Centrum Wiskunde & Informatica

Quantum Authentication and Encryption with Key Recycling

Or: How to Re-use a One-Time Pad Even if P = NP — Safely & Feasibly

Serge Fehr
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Louis Salvail
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Encryption & Authentication

- Schemes with **information theoretic security**
  - One-time pad: $E_k(m) = m + k$
  - Universal hashing, e.g.: $MAC_{A,b}(m) = Am + b$

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- Reason:
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  - Thus, key has to be refreshed **even if not under attack**

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- Would allow for:
  - unbounded safe re-use of the key
  - as long as not under attack

Known Results - and our Results

General idea goes back to

- [Bennett, Brassard & Breidbart 1982]:
  - proposed a simple scheme
  - gave hand-wavy arguments for its security

Quantum Cryptography II:
How to re-use a one-time pad safely even if P=NP

Charles H. Bennett (IBM Yorktown) *
Gilles Brassard (Univ. de Montreal) †
Seth Breidbart (Box 1526, NY 10268)

November 1982

Abstract

When elementary quantum systems, such as polarized photons, are used to transmit digital information, the uncertainty principle gives rise to novel cryptographic phenomena unachievable with traditional transmission media, e.g. a communications channel on which it is impossible in principle to eavesdrop without a high probability of being detected. With such a channel, a one-time pad can safely be reused many times as long as no eavesdrop is detected, and, planning ahead, part of the capacity of these uncompromised transmissions can be used to send fresh random bits with which to replace the one-time pad when an eavesdrop finally is detected. Unlike other schemes for

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Related line of work:

encryption/authentication of **quantum messages**

Some also offer key recycling and/or other features (see e.g. Portmann's talk)

But, in all of those: honest users need quantum computer (even when restricting to classical messages)

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- Our main motivation: intellectual interest

Road Map

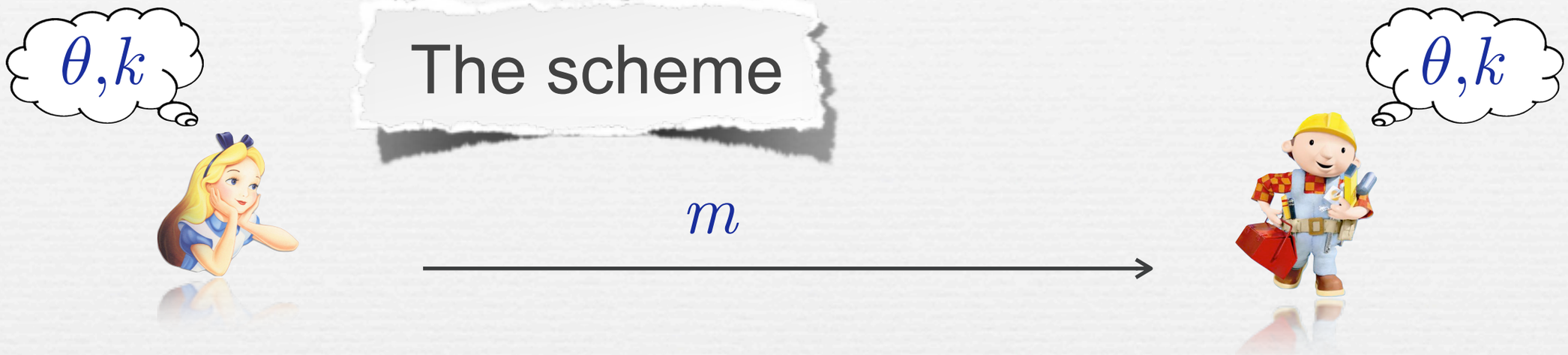
- Introduction
- The basic scheme and its analysis
- Extensions and open problem(s)

Authentication with Key-Recycling

The scheme

m

Authentication with Key-Recycling



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$x \leftarrow \{0,1\}^n$

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θ	+	×	×	+	×	+	⋯	×	×	+
x	1	1	0	0	1	0	⋯	1	0	1
$H^\theta \|x\rangle$	↕	↘	↙	↔	↘	↔	⋯	↘	↙	↕

