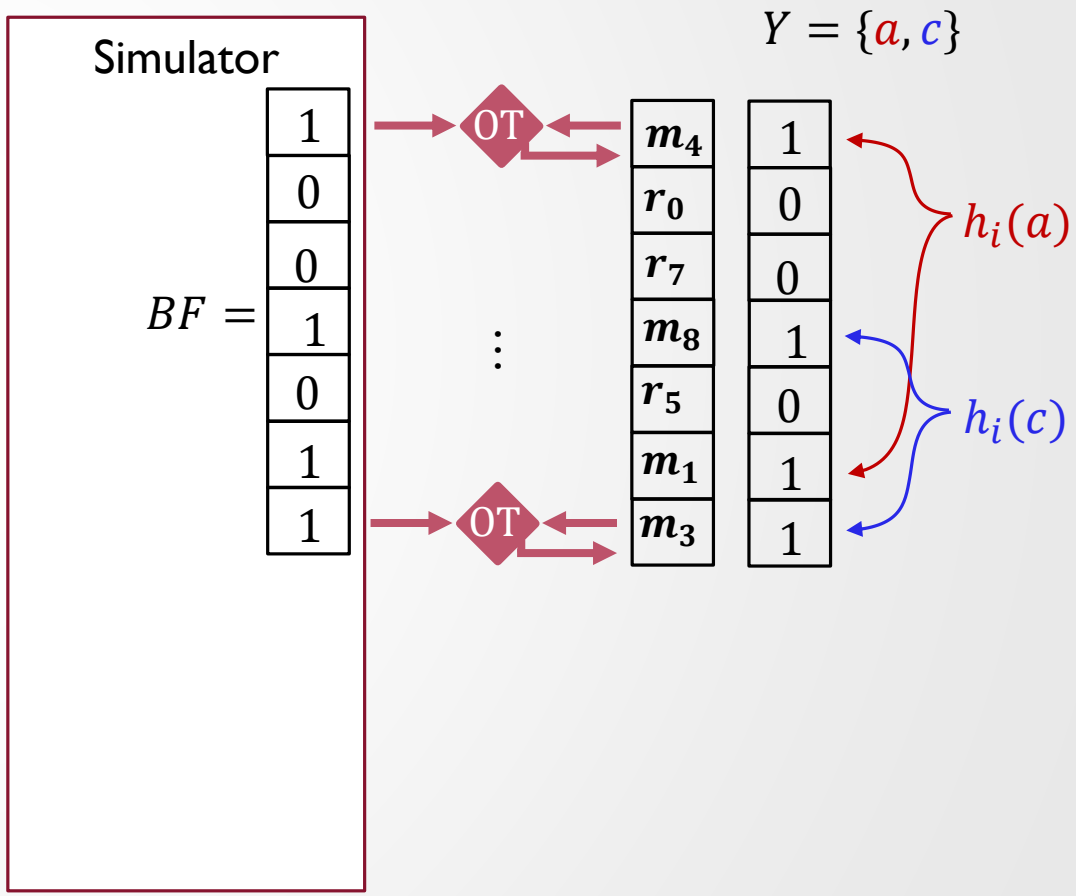

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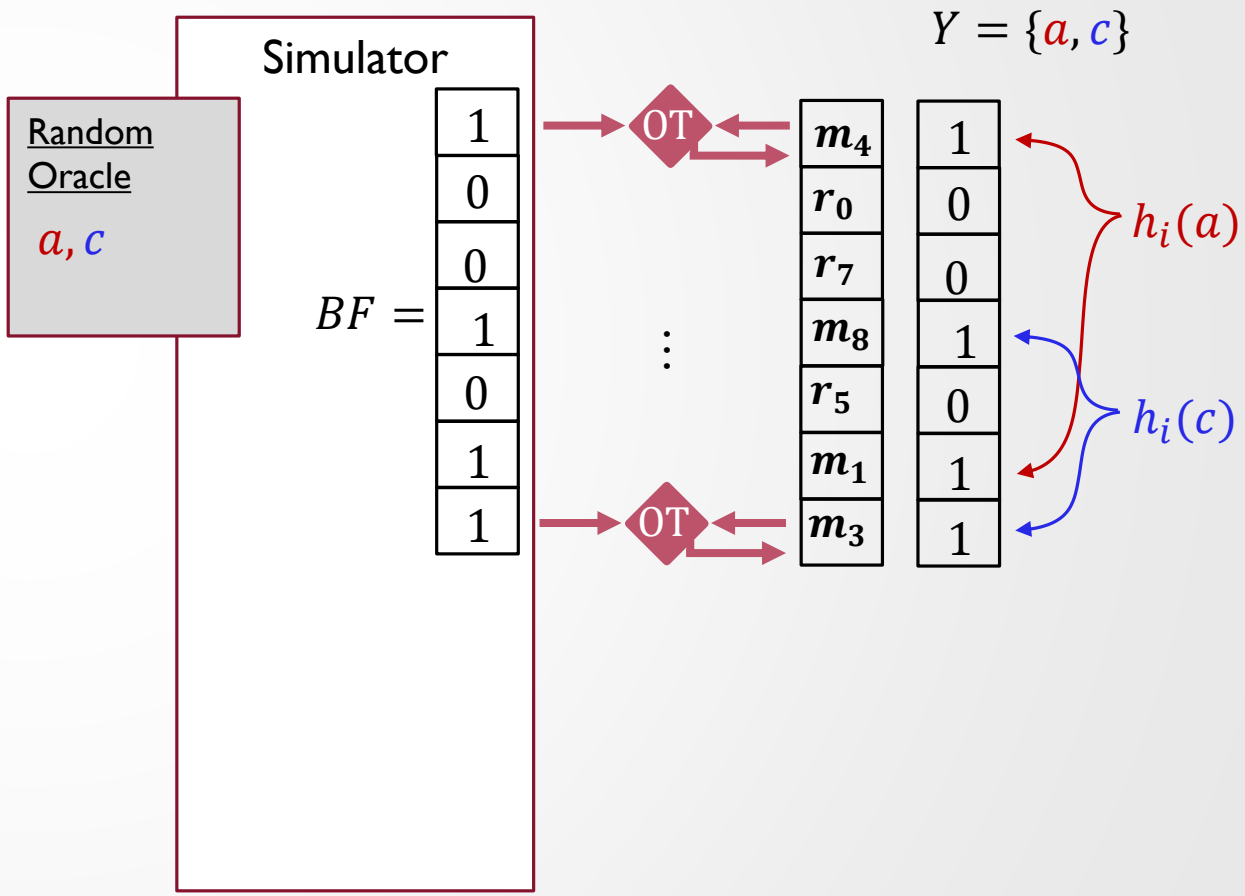

