Analysis of the Blockchain Protocol in Asynchronous Networks

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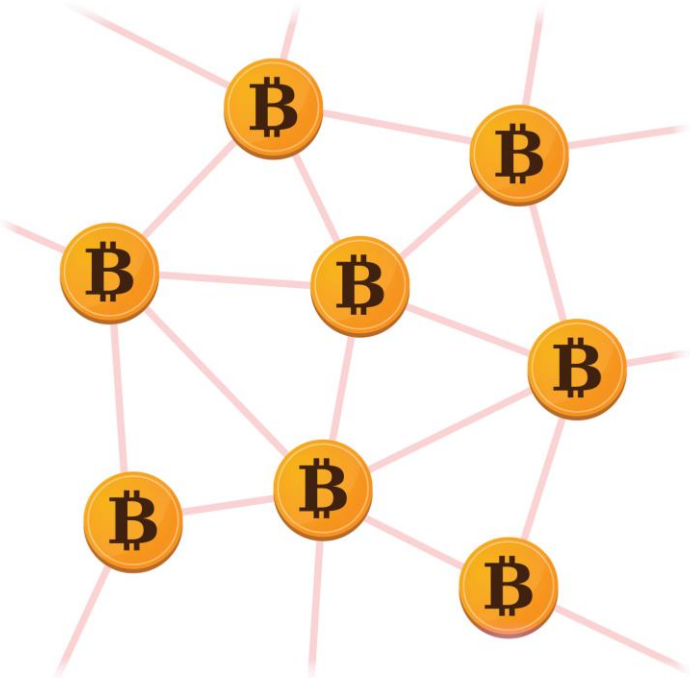
Traditional distributed systems:
The “Permissioned” Model

- **Consistency**
- **Liveness**

The “Permissionless” Model: Bitcoin/Blockchain

The Times 03/Jan/2009
Chancellor on brink of second bailout for banks.



The “Permissionless” Model

- Nodes do not know each other a-priori
- Nodes come and go
- ANYONE can join
- No network synchronization

The “Permissionless” Model

- Strong impossibility results known in the “permissionless” (“unauthenticated”) model [BCLPR05]
    - **Consistency** is impossible
    - Sybil attacks unavoidable.
        - [BCLPR05] defined “weakened” security model (w/o consistency)

Nakamoto's Blockchain [Nak'08]

Prevents Sybil attacks with Proofs-of-Work Puzzles [DN'92]

Claims blockchain achieves "public ledger" assuming "honest majority of **computing power**":

- **Consistency**: everyone sees the same history
- **Liveness**: everyone can add new transactions

Nakamoto’s Blockchain [Nak’08]

Prevents Sybil attacks with Proofs-of-Work Puzzles [DN’92]

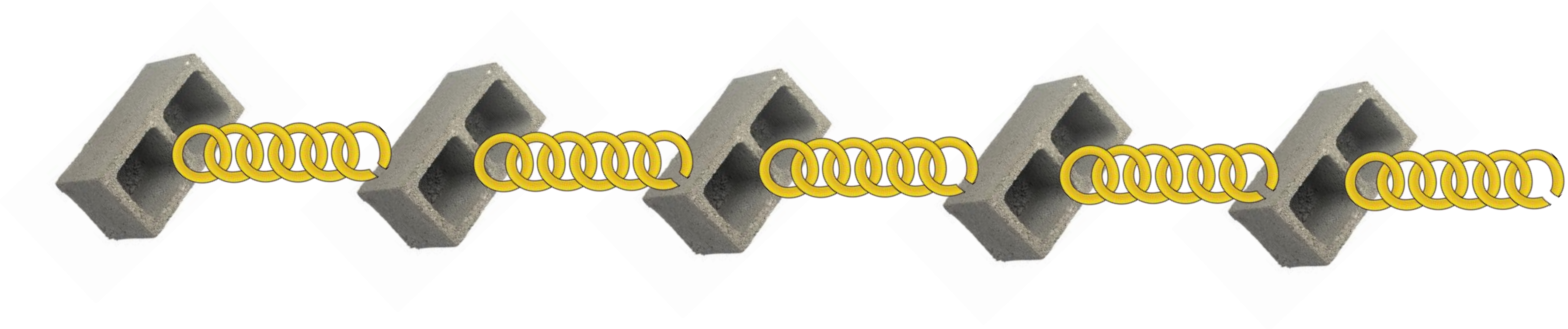
2 amazing aspects:

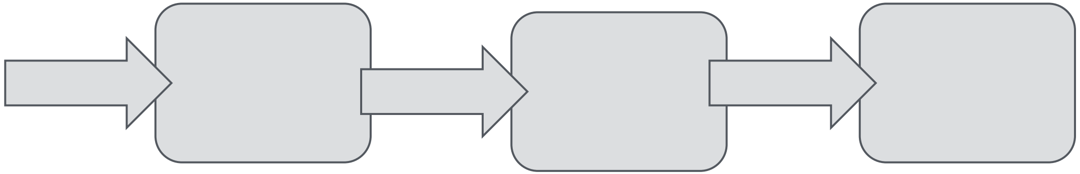
- Overcomes permissionless barrier [BCLPR’05]
- Overcomes ⅓ barrier even in permissioned setting [LSP’83]