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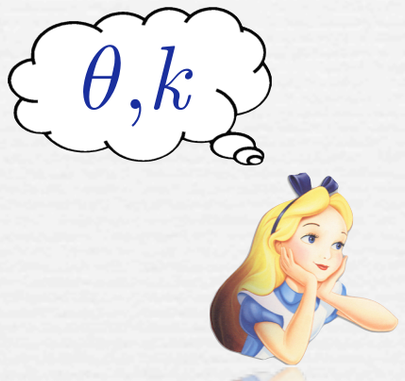

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Claims (informal)

- Offers authentication security

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- If Bob rejects then θ (only) must be refreshed

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If Eve gets to see authentication tags

$$t_i = MAC_k(m_i) = Am_i + b$$

for **known** messages $m_1, m_2, \ldots$ and a fixed key $k = (A,b)$,
and so accumulates (linear) info on k and can solve for it.

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But **here**: authenticated message $m\|x$ is partly **unknown**, since $H^\theta|x\rangle$ **hides** x (to some extent) when θ is unknown.

An “Attack”


θ, k
$x \leftarrow \{0,1\}^n$
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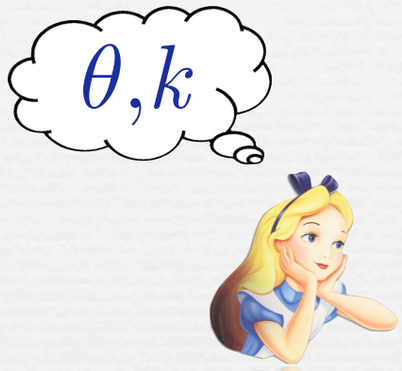

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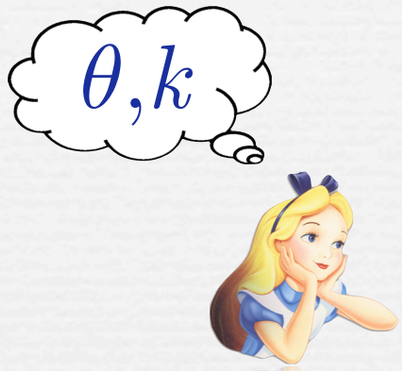

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If $\theta_1 = 0$ then

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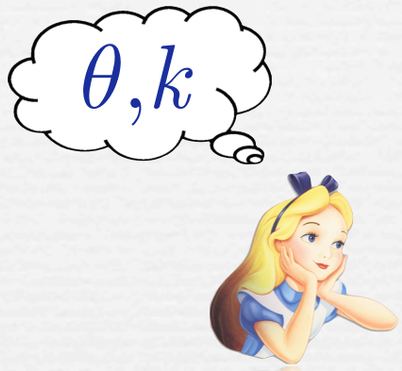

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No need to worry:

The more info she tries to learn the more likely she fails

Insight Gained

- **Cannot** expect to prove:

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- **Cannot** expect to prove:

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- But then:
  - may not be necessary for the key to stay random
  - high uncertainty might be sufficient

Formal Statement - Informally Stated

Theorem. If before the execution:

$$\text{Guess}(\theta | \textit{Eve's view}) \approx 0 \quad \& \quad \delta(k, \textit{unif} | \theta, \textit{Eve's view}) \approx 0$$

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Thus: starting off with a, say, uniformly random key (θ,k) , the (possibly refreshed) key can be re-used over and over.

Proof - The Easy Part

Note: Eve's view E' after the execution consists of

- her old view E
- the tag $t = A\left[\frac{m}{x}\right] + b$
- whatever she kept of $H^\theta|x\rangle$, call it Q
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θ' = θ

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Proving $\delta(k, unif\,|\theta', E') \approx 0$ is more involved. Builds up on techniques from [Tomamichel,Fehr,Kaniewski,Wehner '13].

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- Tolerate **noise** in the quantum communication
  - Straightforward error-correction does not work
  - Error-correction "*without leaking partial info*" by Dodis and Smith comes to the rescue

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$$H^\theta|x\rangle$$

*We still **expect** this to be small, but cannot **prove** it*

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