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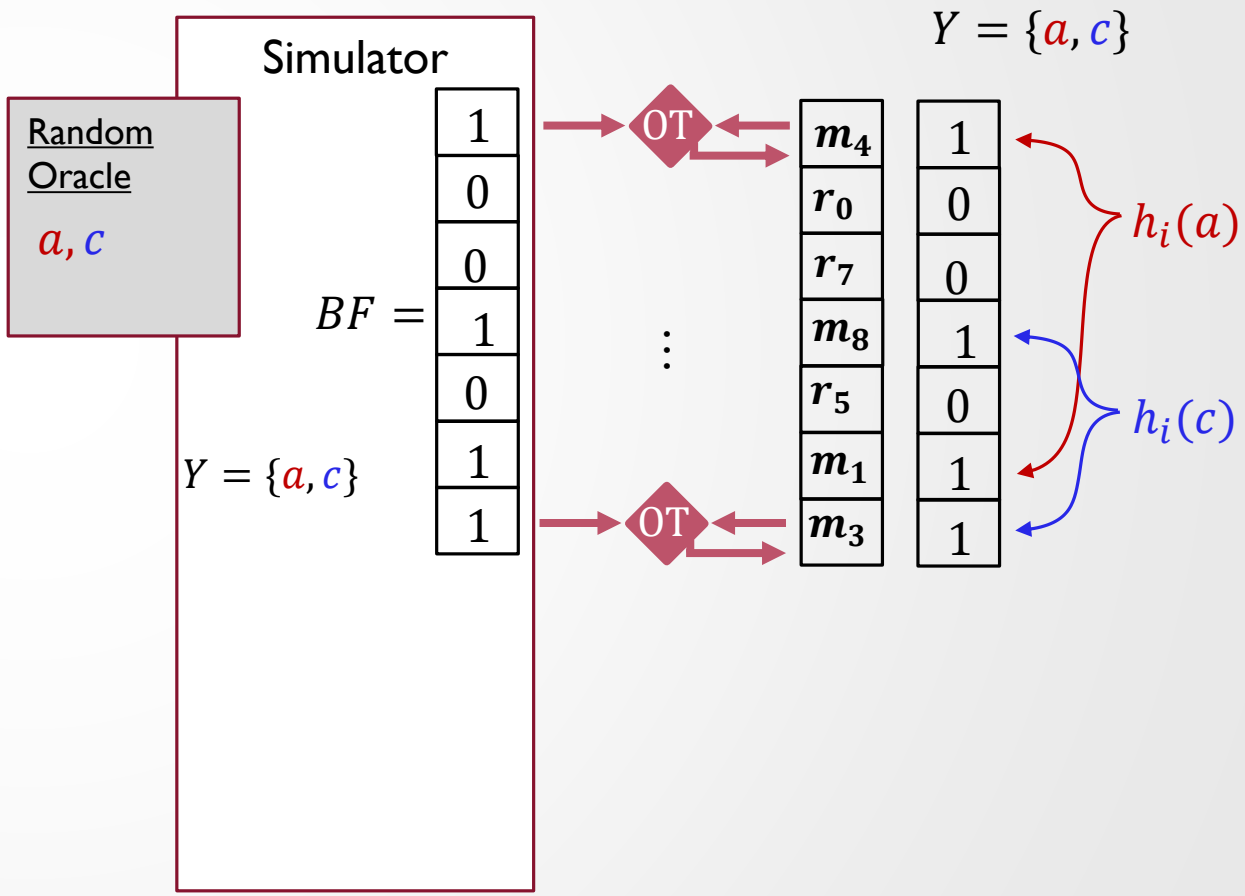

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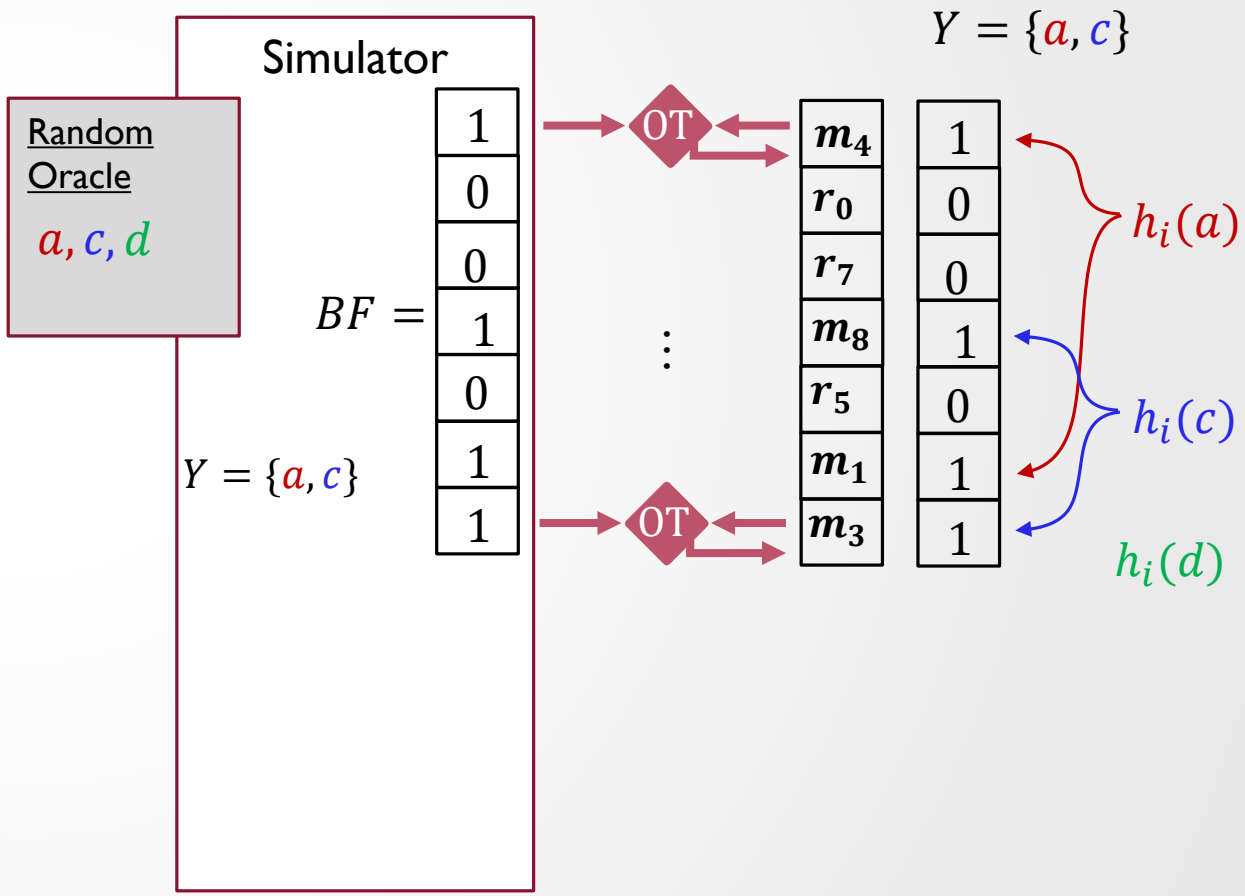