- **WHAT IS** a blockchain?
  - no definition of an “abstract blockchain”

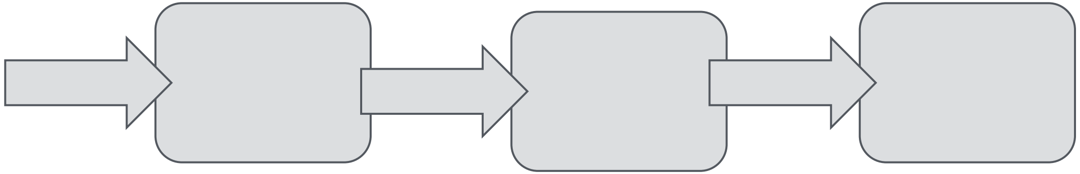
- Does Nakamoto’s protocol achieve **CONSISTENCY**?
  - “Specific attacks” don’t work [N’08,**GKL’15**, SZ’15]
  - 49.1% attack (with 10s network delays) claimed [DW’14]

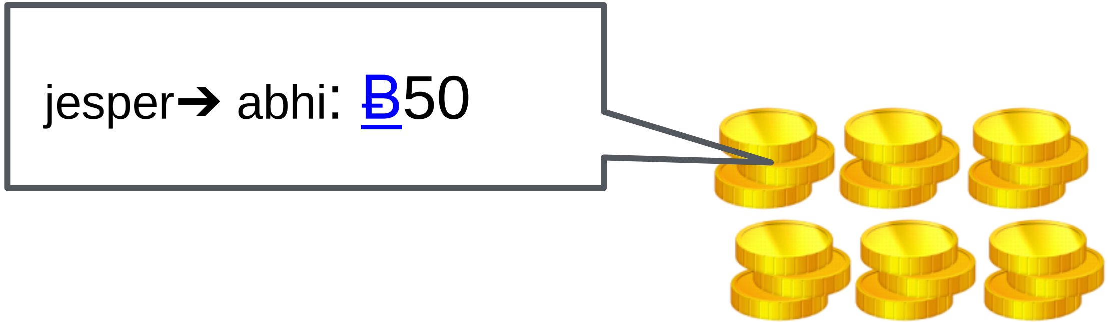
What is a blockchain?





How to build a “blockchain”





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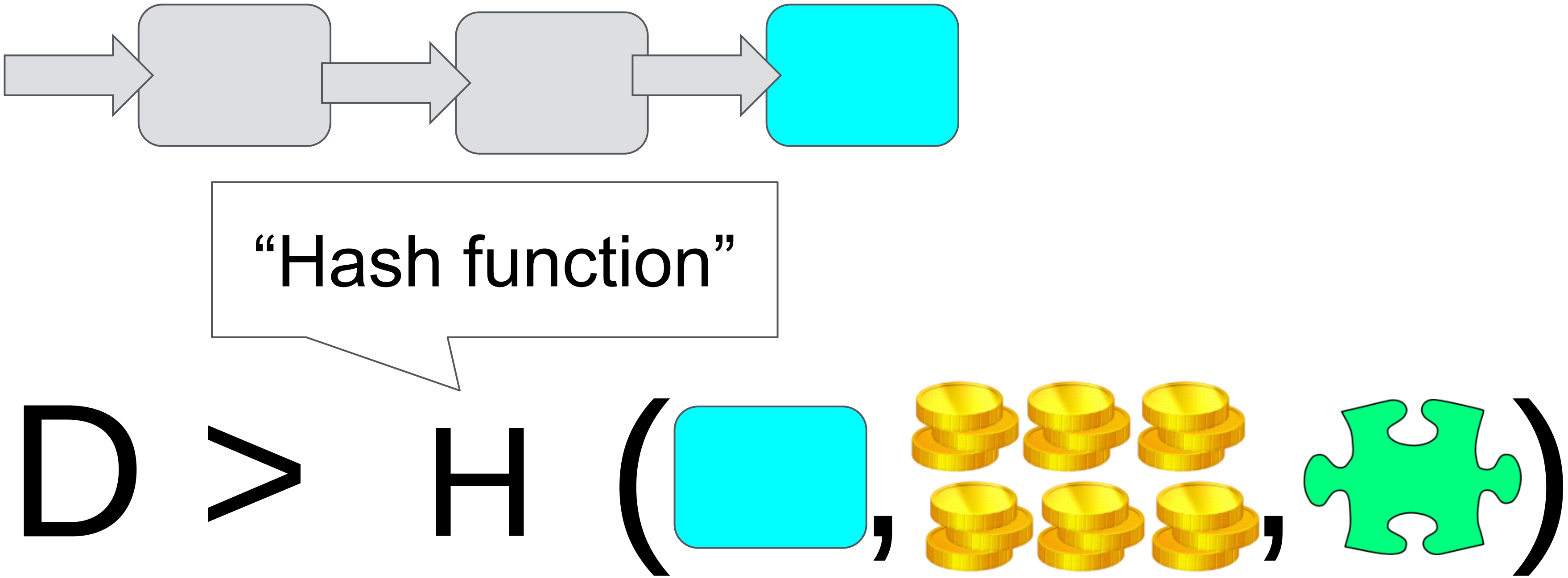