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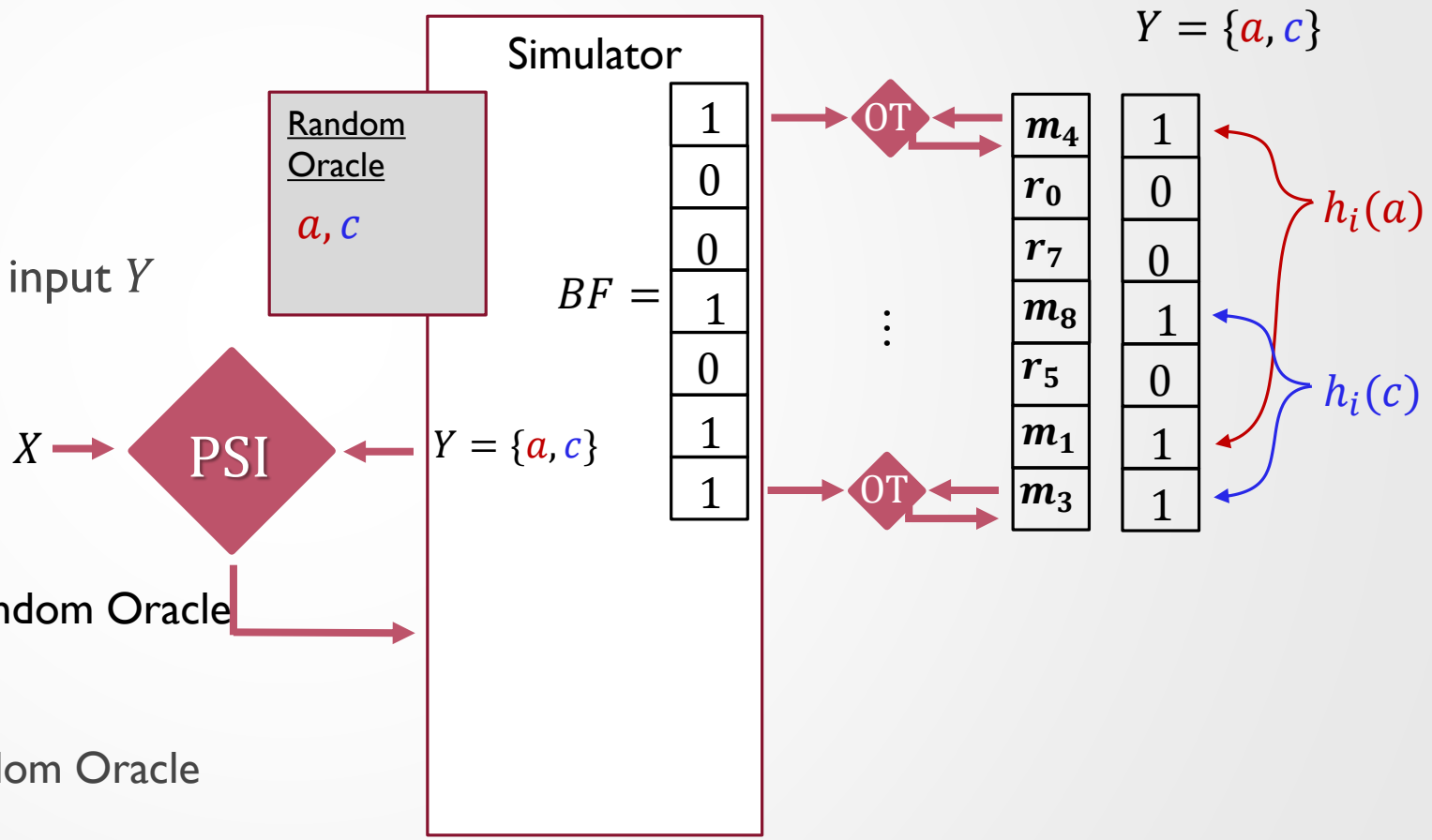

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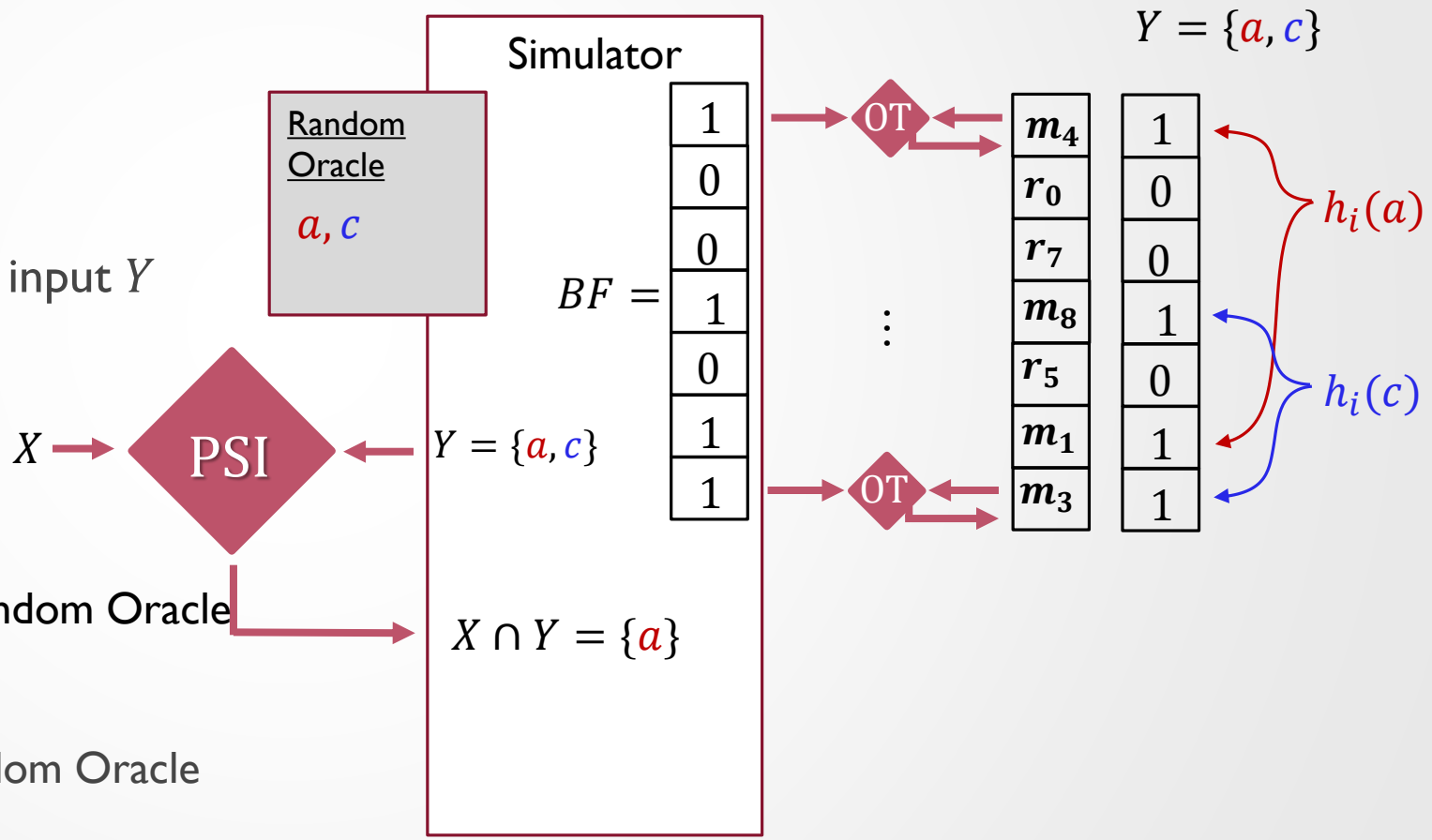

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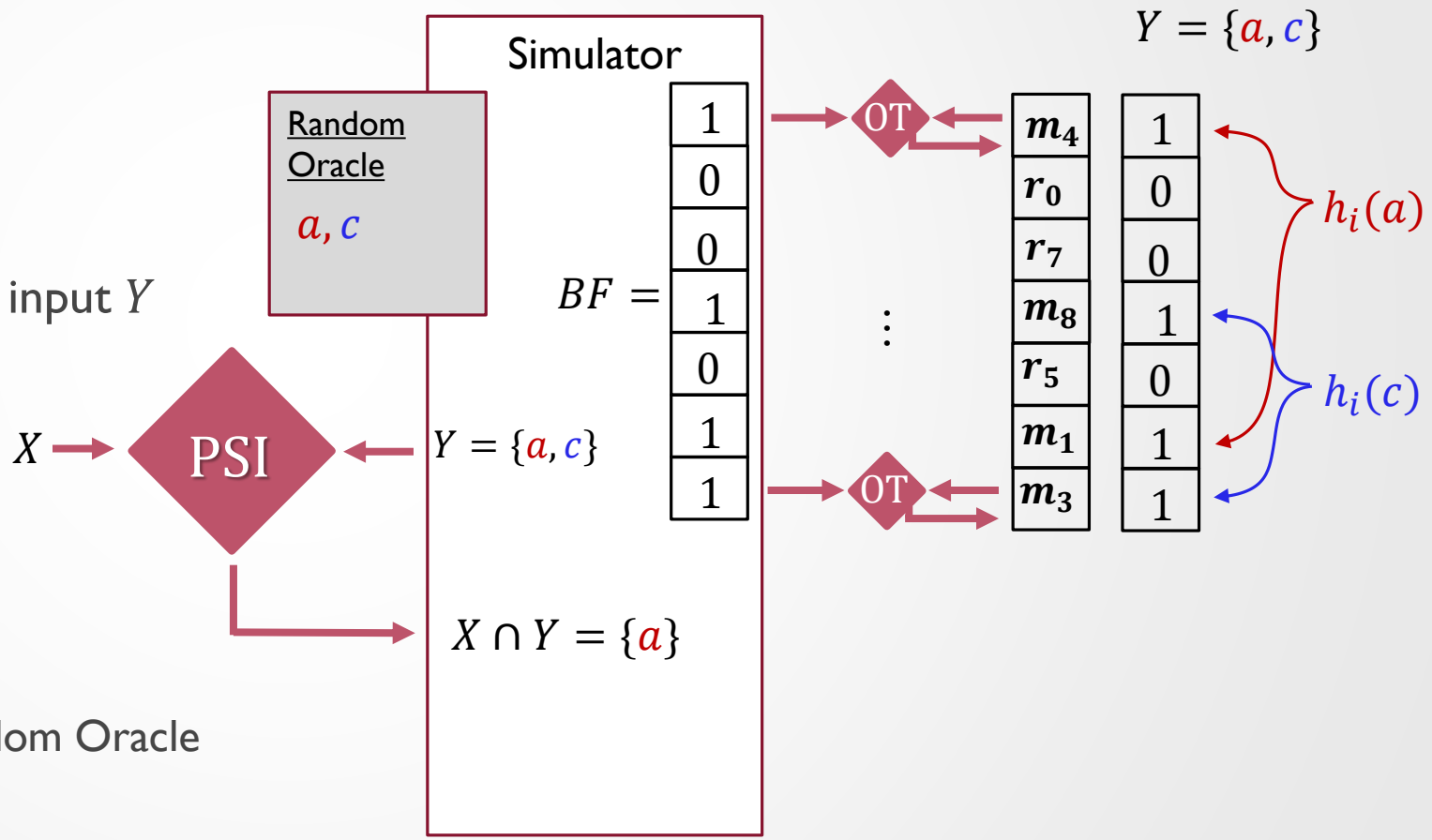

# Generalized Encodings

[RindalRosulek17]

Bloom filter of size $\sim 2n\kappa$ allows a Receiver to insert $n$ items

$$F(a) = \boldsymbol{m_4} \oplus \boldsymbol{m_1}$$
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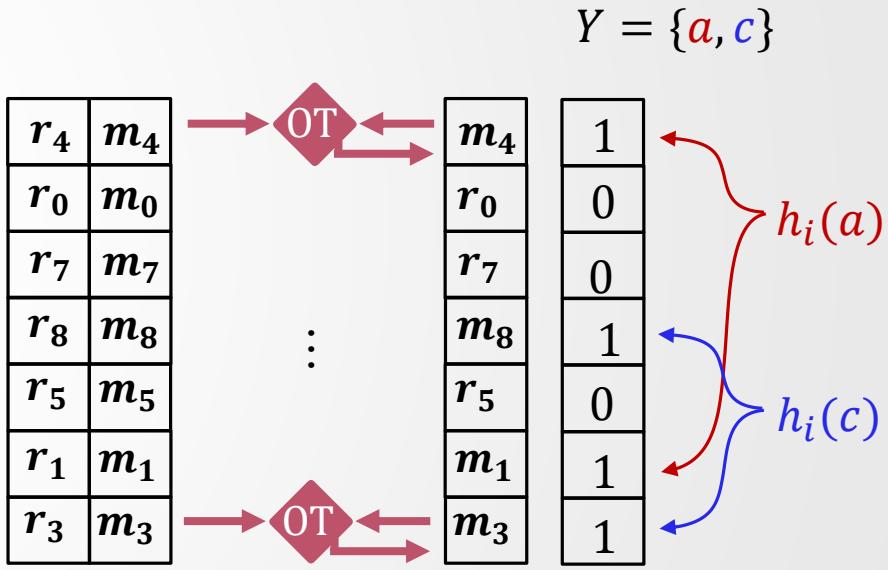
- $$F(\cdot)$$
- View Bloom filter protocol as an OPRF

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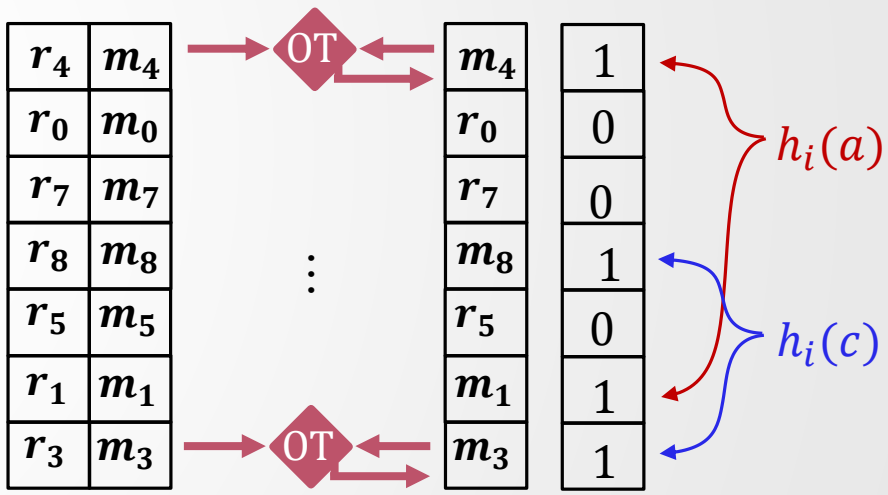


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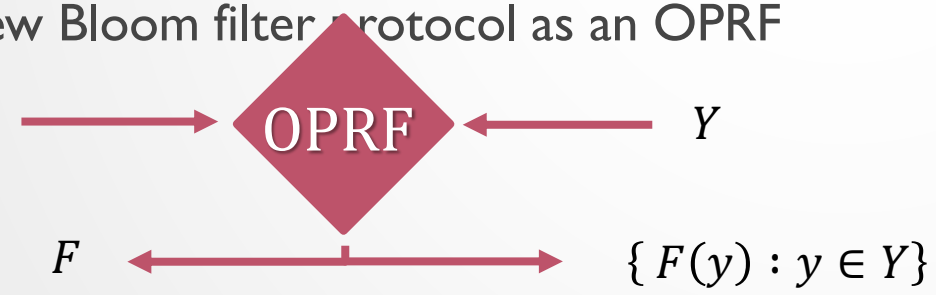