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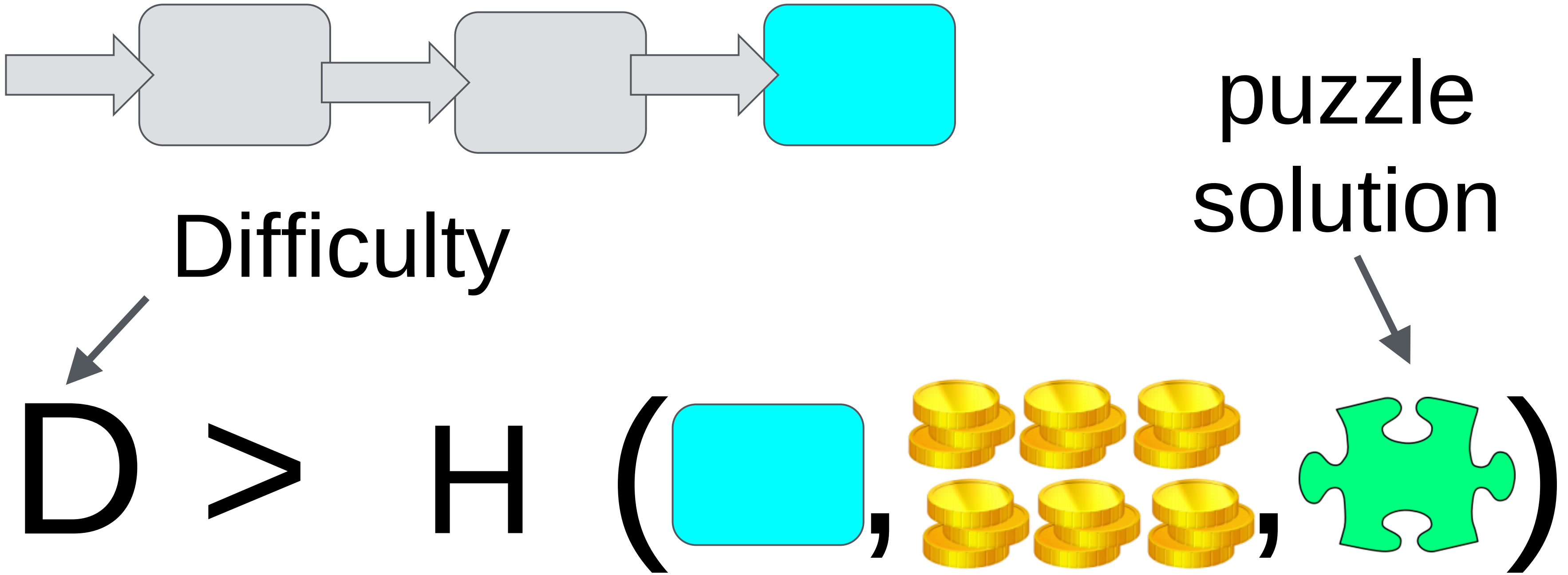
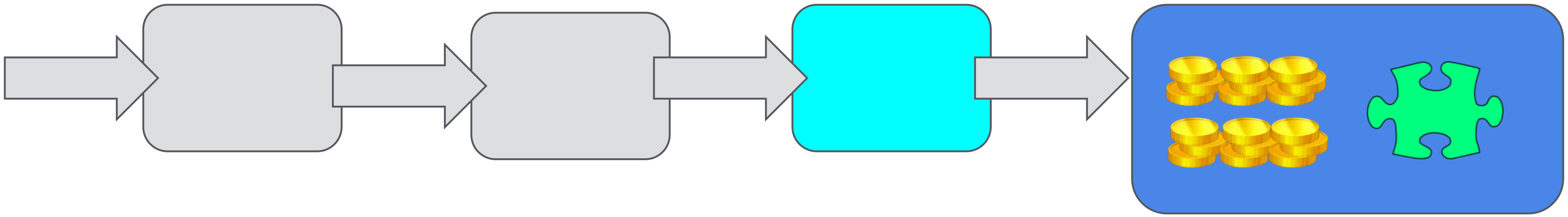
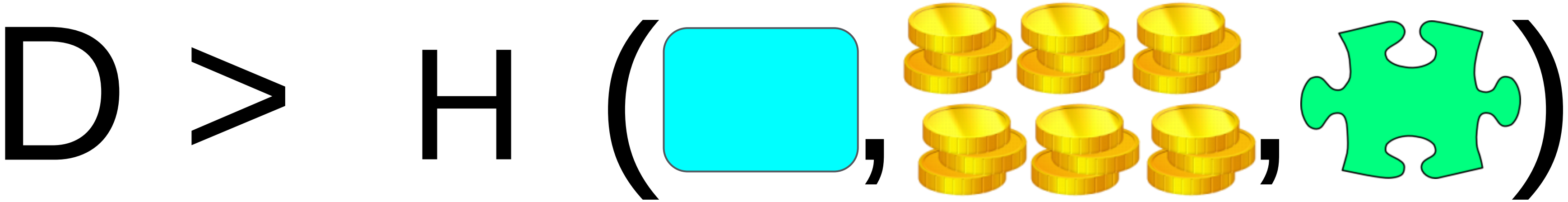
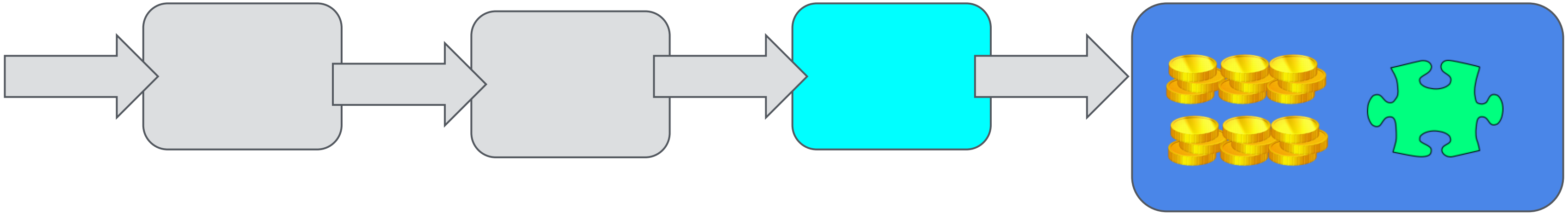


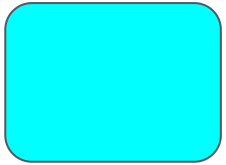
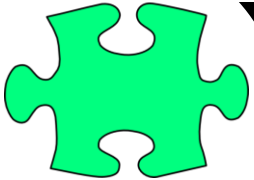
Search for a puzzle solution



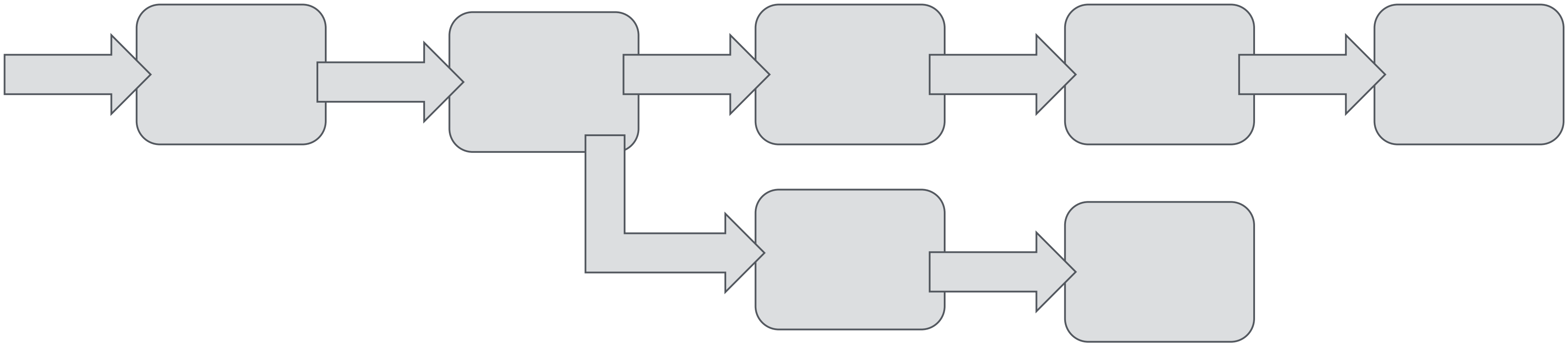


We found a new block



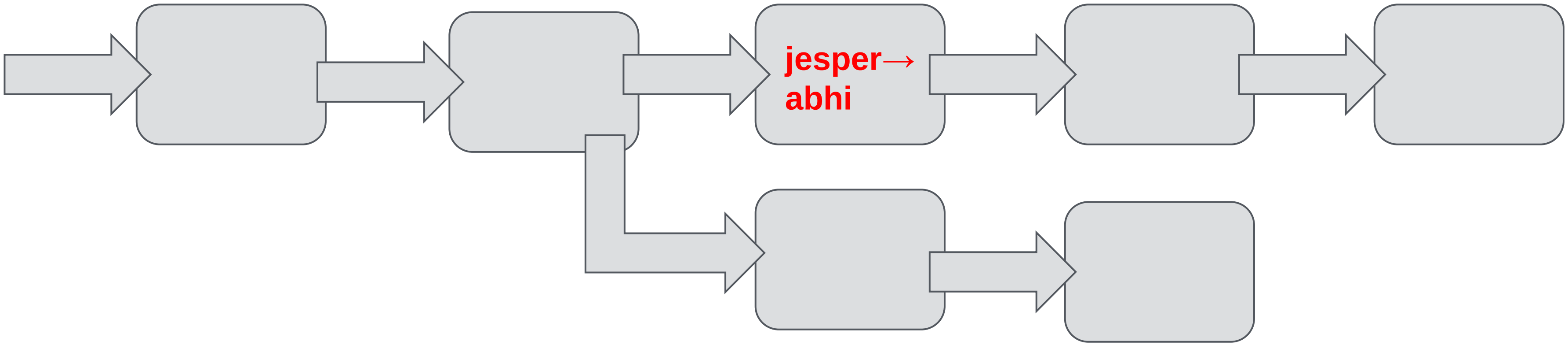
D > H (, , )

Best way to find a solution is brute-force search: model H as RO

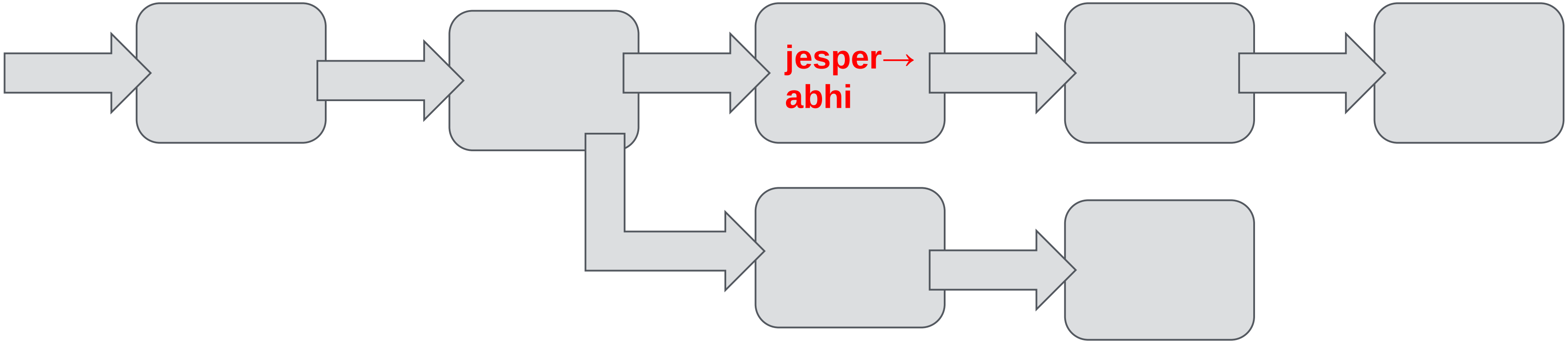


Honest nodes only “believe”