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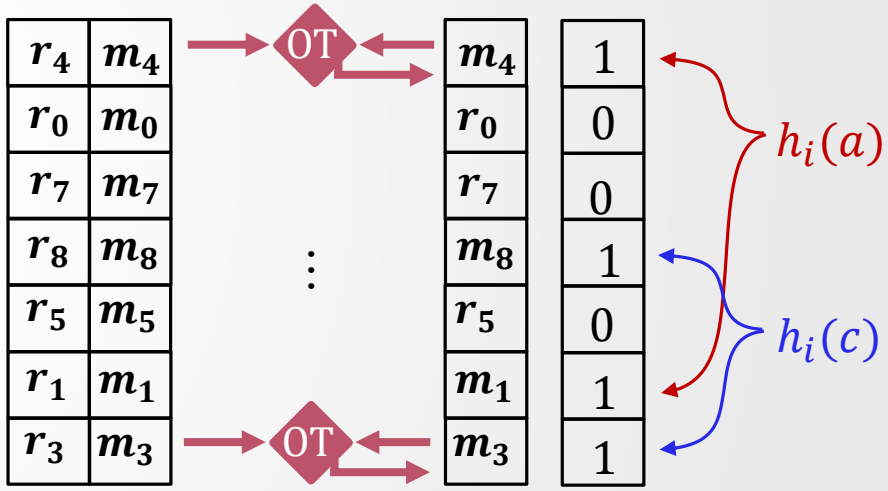


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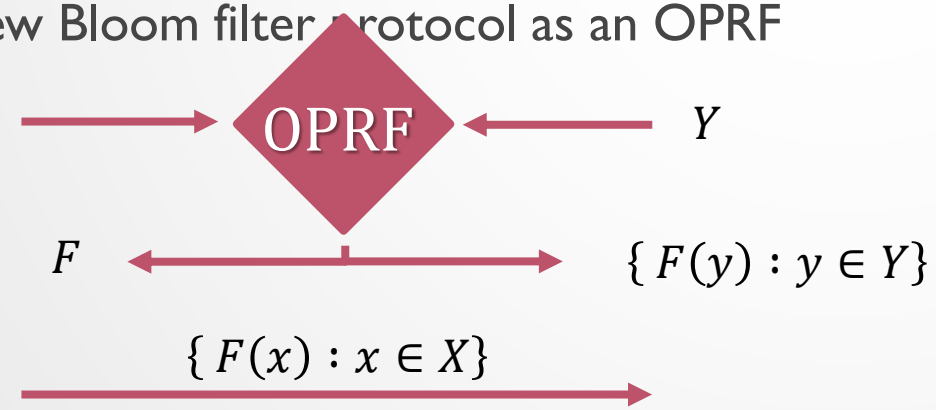


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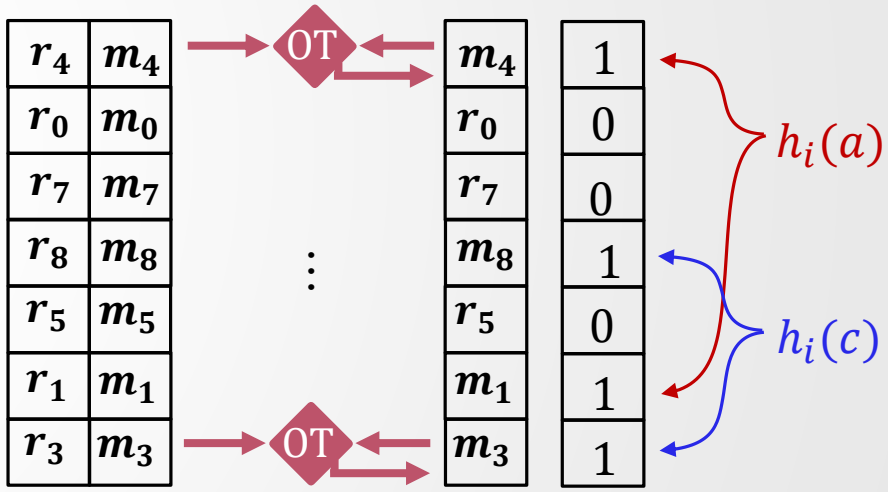


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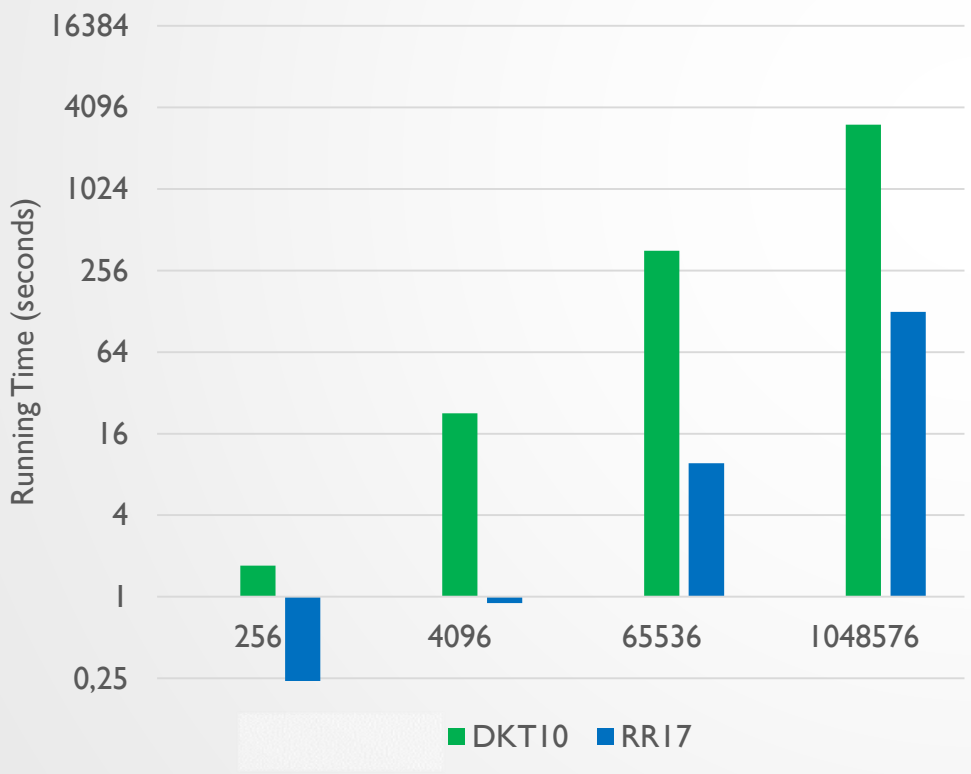
# Comparison – De Cristofaro, Kim, Tsudik10

- DKT10 - Malicious Diffie-Hellman style approach: $x^{\alpha\beta} = y^{\beta\alpha}$



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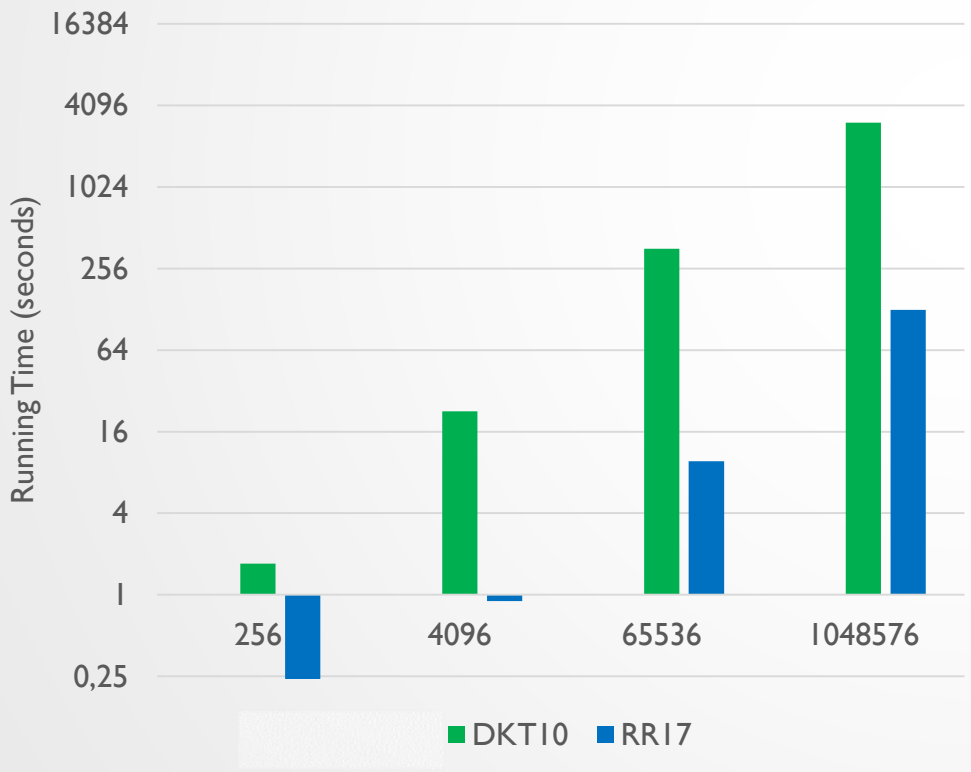