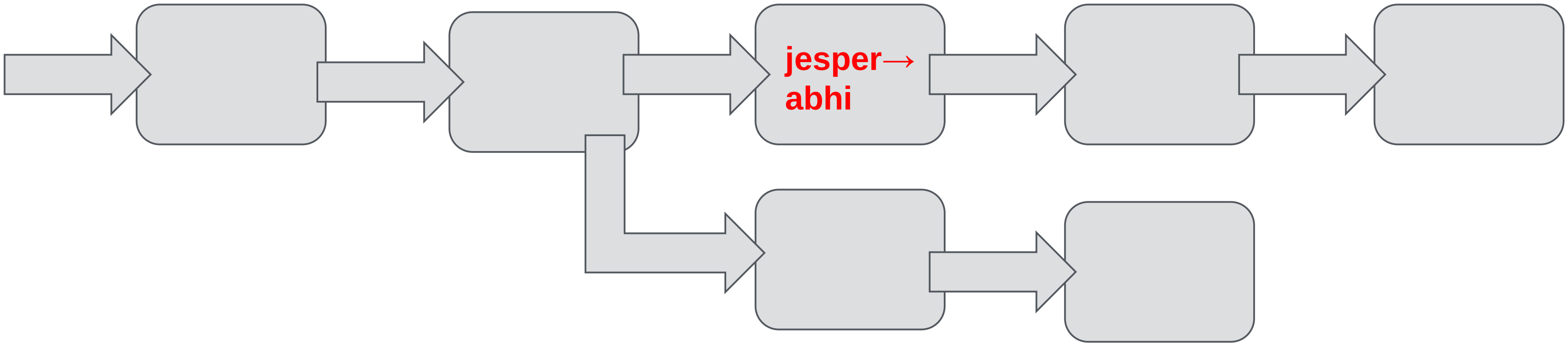
longest chain



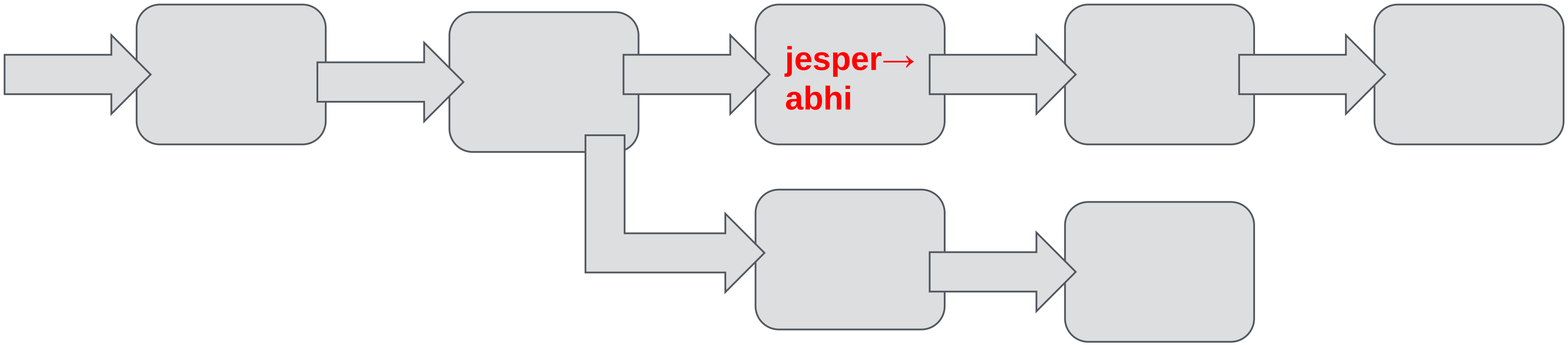
Jesper wants to erase this transaction



For Jesper to erase his transaction, he has to find a longer chain



"If transaction is sufficiently deep, he cannot do this unless he has majority hashpower"



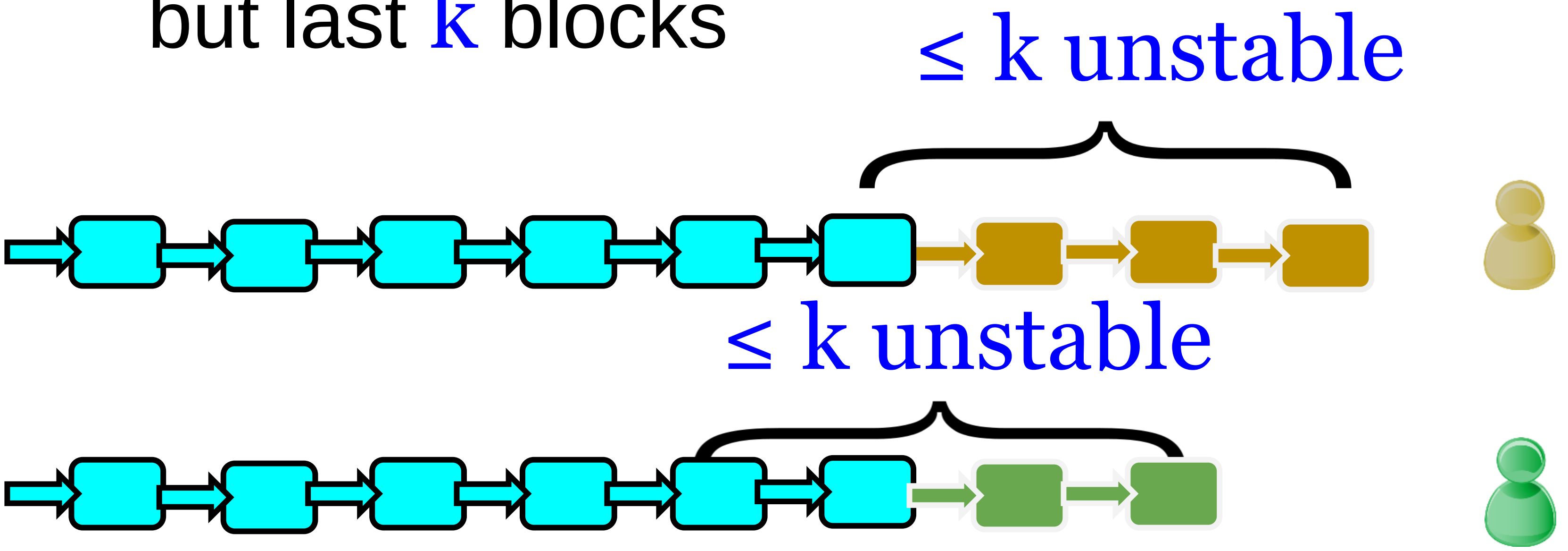
"If transaction is sufficiently deep, he cannot do this unless he has majority hashpower"