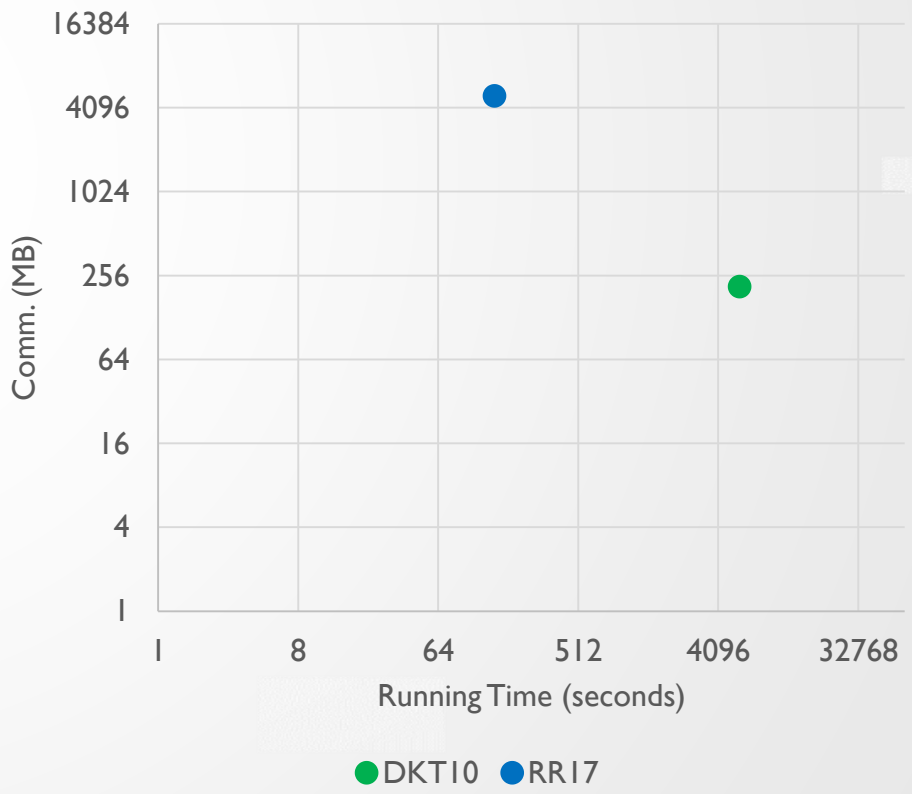

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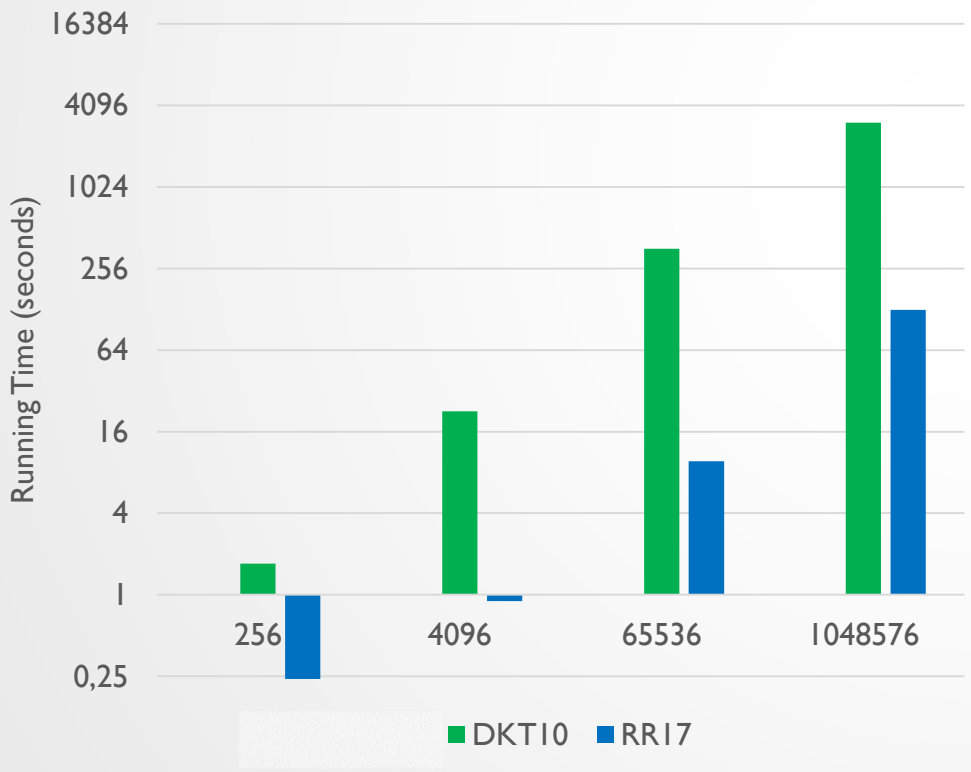