- [Nak'08]: "simply trying to mine alternative chain fails"
- [**GKL'15**]: in synchronous network
- [SZ'15]: "non-withholding attacks" fail also with Δ-delays

Blockchain abstraction (a la GKL,KL)

w/ prob exp(-k)

1 Consistency: Honest nodes agree on all but last k blocks



Blockchain abst… w/ prob $\exp(-k)$

Future-self consistency

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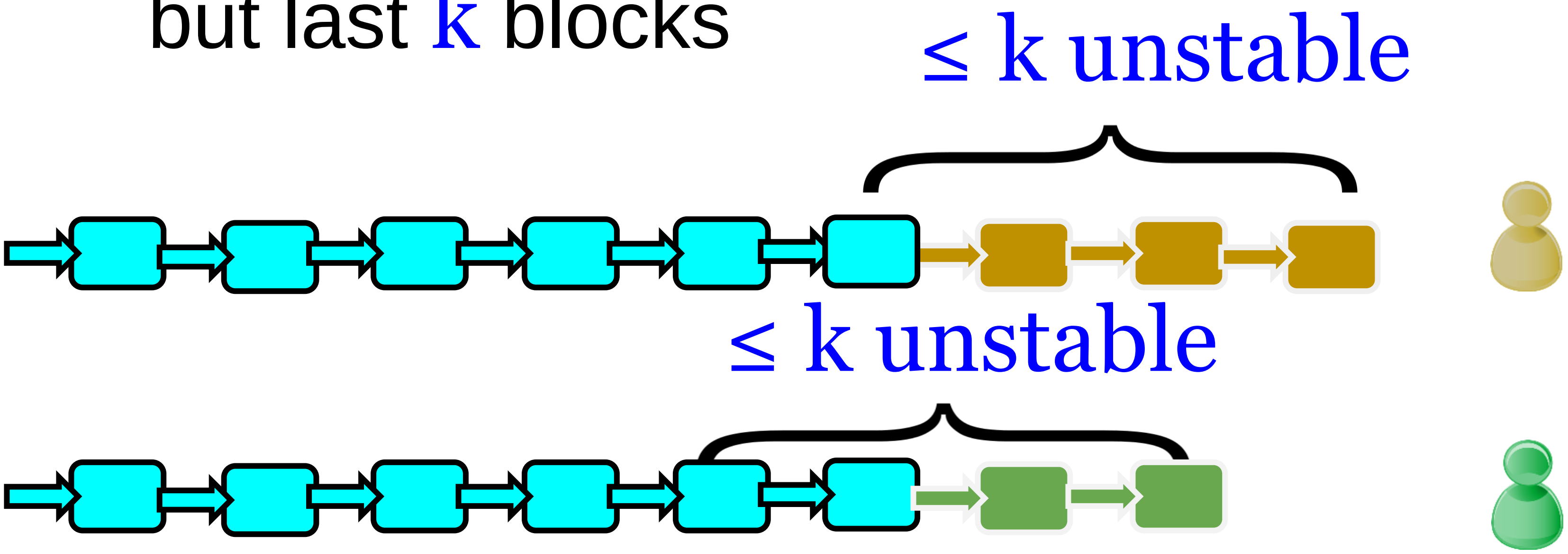

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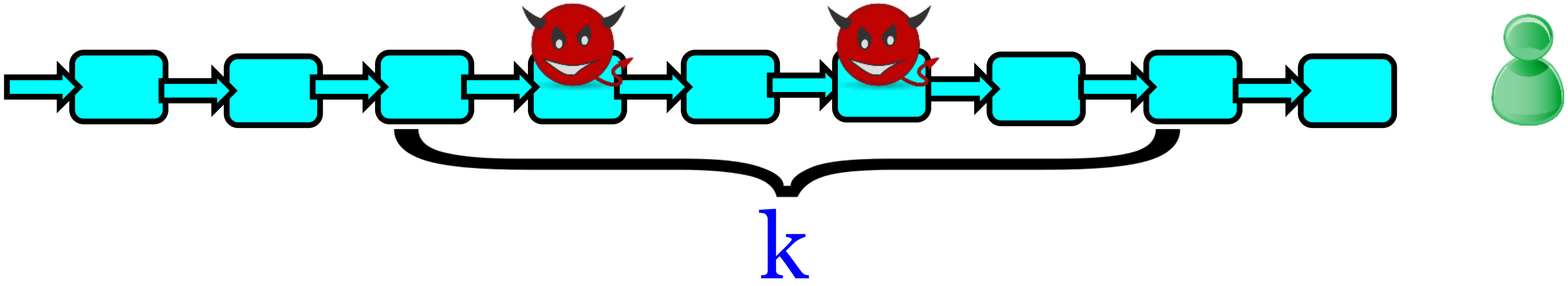

Blockchain abstraction

w/ prob exp(-k)

1. Consistency: Honest nodes agree on all but last k blocks
2. Chain quality: Any consecutive k blocks contain “sufficiently many” honest blocks
3. Chain growth: Chain grows at a steady rate

Blockchain implies “state machine replication” in the permissionless model

1. Consistency
2. Chain quality
3. Chain growth

Traditional

“state machine replication”

1. Consistency
2. Liveness

Theorem:

For every $\rho<1/2$, if "mining difficulty" is appropriately set (as a function of the network delay Δ, and total mining power), Nakamoto's blockchain guarantees:

- Consistency
- Chain quality: $1 - \rho/(1-\rho)$
- Chain growth: $O(1/\Delta)$

where ρ adv's fraction of hashpower, and **adv controls the network**

Theorem:

For every $\rho<1/3$, if "mining difficulty" is appropriately set (as a function of the network delay Δ, and total mining power), Nakamoto's blockchain guarantees:

- Consistency
- Chain quality: $1 - (1/3)/(2/3) = 1/2$
- Chain growth: $O(1/\Delta)$

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"Blocks are found SLOWER than Δ"

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"Blocktime" >> Δ

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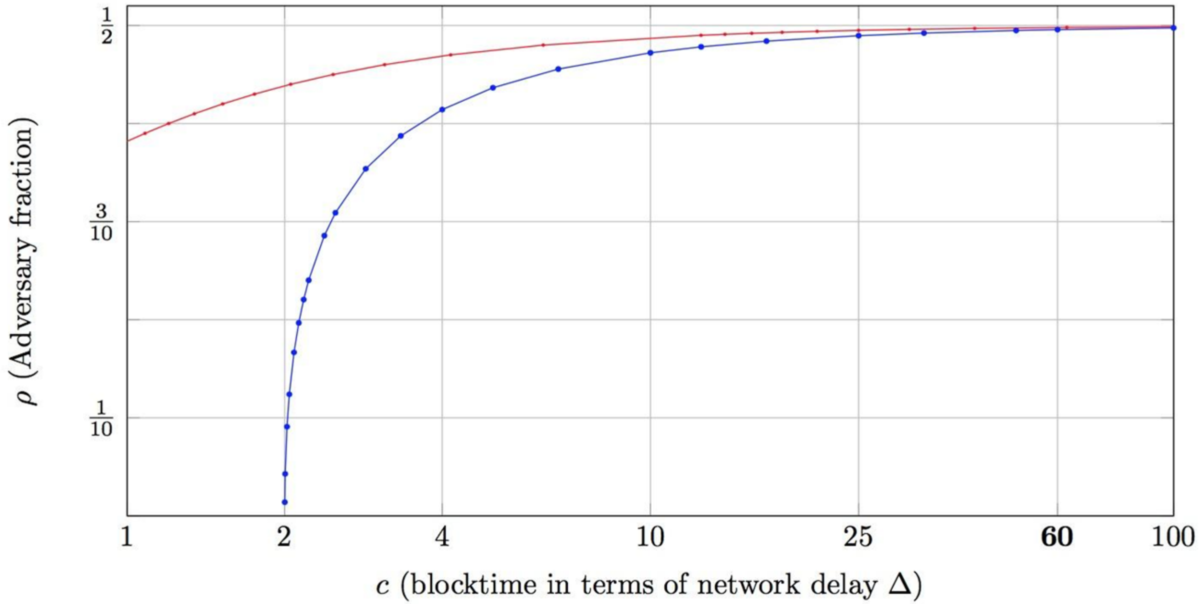
“Appropriately set”



When c = 60 (10 min blocktime, 10s network delays)

Secure: ρ < 49.57 (contradicts [DW’14]’attack!)

Attack: ρ > 49.79

“Appropriately set”

$$\alpha(1 - 2(\Delta + 1)\alpha) > \beta.$$

Mining rate of honest players

Network Delay

Mining rate of Adv

Theorem [Security of Nakamoto]

For every $\rho<1/2$, if mining difficulty is appropriately set (as a function of the network delay, and total mining power), Nakamoto's blockchain guarantees a) consistency, b) chain quality $1 - \rho/(1-\rho)$, and c) Chain growth: $O(1/\Delta)$

Theorem [Blatant attack]:

For every $\rho>0$, for every mining difficulty, there exists a network delay such that Nakamoto's blockchain is inconsistent and has 0 chain quality

Nakamoto's protocol achieves **strong robustness properties**:

- assuming "honest majority of computational power"
- assuming puzzle difficulty is appropriately set as a function of network delay Δ

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BUT 1: Blocktime need to be rougly 10 * Δ to handle ρ> 0.45 ; thus, **slow confirmation times**

BUT 2: **not fair, not incentive compatible!**

Follow-up Works

Incentive Compatibility: **The Fruit Chain** [PS'17]

All use our abstraction of a blockchain, as well as our analysis of Naka

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Fast confirmation:

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