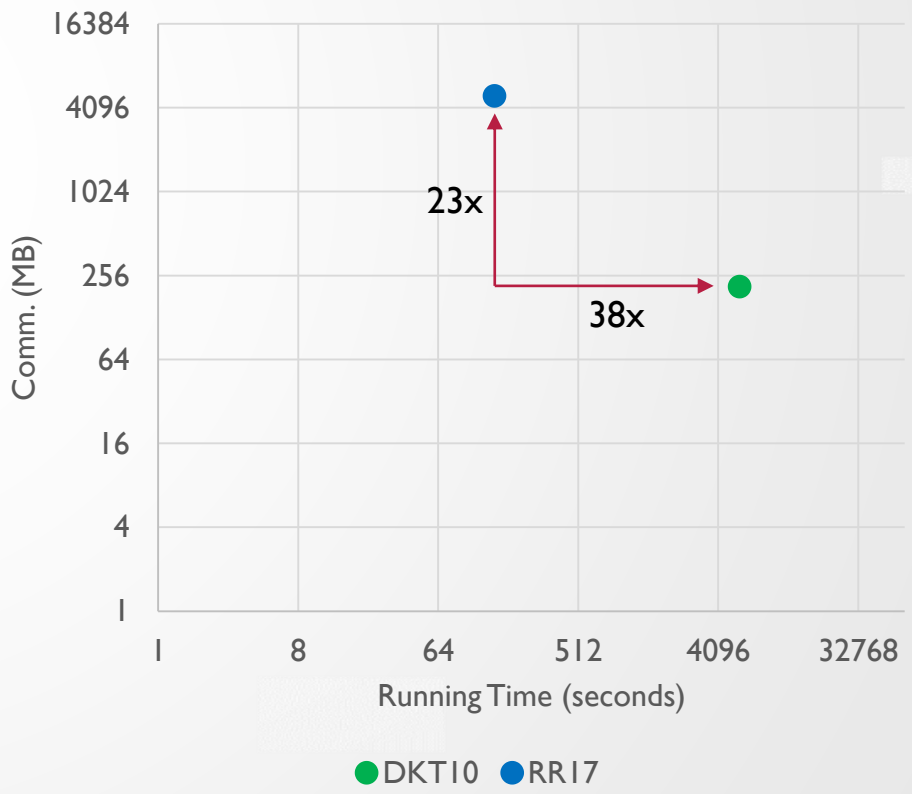

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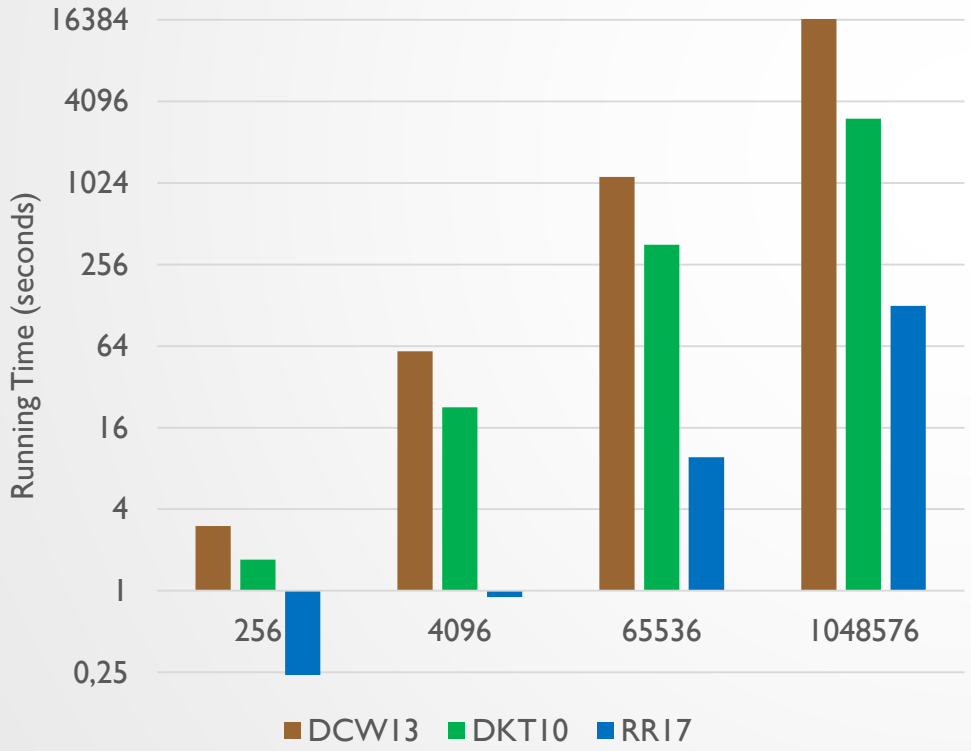


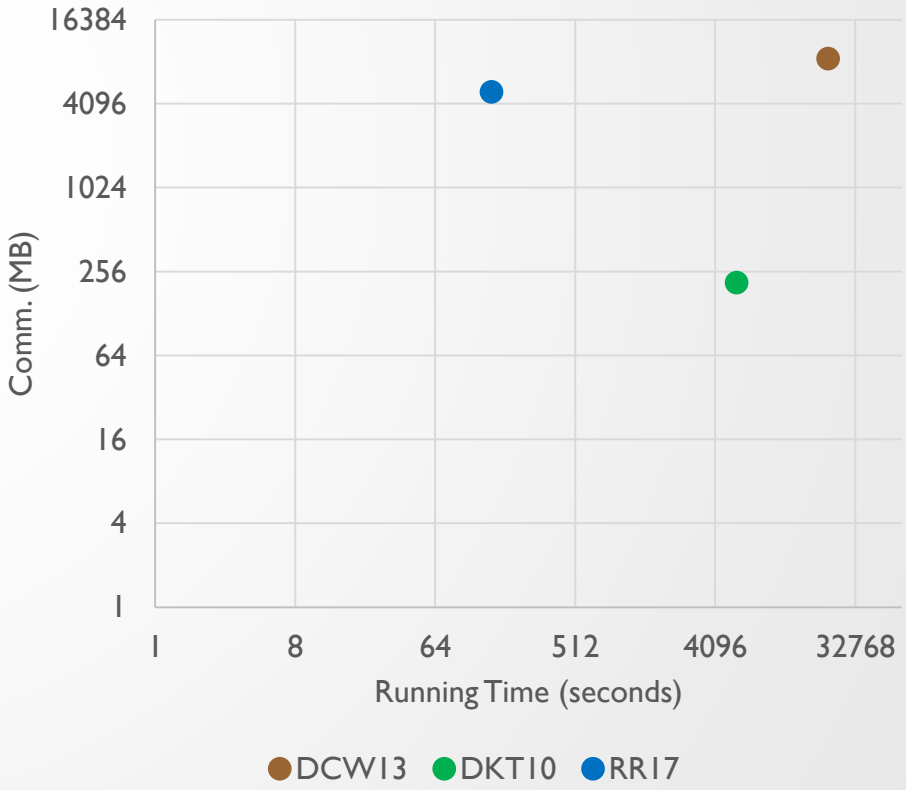

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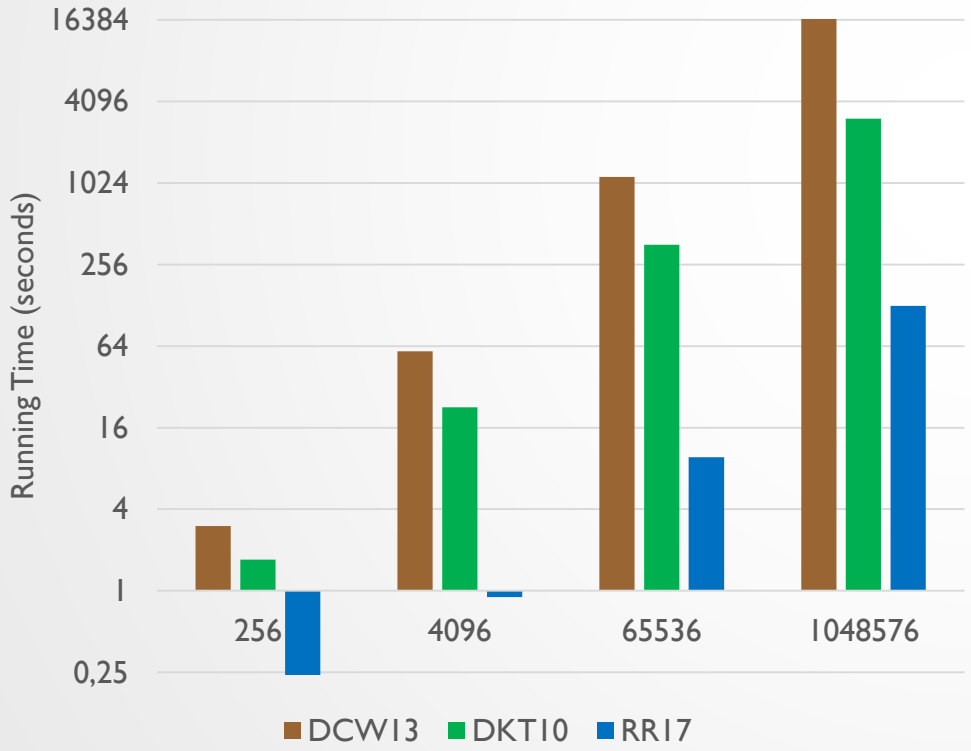


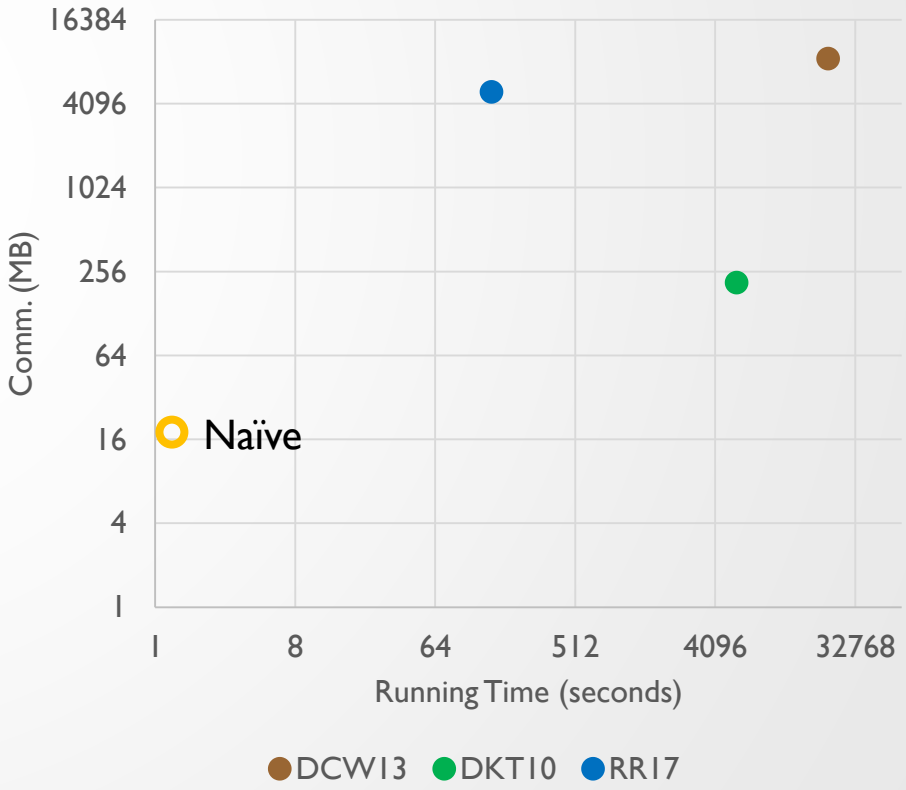

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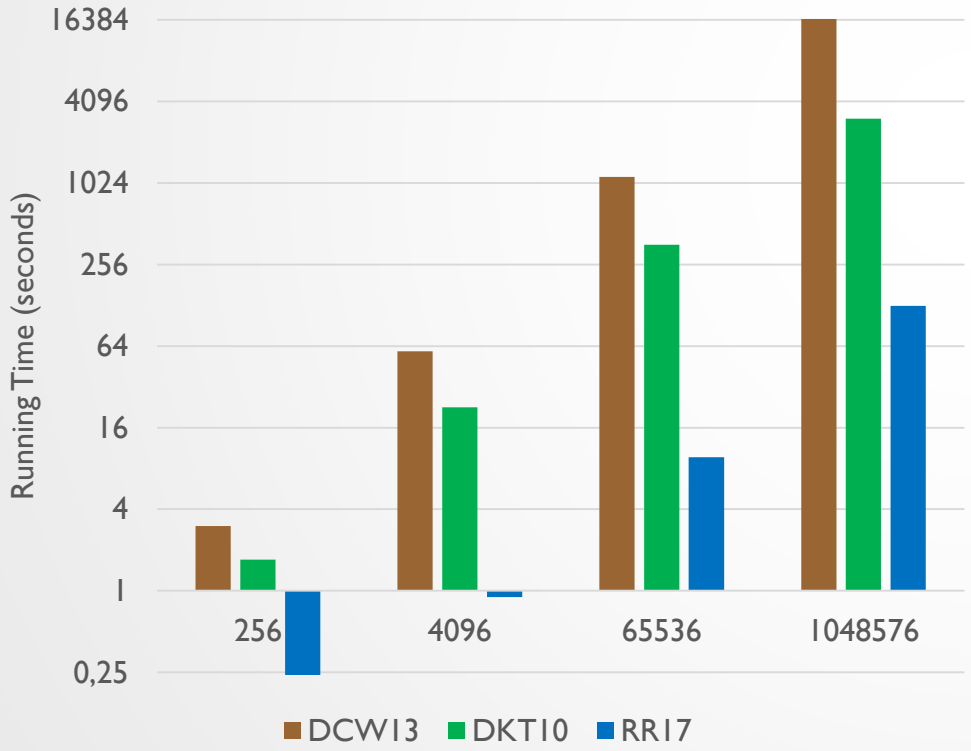
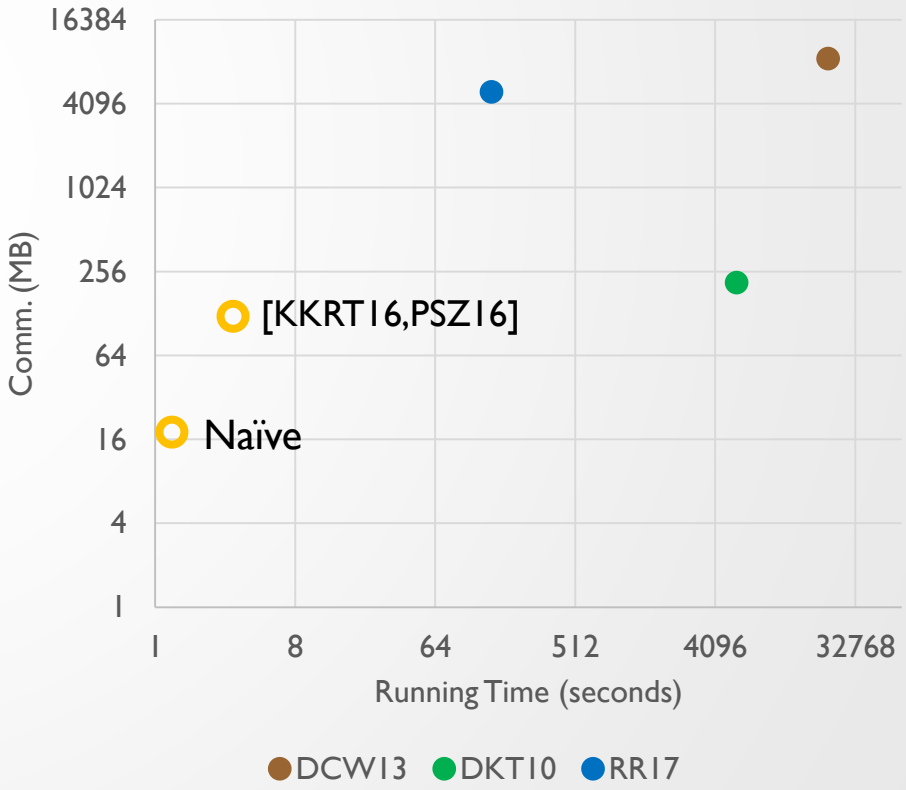
- DKT10 - Malicious Diffie-Hellman style approach: $x^{\alpha\beta} = y^{\beta\alpha}$



# *The End*

**Peter Rindal**
Mike Rosulek

Oregon State UNIVERSITY OSU